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Robust H_∞ Filtering for LPV Discrete-Time State-Delayed Systems

Junling Wang, Changhong Wang, Huijun Gao

Space Control and Inertial Technology Research Centre
Harbin Institute of Technology, Harbin 150001, P.R.China

Abstract—This paper examines the problems of robust H_∞ filtering design for linear parameter-varying discrete-time systems with time-varying state delay. We present new H_∞ performance criteria that depend on the parameters and the delay-varying magnitude using appropriately selected Lyapunov-Krasovskii functional. Then the corresponding filter can be obtained from the solution of convex optimization problems in terms of parameterized linear matrix inequalities, which can be solved via efficient interior-point algorithms. And the admissible filter guarantees a prescribed H_∞ noise attenuation level, relating exogenous signals to the estimation error for all possible parameters that vary in a compact set. A numerical example illustrates the feasibility of the proposed methodologies.

I. INTRODUCTION

Linear parameter-varying (LPV) systems have received considerable attention recently [1]-[8]. LPV systems are linear systems that depend on time-varying parameters, whose values are not known *a priori*, but can be measured in real time. In contrast to continuous-time cases, LPV discrete-time systems [6]-[8] received relatively less attention despite their importance in digital control and signal processing applications.

Since time delay often appears in many control systems either in the state, the control input, or the measurements, and is, in many cases, a source of instability, the stability issue and the performance of LPV systems with delay are of theoretical and practical importance. Many results of the stability analysis for LPV time-delayed systems both on continuous-time cases [4]-[5] and discrete-time case [7] have been obtained. However, it is worth noting that the filters design for LPV time-delay systems [8] is still very limited, especially for LPV discrete time-delayed systems.

In this paper, we investigate the H_∞ filtering problem for LPV discrete-time systems that include time-varying state delay based on the references [8]-[10]. Using parameter-dependent Lyapunov-Krasovskii functional, we obtain a new H_∞ performance criterion that depends on the parameter and the delay-varying magnitude. Then we further modify the obtained criterion by adopting the idea [11] of decoupling between the positive matrices and the system matrices by the introduction of addition slack variable to obtain another parameterized linear matrix inequalities (PLMIs) representation. And the corresponding filter design problems are finally cast into convex optimization problems. The obtained filter design procedure is shown, via a numerical example, to be effective.

The notation used throughout the paper is fairly standard. The superscript "T" stands for matrix transposition, R^n denotes the n dimensional Euclidean space, $R^{m\times n}$ is the set of all $m\times n$ real matrices, and the notation $P>0$ for $P\in R^{n\times n}$ means that P is symmetric and positive definite. In addition, in symmetric block matrices or long matrix expressions, we use * as an ellipsis for the terms that are introduced by symmetry and $diag\{\cdots\}$ stands for a block-diagonal matrix.

II. PROBLEM FORMULATION

Consider the following LPV discrete time-delayed system presented in state-space form by:

$$\begin{aligned} x(k+1) &= A(\rho(k))x(k)+A_d(\rho(k))x(k-d(k))+B(\rho(k))\omega(k) \\ y(k) &= C(\rho(k))x(k)+C_d(\rho(k))x(k-d(k))+D(\rho(k))\omega(k) \\ z(k) &= H(\rho(k))x(k)+H_d(\rho(k))x(k-d(k))+L(\rho(k))\omega(k) \end{aligned} \tag{1}$$

where $x(k)\in R^n$ is the state; $y(k)\in R^m$ is the measured output; $z(k)\in R^p$ is the signal to be estimated; $\omega(k)\in R^l$ is the noise input; $\rho(k)=(\rho_1(k),\ldots,\rho_S(k))$ is a vector of time-varying parameters which belong to a compact set $\Im\in R^S$; $d(k)>0$ is time-varying delay. It is assumed that there exist two positive constants d_m and d_M such that the following inequality holds

$$d_m\le d(k)\le d_M,\quad \forall k\ge 0 \tag{2}$$

And the system matrices $A(\cdot)$, $A_d(\cdot)$, $B(\cdot)$, $C(\cdot)$, $C_d(\cdot)$, $D(\cdot)$, $H(\cdot)$, $H_d(\cdot)$, $L(\cdot)$ are known functions of $\rho(\cdot)$. For simplicity, ρ_k denotes the time-varying parameter vector $\rho(k)$ throughout the paper.

Here we are interested in designing an estimator or full-order filter described by:

$$\begin{aligned} x_F(k+1) &= A_F(\rho_k)x_F(k)+B_F(\rho_k)y(k),\ x_F(0)=0 \\ z_F(k) &= C_F(\rho_k)x_F(k)+D_F(\rho_k)y(k) \end{aligned} \tag{3}$$

Augmenting the model of (1) to include the states of the filter, we obtain the filtering error system as follows:

$$\begin{aligned} \xi(k+1) &= \bar{A}(\rho_k)\xi(k)+\bar{A}_d(\rho_k)K\xi(k-d(k))+\bar{B}(\rho_k)\omega(k) \\ e(k) &= \bar{C}(\rho_k)\xi(k)+\bar{C}_d(\rho_k)K\xi(k-d(k))+\bar{D}(\rho_k)\omega(k) \end{aligned} \tag{4}$$

where

$$\xi(k)=\{x^T(k),x_F^T(k)\}^T,\quad e(k)=z(k)-z_F(k)$$

$$
\begin{aligned}
&\bar{A}(\rho_k) = \begin{bmatrix} A(\rho_k) & 0 \\ B_F(\rho_k)C(\rho_k) & A_F(\rho_k) \end{bmatrix}, \\
&\bar{A}_d(\rho_k) = \begin{bmatrix} A_d(\rho_k) \\ B_F(\rho_k)C_d(\rho_k) \end{bmatrix}, \quad \bar{B}(\rho_k) = \begin{bmatrix} B(\rho_k) \\ B_F(\rho_k)D(\rho_k) \end{bmatrix}, \\
&\bar{C}_d(\rho_k) = H_d(\rho_k) - D_F(\rho_k)C_d(\rho_k), \\
&\bar{D}(\rho_k) = L(\rho_k) - D_F(\rho_k)D(\rho_k), \quad K = \begin{bmatrix} I & 0 \end{bmatrix}
\end{aligned} \tag{5}
$$

and I denotes an identity matrix with an appropriate dimension.

Our objective is to develop a robust H_∞ filter of the form (3) such that for all admissible parameter trajectories:

(a) The filtering error system (4) is asymptotically stable.

(b) The filtering error system (4) guarantees, under zero-initial condition,

$$
\|e\|_2 \le \gamma \|\omega\|_2 \tag{6}
$$

for all nonzero $\omega \in l_2[0,\infty)$ and a given positive constant γ.

III. Robust H_∞ Filtering Analysis

In this section, we will derive new H_∞ performance criteria for filtering analysis and synthesis of system (1).

Theorem 1: Consider the system of (1). For a prescribed $\gamma > 0$, if there exist matrices $0 < P^T(\rho) = P(\rho) \in R^{2n\times 2n}$, $0 < Q^T = Q \in R^{n\times n}$ that satisfy the following PLMI

$$
\begin{bmatrix}
-P(\rho_k)+d_\Delta K^T QK & * & * & * & * \\
0 & -Q & * & * & * \\
0 & 0 & -\gamma^2 I & * & * \\
P(\rho_{k+1})\bar{A}(\rho_k) & P(\rho_{k+1})\bar{A}_d(\rho_k) & P(\rho_{k+1})\bar{B}(\rho_k) & -P(\rho_{k+1}) & * \\
\bar{C}(\rho_k) & \bar{C}_d(\rho_k) & \bar{D}(\rho_k) & 0 & -I
\end{bmatrix} < 0 \tag{7}
$$

for all ρ then (4) is asymptotically stable with a H_∞ noise attenuation level γ. Where $d_\Delta = d_M - d_m + 1$.

Proof: Construct a Lyapunov-Krasovskii functional as

$$
\begin{aligned}
V(\xi(k)) &:= V_1 + V_2 + V_3 \\
V_1 &:= \xi^T(k)P(\rho_k)\xi(k) \\
V_2 &:= \sum_{i=k-d(t)}^{k-1} \xi^T(i)K^T QK\xi(i) \\
V_3 &:= \sum_{j=-d_M+2}^{-dm+1} \sum_{i=k+j-1}^{k-1} \xi^T(i)K^T QK\xi(i)
\end{aligned} \tag{8}
$$

where $P(\rho_k) > 0$, $Q > 0$.

Define $\Delta V := V(\xi(k+1)) - V(\xi(k))$, and along the trajectory of system (4) under the zero disturbance input, we have

$$
\begin{aligned}
\Delta V_1 = {}& \xi^T(k)\left[\bar{A}^T(\rho_k)P(\rho_{k+1})\bar{A}(\rho_k) - P(\rho_k)\right]\xi(k) \\
&+ 2\xi^T(k)\bar{A}^T(\rho_k)P(\rho_{k+1})\bar{A}_d(\rho)K\xi(k-d(k)) \\
&+ \xi^T(k-d(k))K^T\bar{A}_d^T(\rho_k)P(\rho_{k+1})\bar{A}_d(\rho)K\xi(k-d(k))
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
\Delta V_2 = {}& \xi^T(k)K^T QK\xi(k) - \xi^T(k-d(k))K^T QK\xi(k-d(k)) \\
&+ \sum_{i=k-d(k+1)+1}^{k-1} \xi^T(i)K^T QK\xi(i) - \sum_{i=k-d(k)+1}^{k-1} \xi^T(i)K^T QK\xi(i)
\end{aligned} \tag{10}
$$

where

$$
\begin{aligned}
\sum_{i=k-d(k+1)+1}^{k-1} \xi^T(i)K^T QK\xi(i) = {}& \sum_{i=k-d_m+1}^{k-1} \xi^T(i)K^T QK\xi(i) \\
&+ \sum_{i=k-d(k+1)+1}^{k-d_m} \xi^T(i)K^T QK\xi(i)
\end{aligned} \tag{11}
$$

$$
\sum_{i=k-d(k)+1}^{k-1} \xi^T(i)K^T QK\xi(i) \ge \sum_{i=k-d_M+1}^{k-1} \xi^T(i)K^T QK\xi(i) \tag{12}
$$

Then

$$
\begin{aligned}
\Delta V_2 \le {}& \xi^T(k)K^T QK\xi(k) - \xi^T(k-d(k))K^T QK\xi(k-d(k)) \\
&+ \sum_{i=k-d_M+1}^{k-d_m} \xi^T(i)K^T QK\xi(i)
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
\Delta V_3 &= \sum_{j=-d_M+2}^{-d_m+1} \sum_{i=k+j}^{k} \xi^T(k)K^T QK\xi(k) - \sum_{j=-d_M+2}^{-d_m+1} \sum_{i=k+j-1}^{k-1} \xi^T(k)K^T QK\xi(k) \\
&= \sum_{j=-d_M+2}^{-d_m+1} \left[\xi^T(k)K^T QK\xi(k) - \xi^T(k+j-1)K^T QK\xi(k+j-1)\right] \\
&= (d_M - d_m)\xi^T(k)K^T QK\xi(k) - \sum_{i=k-d_M+1}^{k-d_m} \xi^T(i)K^T QK\xi(i)
\end{aligned} \tag{14}
$$

Therefore, from (9)-(14) we can obtain that

$$
\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3 \le \bar{\xi}^T(k)\mathrm{M}\bar{\xi}(k)
$$

where

$$
\bar{\xi}(k) := \begin{bmatrix} \xi^T(k) & \xi^T(k-d(k))K^T \end{bmatrix}^T = \begin{bmatrix} \xi^T(k) & x^T(k-d(k)) \end{bmatrix}^T
$$

$$
\mathrm{M} := \begin{bmatrix} \bar{A}^T(\rho_k)P(\rho_{k+1})\bar{A}(\rho_k) - P(\rho_k) + d_\Delta K^T QK & * \\ \bar{A}_d^T(\rho_k)P(\rho_{k+1})\bar{A}(\rho_k) & \bar{A}_d^T(\rho_k)P(\rho_{k+1})\bar{A}_d(\rho_k) - Q \end{bmatrix}
$$

Using the Schur complement [12], PLMI (7) implies M<0. Then from the Lyapunov-Krasovskii stability theorem, we can conclude that the filtering error system (4) is asymptotically stable.

Now, to establish the H_∞ performance for the filtering error system, assume zero-initial condition and consider the following index

$$J := \sum_{k=0}^{\infty}\left[e^T(k)e(k)-\gamma^2\omega^T(k)\omega(k)\right] \quad (15)$$

Under zero initial condition, $V(\xi(k))|_{k=0}=0$ and we have

$$\begin{aligned} J &\le \sum_{k=0}^{\infty}\left[e^T(k)e(k)-\gamma^2\omega^T(k)\omega(k)\right]+V(\xi(k))|_{k=\infty}-V(\xi(k))|_{k=0} \\ &= \sum_{k=0}^{\infty}\left[e^T(k)e(k)-\gamma^2\omega^T(k)\omega(k)+\Delta V(\xi(k))\right] \\ &= \sum_{k=0}^{\infty}\lambda^T(k)\Xi\lambda(k) \end{aligned} \quad (16)$$

where

$\lambda(k) := \left[\xi^T(k) \quad \xi^T(k-d(k))K^T \quad \omega^T(k)\right]^T = \left[\xi^T(k) \quad x^T(k-d(k)) \quad \omega^T(k)\right]^T$

$\Xi :=$

$$\begin{bmatrix} \begin{pmatrix} \bar{A}^T(\rho_k)P(\rho_{k+1})\bar{A}(\rho_k) \\ -P(\rho_k)+d_\Delta K^TQK \\ +\bar{C}^T(\rho_k)\bar{C}(\rho_k) \end{pmatrix} & * & * \\ \begin{pmatrix} \bar{A}_d^T(\rho_k)P(\rho_{k+1})\bar{A}(\rho_k) \\ +\bar{C}_d^T(\rho_k)\bar{C}(\rho_k) \end{pmatrix} & \begin{pmatrix} \bar{A}_d^T(\rho_k)P(\rho_{k+1})\bar{A}_d(\rho_k) \\ -Q+\bar{C}_d^T(\rho_k)\bar{C}_d(\rho_k) \end{pmatrix} & * \\ \begin{pmatrix} \bar{B}^T(\rho_k)P(\rho_{k+1})\bar{A}(\rho_k) \\ +\bar{D}^T(\rho_k)\bar{C}(\rho_k) \end{pmatrix} & \begin{pmatrix} \bar{B}^T(\rho_k)P(\rho_{k+1})\bar{A}_d(\rho_k) \\ +\bar{D}^T(\rho_k)\bar{C}_d(\rho_k) \end{pmatrix} & \begin{pmatrix} \bar{B}^T(\rho_k)P(\rho_{k+1})\bar{B}(\rho_k) \\ +\bar{D}^T(\rho_k)\bar{D}(\rho_k)-\gamma^2 I \end{pmatrix} \end{bmatrix}$$

By Schur complement, PLMI (7) guarantees $\Xi<0$, therefore $J \le 0$ and $\|e\|_2 \le \gamma\|\omega\|_2$. The proof is completed.

Remark 1: It should be noted that the condition presented in Theorem 1 contains product terms between positive matrices and system matrices when (5) is considered. In the following, we will give an improved version of Theorem 1 by introducing a slack variable to decouple these product terms, which is more easily tractable for handling the filtering problems.

Theorem 2: Consider the system of (1). For a prescribed $\gamma>0$, if there exist matrices $0<P^T(\rho_k)=P(\rho_k)\in R^{2n\times 2n}$, $0<Q^T=Q\in R^{n\times n}$, $W\in R^{2n\times 2n}$ satisfying

$$\begin{bmatrix} -P(\rho_k)+d_\Delta K^TQK & * & * & * & * \\ 0 & -Q & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * \\ W^T\bar{A}(\rho_k) & W^T\bar{A}_d(\rho_k) & W^T\bar{B}(\rho_k) & P(\rho_{k+1})-(W+W^T) & * \\ \bar{C}(\rho_k) & \bar{C}_d(\rho_k) & \bar{D}(\rho_k) & 0 & -I \end{bmatrix}<0 \quad (17)$$

for all ρ, the filtering error system (4) is asymptotically stable with a H_∞ noise attenuation level γ.

Proof: we will prove the theorem by showing the equivalence between (7) and (17). If (7) holds, (17) is readily established by choosing $W=W^T=P(\rho_{k+1})$. On the other hand, if (17) holds, we can explore the facts $W^T+W-P(\rho_{k+1})>0$ so that W is a nonsingular matrix. In addition, we have $(P(\rho_{k+1})-W)^TP^{-1}(\rho_{k+1})(P(\rho_{k+1})-W)\ge 0$, which implies that $-W^TP^{-1}(\rho_{k+1})W\le P(\rho_{k+1})-W^T-W$. Therefore we can conclude from (17) that

$$\begin{bmatrix} -P(\rho_k)+d_\Delta K^TQK & * & * & * & * \\ 0 & -Q & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * \\ W^T\bar{A}(\rho_k) & W^T\bar{A}_d(\rho_k) & W^T\bar{B}(\rho_k) & -W^TP(\rho_{k+1})W & * \\ \bar{C}(\rho_k) & \bar{C}_d(\rho_k) & \bar{D}(\rho_k) & 0 & -I \end{bmatrix}<0 \quad (18)$$

Performing congruence transformation to (18) by $diag\{I,I,I,W^{-1},I\}$ yields (7), and then the proof is completed.

Now, considering the delay-free LPV system

$$\begin{aligned} x(k+1) &= A(\rho(k))x(k)+B(\rho(k))\omega(k) \\ y(k) &= C(\rho(k))x(k)+D(\rho(k))\omega(k) \\ z(k) &= H(\rho(k))x(k)+L(\rho(k))\omega(k) \end{aligned} \quad (19)$$

the following two corollaries establish the counterparts of Theorem 1 and 2, respectively.

Corollary 1: Consider the delay-free system of (19). The system (4) is asymptotically stable with a H_∞ noise attenuation level γ if there exist a matrix $0<P^T(\rho)=P(\rho)\in R^{2n\times 2n}$ satisfying the PLMI

$$\begin{bmatrix} -P(\rho_k) & * & * & * & * \\ 0 & -Q & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * \\ P(\rho_{k+1})\bar{A}(\rho_k) & P(\rho_{k+1})\bar{A}_d(\rho_k) & P(\rho_{k+1})\bar{B}(\rho_k) & -P(\rho_{k+1}) & * \\ \bar{C}(\rho_k) & \bar{C}_d(\rho_k) & \bar{D}(\rho_k) & 0 & -I \end{bmatrix}<0 \quad (20)$$

Corollary 2: Consider the delay-free system of (19). The system (4) is asymptotically stable with a H_∞ noise attenuation level γ if there exist two matrices $W\in R^{2n\times 2n}$, $0<P^T(\rho)=P(\rho)\in R^{2n\times 2n}$ satisfying the PLMI

$$\begin{bmatrix} -P(\rho_k) & * & * & * & * \\ 0 & -Q & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * \\ W^T\bar{A}(\rho_k) & W^T\bar{A}_d(\rho_k) & W^T\bar{B}(\rho_k) & P(\rho_{k+1})-(W+W^T) & * \\ \bar{C}(\rho_k) & \bar{C}_d(\rho_k) & \bar{D}(\rho_k) & 0 & -I \end{bmatrix}<0 \quad (21)$$

IV. Robust H_∞ Filtering Design

In this section, based on Theorem 2, we will develop linear filter of form (3) assuring robust H_∞ performance for discrete time-delayed LPV system (1).

The following theorem provides sufficient conditions for the existence of delay-dependent H_∞ filters.

Theorem 3: Consider the system of (1). For a prescribed $\gamma > 0$, if there exist matrices $E \in R^{n\times n}$, $F \in R^{n\times n}$, $U \in R^{n\times n}$, $\overline{P}_1^T(\rho) = \overline{P}_1(\rho) \in R^{n\times n}$, $\overline{P}_3^T(\rho) = \overline{P}_3(\rho) \in R^{n\times n}$, $\overline{P}_2(\rho) \in R^{n\times n}$, $0 < Q^T = Q \in R^{n\times n}$, $\overline{A}_F(\rho) \in R^{n\times n}$, $\overline{B}_F(\rho) \in R^{n\times m}$, $\overline{C}_F(\rho) \in R^{p\times n}$ and $\overline{D}_F(\rho) \in R^{p\times m}$ that satisfy the following inequalities (22-23) for all ρ then an admissible H_∞ filter of the form (3) exists.

$$\begin{bmatrix} \overline{P}_1(\rho_k) & * \\ \overline{P}_2^T(\rho_k) & \overline{P}_3(\rho_k) \end{bmatrix} > 0 \tag{23}$$

Proof: First let some matrix variables in Theorem 2 be partitioned as

$$W := \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix} \quad V := W^{-1} = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} \tag{24}$$

Now we introduce matrices

$$J_V := \begin{bmatrix} I & I \\ 0 & V_{21}V_{11}^{-1} \end{bmatrix}, \quad J_2 := diag\{J_V, I, I, J_V, I\},$$
$$\overline{P}(\rho) := \begin{bmatrix} \overline{P}_1(\rho) & * \\ \overline{P}_2^T(\rho) & \overline{P}_3(\rho) \end{bmatrix} = J_V^T \begin{bmatrix} P_1(\rho) & * \\ P_2^T(\rho) & P_3(\rho) \end{bmatrix} J_V,$$
$$E := W_{11}, \; F := V_{11}^{-1}, \; U := V_{11}^{-T} V_{21}^T W_{21},$$
$$\overline{A}_F(\rho) := W_{21}^T A_F(\rho) V_{21} V_{11}^{-1}, \; \overline{B}_F(\rho) := W_{21}^T B_F(\rho),$$
$$\overline{C}_F(\rho) := C_F(\rho) V_{21} V_{11}^{-1}, \; \overline{D}_F(\rho) := D_F(\rho) \tag{25}$$

Then performing congruence transformation to (17) by J_2, it can be readily established that (22)-(23) are equivalent to (17).

Therefore, from Theorem 3 we can conclude that the filter with a state-space realization $(A_F(\rho), B_F(\rho), C_F(\rho), D_F(\rho))$ defined in (25) guarantees that the filtering error system (4) has a H_∞ noise attenuation level γ. □

Remark 2: Notice that the PLMI conditions (22)-(23) correspond to infinite-dimensional convex problems due to their parametric dependence. Using the gridding technique and the appropriate basis functions [3], infinite-dimensional PLMIs can be transformed to finite-dimensional ones, which can be solved numerically using convex optimization technique. Hence, by choosing appropriate basis function $\{f_j(\rho)\}_{j=1}^{s}$ such that

$$\overline{P}(\rho) = \begin{bmatrix} \overline{P}_1(\rho) & * \\ \overline{P}_2^T(\rho) & \overline{P}_3(\rho) \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{n_f} f_j(\rho)\overline{P}_{1j} & * \\ \sum_{j=1}^{n_f} f_j(\rho)\overline{P}_{2j}^T & \sum_{j=1}^{n_f} f_j(\rho)\overline{P}_{3j} \end{bmatrix} > 0 \tag{26}$$

PLMI can be approximated.

Remark 3: Theorem 3 casts the full-order robust H_∞ filtering problem for system (1) into PLMIs feasibility test, and any feasible solution to (22) and (26) will yield a suitable filter. If we can find an admissible robust H_∞ filter for system (1), the filter matrices can be calculated from the definition (25). However, there seem to be no systematic ways to determine the matrices V_{21} and V_{22} needed for the filter matrices. To deal with such a problem, first of all, let us denote the filter transfer function from $y(k)$ to $z_F(k)$ by (27).

$$\begin{bmatrix} -\overline{P}_1(\rho_k) + d_\Delta Q & * & * & * & * & * & * \\ -\overline{P}_2^T(\rho_k) + d_\Delta Q & -\overline{P}_3(\rho_k) + d_\Delta Q & * & * & * & * & * \\ 0 & 0 & -Q & * & * & * & * \\ 0 & 0 & 0 & -\gamma^2 I & * & * & * \\ E^T A(\rho_k) + \overline{B}_F(\rho_k)C(\rho_k) & E^T A(\rho_k) + \overline{B}_F(\rho_k)C(\rho_k) + \overline{A}_F(\rho_k) & E^T A_d(\rho_k) + \overline{B}_F(\rho_k)C_d(\rho_k) & E^T B(\rho_k) + \overline{B}_F(\rho_k)D(\rho_k) & \overline{P}_1(\rho_{k+1}) - E - E^T & * & * \\ F^T A(\rho_k) & F^T A(\rho_k) & F^T A_d(\rho_k) & F^T A(\rho_k) & \overline{P}_2^T(\rho_{k+1}) - E - F^T - U & \overline{P}_3(\rho_{k+1}) - F - F^T & * \\ H(\rho_k) - \overline{D}_F(\rho_k)C(\rho_k) & H(\rho_k) - \overline{D}_F(\rho_k)C(\rho_k) - \overline{C}_F(\rho_k) & H_d(\rho_k) - \overline{D}_F(\rho_k)C_d(\rho_k) & L(\rho_k) - \overline{D}_F(\rho_k)D(\rho_k) & 0 & 0 & -I \end{bmatrix} < 0 \tag{22}$$

$$T_{z_F y} = C_F(\rho)(zI - A_F(\rho))^{-1} B_F(\rho) + D_F(\rho) \tag{27}$$

Substituting the filter matrices with (25) and considering the relationship $U = V_{11}^{-T} V_{21}^{T} W_{21}$ yields

$$\begin{aligned} T_{z_F y} &= \overline{C}_F(\rho) F^{-1} V_{21}^{-1} (zI - W_{21}^{-T} \overline{A}_F(\rho) F^{-1} V_{21}^{-1})^{-1} W_{21}^{-T} \overline{B}_F(\rho) + \overline{D}_F(\rho) \\ &= \overline{C}_F(\rho)\left[zW_{21}^{T} V_{21} F - \overline{A}_F(\rho)\right]^{-1} \overline{B}_F(\rho) + \overline{D}_F(\rho) \\ &= \overline{C}_F(\rho)[zI - U^{-T}\overline{A}_F(\rho)]^{-1} U^{-T} \overline{B}_F(\rho) + \overline{D}_F(\rho) \end{aligned}$$

Therefore, an admissible filter is given by

$$\begin{aligned} &A_F(\rho) = U^{-T}\overline{A}_F(\rho), \quad C_F(\rho) = \overline{C}_F(\rho), \\ &B_F(\rho) = U^{-T}\overline{B}_F(\rho), \quad D_F(\rho) = \overline{D}_F(\rho) \end{aligned} \tag{28}$$

Remark 4: Note that (22) and (26) are PLMIs not only over the matrix variables, bust also over the scalar γ^2. This implies that the scalar γ^2 can be included as one of the optimization variables for LMI (22) and (26) to obtain the minimum noise attenuation level. Then the minimum guaranteed cost of robust parameter-dependent H_∞ filter can be readily found by solving the following convex optimization problem:

$$\textit{Minimize } \gamma^2 \textit{ subject to } (22) \text{ and } (26) \tag{29}$$

Remark 5: It can be shown that the time-varying delay of LPV system (1) is constant delay for $d_\Delta = 1$.

V. An Illustrative Example

Consider the following discrete-time LPV system with a state-delay.

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 0 & 0.3 \\ -0.2 & 0.5\rho_1(k) \end{bmatrix} x(k) + \begin{bmatrix} 0 & 0 \\ 0.1 & 0.1\rho_2(k) \end{bmatrix} x(k-d(k)) \\ &\quad + \begin{bmatrix} -0.45 \\ 0.35 \end{bmatrix} \omega(k) \\ y(k) &= \begin{bmatrix} 0.35 & -0.65 \end{bmatrix} x(k) + \begin{bmatrix} 0.2 & 0 \end{bmatrix} x(k-d(k)) + 0.2\omega(k) \\ z(k) &= \begin{bmatrix} 1 & 2 \end{bmatrix} x(k) \end{aligned} \tag{30}$$

where $\rho_1 = \sin k$ and $\rho_2 = \cos k$ are time-varying parameters satisfying

$$-1 \le \rho_1 \le 1, \; -1 \le \rho_2 \le 1$$

Our objective is to design a robust H_∞ filter. First we choose appropriate basis functions

$$f_1(\rho) = 1, \; f_2(\rho) = \rho_1, \; f_3(\rho) = \rho_2$$

Gridding the parameter space uniformly using 9×9 grids. The minimum noise attenuation level obtained by solving convex optimization problem for different $d_\Delta = d_M - d_m + 1$ are shown in Table I.

Table I The Minimum Guaranteed Cost for Different Delay-Varying Magnitude

$d_\Delta = d_M - d_m + 1$	Minimum Guaranteed Cost γ^*
1	0.5458
2	0.6653
3	0.7709
4	0.8713

From Table I, we can see that the effect of the delay-varying magnitude on the attainable the minimum guaranteed cost. For the minimum noise attenuation level $\gamma^* = 0.5458$ with $d_\Delta = 1$ (constant delay case) we obtain the corresponding parameter-dependent filter (31) given by

$$\begin{aligned} A_F(\rho) &= \begin{bmatrix} 0.1504\text{-}0.6642\rho_1 + 0.1361\rho_2 & 0.1345\text{-}0.5637\rho_1 + 0.1504\rho_2 \\ -0.1599 + 0.8713\rho_1 \text{-}0.1527\rho_2 & -0.1384 + 0.7464\rho_1 \text{-}0.1730\rho_2 \end{bmatrix}, \\ B_F(\rho) &= \begin{bmatrix} 1.4785 + 1.4867\rho_1 - 0.2638\rho_2 \\ -2.0584\text{-}1.9976\rho_1 + 0.2937\rho_2 \end{bmatrix}, \\ C_F(\rho) &= \begin{bmatrix} 1.3436\text{-}0.1084\rho_1 - 0.0090\rho_2 & 1.1981 + 0.0336\rho_1 + 0.0875\rho_2 \end{bmatrix}, \\ D_F(\rho) &= -1.0074 - 0.1122\rho_1 + 0.1683\rho_2 \end{aligned} \tag{31}$$

And for $\gamma^* = 0.8713$ with $d_\Delta = 4$, we obtain the corresponding parameter-dependent filter

$$\begin{aligned} A_F(\rho) &= \begin{bmatrix} 0.2301\text{-}0.2991\rho_1 + 0.1022\rho_2 & 0.1591\text{-}0.2126\rho_1 + 0.0286\rho_2 \\ -0.2036 + 0.5297\rho_1 \text{-}0.2144\rho_2 & -0.1663 + 0.3861\rho_1 \text{-}0.0587\rho_2 \end{bmatrix}, \\ B_F(\rho) &= \begin{bmatrix} 0.0393 + 0.4381\rho_1 - 0.0644\rho_2 \\ -0.7964\text{-}0.8936\rho_1 + 0.1642\rho_2 \end{bmatrix}, \\ C_F(\rho) &= \begin{bmatrix} 1.3014\text{-}0.0880\rho_1 - 0.0569\rho_2 & 0.9537 + 0.2400\rho_1 + 0.0021\rho_2 \end{bmatrix}, \\ D_F(\rho) &= -1.1807 + 0.0243\rho_1 + 0.2122\rho_2 \end{aligned} \tag{32}$$

Then we analyze the disturbance attenuation level of the filtering error system by connecting the two obtained filters to the original system respectively. Here we assume $\omega(k)$ to be (33).

$$\omega(k) = \begin{cases} 2, & 20 \le k \le 30 \\ -2, & 50 \le k \le 60 \\ 0, & else \end{cases} \tag{33}$$

Fig.1 presents the simulation curves of estimating the signal $z(k)$ by the two filters respectively. We can see that

$\omega(k)$ drives $z_F(k)$ to deviate from $z(k)$. However, when $\omega(k)$ is zero, the deviation tends to be zero due to the asymptotically stability of the filter error system. Now we will further analyze the H_∞ performance. Fig.2 shows the changing curves of the disturbance signal and the filtering error signal. From (33) and Fig.2, we obtain that $\|\omega\|_2 = \sqrt{\sum_{k=0}^{\infty} \omega^T(k)\omega(k)} = 9.3808$, $\|e\|_2 = \sqrt{\sum_{k=0}^{\infty} e^T(k)e(k)} = 2.1634$, then it can be easily established that $\|e\|_2 / \|\omega\|_2 = 0.2306 < \gamma^* = 0.8713$, therefore, the H_∞ filter (32) can guarantee the prescribed noise disturbance attenuation level.

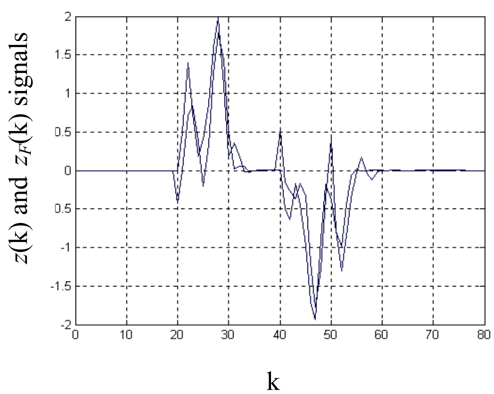

Fig.1 z(k) and z_F(k) signals of the filter error system with time-varying state delay

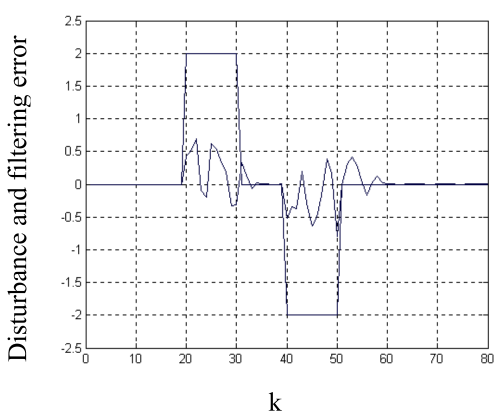

Fig.2 Disturbance and filtering error

VI. CONCLUDING REMARKS

In this paper, robust H_∞ filters design is proposed for LPV discrete-time systems with constant and time-varying state delay. The filtering problems have been solved and cast into convex optimization problems in terms of PLMIs, which can be solved via efficient interior-point algorithms.

A numerical example has shown the feasibility applicability of the proposed designs.

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