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A Robust Controller for an Electro-Mechanical Fin Actuator

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Abstract—**The Objective of this paper is to realize a robust controller for an electro-mechanical fin servo system of a missile. In this paper, we design the robust controller using an H_∞ optimization method and a disturbance observer, in order to improve the overall performance of the fin servo system. The H_∞ controller is designed using the mixed sensitivity H_∞ control method, based on the 2-Riccati state-space approach of Glover and Doyle. The newly proposed disturbance observer is applied to the fin servo system and it consists of three elements: a time delay estimation algorithm part, an anti-filtering compensator(AFC) part and a low pass filter(LPF) part. The effectiveness of this control scheme is verified through simulations and experiments.**

I. Introduction

ELECTRO-MECHANICAL servo systems have been steadily used in fin position servo systems of guided missiles, because of their momentary overdrive capability, low quiescent power/low maintenance characteristics and long-term storability. During a flight, fin position servo systems have many uncertainties due to disturbances, parameter variations, electrical noises and so on. Furthermore, fin position servo systems are subjected to aerodynamic load disturbances, which are the function of such parameters as the deflection angle of the control fin, the angle of attack and Mach number. Consequently, the conventional control approaches based on a linearized model near the operating point of interest may not guarantee the satisfactory control performance for electro-mechanical fin position servo systems.

Recently several robust control methods are available to solve the above problems. H_∞ control theory can be one of the most powerful solutions for such problems. However, there is a trade-off between the robust stability and the robust performance. To compensate this trade-off we design a two-degree of freedom controller using an H_∞ controller and a disturbance observer. The feedforward H_∞ controller is designed to improve the command tracking performance and guarantee the robust performance in the presence of parameter variations and measurement noises. The feedback controller based on the disturbance observer compensates for external disturbances and plant uncertainties.

In this paper, we propose a new disturbance observer scheme. The proposed disturbance observer consists of three components: a time delay estimation algorithm part, an anti-filtering compensator(AFC) part and a low pass filter(LPF) part. The time delay estimation algorithm part is based on the concept that plant uncertainties are estimated from the time delayed information[1]. The proposed disturbance observer calculates the additional control input to compensate plant uncertainties from the time delayed information. Generally sensor output signals are filterd by anti-aliasing filters. The role of the AFC part is to remove the effects of anti-aliasing filters[2]. The LPF part is used to adjust the observer performance between the disturbance suppression and the sensor noise rejection. The proposed disturbance observer has a simple structure and a good performance.

To demonstrate the effectiveness of the designed controller, a series of numerical simulations and experiments are performed on the fin position servo system for various conditions. Simulation and experimental results show that it can improve the performance and robustness of the fin position servo system.

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II. System Modelling

The electro-mechanical fin actuation servo system, which is the object of this research, is composed of a DC Motor, a gearing mechanism, sensors, and a Microprocessor-controlled PWM servo-amplifier.

Under linear assumption, a system model can be obtained. Ignoring the motor/amplifier time constant and the mechanical coupling constant, the system model can be simplified to a second order model. Consequently, the state equation of the fin servo system is expressed as:

$$\begin{bmatrix} \dot{\theta}_m(t) \\ \dot{\omega}_m(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -1/\tau_m \end{bmatrix} \begin{bmatrix} \theta_m(t) \\ \omega_m(t) \end{bmatrix} + \begin{bmatrix} 0 \\ b_m \end{bmatrix} u(t) \quad (1)$$

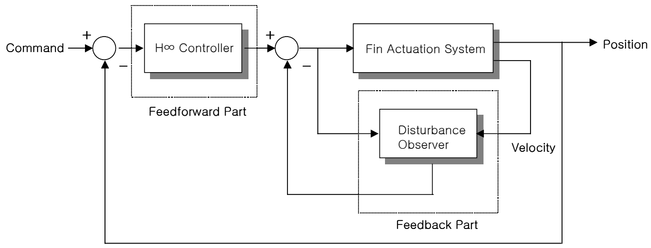


Fig. 1. Controller Structure

$$y(t) = [1/N \quad 0]\begin{bmatrix}\theta_m(t) \\ \omega_m(t)\end{bmatrix} \tag{2}$$

where, $u(t)$ denotes the input voltage to the motor, $y(t)$ the fin angular position, $\theta_m(t)$ the motor angular position, $\omega_m(t)$ the motor angular velocity, N the gear ratio, $\tau_m = (R_m J_e)/(R_m B + K_T K_B)$ the mechanical time constant, $b_m = (K_T)/(R_m J_e)$ the input gain, R_m the motor resistance, J_e the motor inertia, B the motor viscous damping coefficient, K_T the motor torque sensitivity, K_B the motor back EMF constant.

The plant transfer function $P(s)$ from $u(t)$ to $y(t)$ is obtained as:

$$\frac{Y(s)}{U(s)} = P(s) = \frac{b_m / N}{s(s+a)} \tag{3}$$

where, $a = 1/\tau_m$

III. Controller Design

A. Controller Structure

Fig. 1 is the overall structure of the designed servo system.

The designed system has a two-degree of freedom structure using an H∞ control method and a disturbance observer. The feedforward H∞ controller is designed to improve the command tracking performance and guarantee the robust performance in the presence of parameter variations and measurement noises. The feedback controller based on the disturbance observer compensates for external disturbances and plant uncertainties.

The newly proposed disturbance observer is applied to make the closed-loop system robust to plant model uncertainties. The proposed disturbance observer consists of three components: a time delay estimation algorithm part, an AFC part and a LPF part.

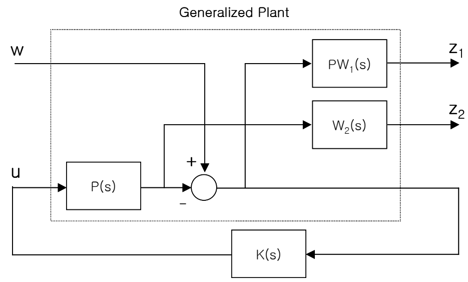


Fig. 2. Mixed Sensitivity Problem

B. H∞ Controller Structure

In this paper, the mixed sensitivity H∞ design method, which utilizes the sensitivity function $S(s)$ and the complementary sensitivity function $T(s)$, is used.

The mixed sensitivity H∞ design method is expressed as:

$$S(s) + T(s) = I \tag{4}$$

$$\|R_{zw}\|_\infty = \left\| \begin{matrix} W_1 S \\ W_2 T \end{matrix} \right\|_\infty \langle \gamma \tag{5}$$

where, W_1 and W_2 are frequency domain weight functions and $\gamma \rangle 0$ is the cost coefficient.

The mixed sensitivity problem is to obtain the stable controller to satisfy (5).

The minimization of $S(s)$ over low to middle frequencies is related to the improved command tracking and disturbance attenuation, while the minimization of $T(s)$ over high frequencies is necessary to provide the robust stability in the presence of sensor noises, modelling errors and plant parameter variations[3].

Fig. 2 is the configuration of the weighted mixed sensitivity problem. For the mixed sensitivity problem, the generalized plant $G(s)$ is mathematically represented as:

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} = \left[\begin{array}{c|c} W_1 & -\rho W_1 P \\ 0 & W_2 P \\ \hline I & -P \end{array}\right] \tag{6}$$

All transfer matrices are then converted to their state-space equivalents, so the generalized plant $G(s)$ may be rewritten as:

$$G = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right] \tag{7}$$

In this paper, we use the G-D(Glover-Doyle) algorithm to get the H∞ controller: only two Algebraic Riccati Equations need to be solved to compute the (sub)optimal controller,

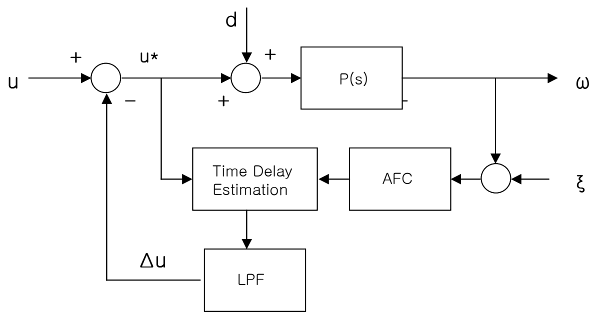


Fig. 3. Structure of the proposed disturbance observer

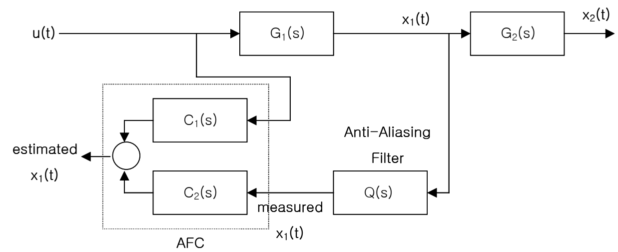


Fig. 4. Structure of the anti-filtering compensator(AFC)

which (if it exists) is often of the same order as the generalized plant.

$$A^T X_\infty + X_\infty A + C_1^T C_1 + X_\infty(\gamma^{-2} B_1 B_1^T - B_2 B_2^T) X_\infty = 0 \quad (8)$$

$$Y_\infty A^T + A Y_\infty + B_1 B_1^T + Y_\infty(\gamma^{-2} C_1^T C_1 - C_2^T C_2) Y_\infty = 0 \quad (9)$$

If a particular suboptimal controller K_{sub} is obtained, then a parametrization of the family of stabilizing controllers K for the plant is given by a LLFT(Lower Linear Fractional Transformation) on a stable contraction Q [4],[5],[6].

C. Disturbance Observer Design

The disturbance observer shown in Fig. 3 is used to cancel the input disturbance with an estimate of its value.

Let us consider a nonlinear system in the form of the following differential equation:

$$\dot{x}(t) = f(x,t) + B(x,t)u(t) + h(x,t) + d(t) \quad (10)$$

where, $x(t)$ denotes the state variable, $u(t)$ the input, $f(x,t)$ the system nonlinearity which is known, $B(x,t)$ the nonlinear control distribution scalar, the range of which is known, $h(x,t)$ the system nonlinearity which is unknown and $d(t)$ the unknown disturbance.

Plant uncertainties, $h(x,t)$ and $d(t)$, are estimated from the time delayed information[1].

If the sampling speed is much faster than that of the system dynamics, the estimation values of plant uncertainties at the present time step are almost equal to the values at the previous time step. For a sufficiently small time delay L, we obtain the following equations:

$$\hat{h}(x,t) + \hat{d}(t) \approx h(x,t-L) + d(t-L) \quad (11)$$

$$\hat{h}(x,t) + \hat{d}(t) \approx \dot{x}(t-L) - f(x,t-L) - B(x,t-L)u(t-L) \quad (12)$$

where, $\hat{h}(x,t)$ and $\hat{d}(t)$ are estimates of $h(x,t)$ and $d(t)$.

We define $\Delta u(t)$ as an additional control input to compensate $h(x,t)$ and $d(t)$.

If we replace $h(x,t)$ and $d(t)$ by $\hat{h}(x,t)$ and $\hat{d}(t)$, $\Delta u(t)$ is obtained as follows:

$$\Delta u(t) = -\frac{\hat{h}(x,t) + \hat{d}(t)}{B(x,t)} = -\frac{\dot{x}(t-L) - f(x,t-L) - B(x,t-L)u(t-L)}{B(x,t)} \quad (13)$$

In Fig. 3 the AFC Block denotes the anti-filtering compensator. Because of sensor noises, sensor output signals are filtered by anti-aliasing filters. The role of the AFC is to cancel the effect of the filter dynamics and estimate the original sensor output signal. Being able to know the transfer function of the anti-aliasing filter, we can design an additional compensator using an observer, which compensates for the influence of the filter[2]. Fig. 4 is the structure of the AFC.

Cancelling the effect of $Q(s)$, the anti-aliasing filter dynamics, the AFC satisfies the following condition:

$$\frac{X(s)}{U(s)} = \frac{\hat{X}(s)}{U(s)} \quad (14)$$

where, $\hat{X}(s) = estimated\ X(s)$

There are many methods to design C_1 and C_2, which satisfy (14). In this paper, we choose a reduced order observer, which uses $u(t)$ and *measured* $x(t)$ as an input.

In Fig. 3 the LPF Block denotes the low pass filter, which plays a role in adjusting the observer performance between the disturbance suppression and the sensor noise rejection. If the cut-off frequency of LPF is high, the estimation performance of the observer about uncertainties is improved. But in this case the stability of the observer in the presence of sensor noises becomes worse. If the cut-off frequency is low, the opposite condition occurs. One can use the various types of LPF. If one increases the order of LPF, the noise rejection performance is improved. But it can make the system more unstable because it can rise the high peak resonance value.

D. Implementation of the designed controller

The characteristic values of the fin position servo system are shown in Table 1.

To design the H_∞ Controller we choose the frequency domain specification as follows: $W_1(s)$ is chosen to minimize the sensitivity function $S(s)$ for frequencies below 50 [rad/sec], while $W_2(s)$ is chosen to penalize the complementary sensitivity function $T(s)$ for frequencies over 80 [rad/sec].

$$W_1(s) = \frac{0.28(0.001s+1)}{0.001s} \tag{15}$$

$$W_2(s) = 1.4\times10^{-4}s^2 \tag{16}$$

The transfer function of the generalized plant $G(s)$ is described as follows:

$$G(s) = \left[\begin{array}{c|c} \frac{0.28(0.001s+1)}{0.001s} & -\frac{100.65(0.001s+1)}{0.001s^3+7.60\times10^{-2}s^2} \\ 0 & \frac{0.05s}{s+75.98} \\ \hline 1 & -\frac{359.48}{s^2+75.98s} \end{array}\right] \tag{17}$$

Consequently we get the H_∞ controller as follows:

$$K(s) = \frac{1.36\times10^4 s^2 + 1.03\times10^6 s + 2.70\times10^{-7}}{s^3 + 8.14\times10^2 s^2 + 9.60\times10^4 s - 6.71\times10^{-10}} \tag{18}$$

The design process of the proposed disturbance observer is explained below.

From (1) the state equation from $u(t)$ to $\omega_m(t)$ is obtained as follows:

$$\dot{\omega}_m(t) = -\frac{1}{\tau_m}\omega_m(t) + b_m u(t) \tag{19}$$

From (13) and (19) we can calculate $\Delta u(t)$, which is the additional control input to compensate the uncertainties.

Our fin servo system has the 2nd order butterworth type anti-aliasing filter in the velocity feedback loop, whose cut-off frequency is 300 [Hz]. So we design the AFC to estimate the original velocity sensor output signal using a reduced order observer.

The state-space equation of the 2nd order butterworth type anti-aliasing filter is expressed as:

$$\begin{bmatrix} \dot{\omega}_s \\ \ddot{\omega}_s \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_f^2 & -2\zeta_f\omega_f \end{bmatrix}\begin{bmatrix} \omega_s \\ \dot{\omega}_s \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_f^2 \end{bmatrix}\omega_m \tag{20}$$

where, ω_s is the measured velocity sensor signal, ζ_f the damping coefficient of the filter, ω_f the natural frequency of the filter.

From (19) and (20) we can get the following equation:

$$\left[\begin{array}{c} \dot{\omega}_s \\ \hline \ddot{\omega}_s \\ \dot{\omega}_m \end{array}\right] = \left[\begin{array}{c|cc} 0 & 1 & 0 \\ \hline -\omega_f^2 & -2\zeta_f\omega_f & \omega_f^2 \\ 0 & 0 & -1/\tau_m \end{array}\right]\left[\begin{array}{c} \omega_s \\ \hline \dot{\omega}_s \\ \omega_m \end{array}\right] + \left[\begin{array}{c} 0 \\ \hline 0 \\ b_m \end{array}\right] u \tag{21}$$

$$\Leftrightarrow \begin{bmatrix} F_{aa} & F_{ab} \\ F_{ba} & F_{bb} \end{bmatrix}\begin{bmatrix} \omega_s \\ \dot{\omega}_s \\ \omega_m \end{bmatrix} + \begin{bmatrix} G_a \\ G_b \end{bmatrix} u$$

Table 1. The characteristic values of the fin servo system

Parameters	Symbols	Values	Units
Motor Resistance	R_m	1.8	Ohms
Motor Inertia	J_e	0.0004	lb-in-sec^2
Motor Viscous Damping Coefficient	B	0.00125	lb-in/rad/sec
Motor torque sensitivity	K_T	0.6812	lb-in/amp
Motor Back EMF Constant	K_B	0.077	volt/rad/sec
Gear Ratio	N	150	-

State variables can be partitioned into two substates: ω_s which can be measured, and $\dot{\omega}_s$ and ω_m which cannot be measured directly. So we can design the reduced observer to estimate ω_m. Consequently, the AFC is described as:

$$\dot{x}_c = (F_{bb} - LF_{ab})\hat{x}_b + (F_{ba} - LF_{aa})y + (G_b - LG_a)u \tag{22}$$

$$\hat{x}_b = x_c + Ly \tag{23}$$

where, $\hat{x}_b = \begin{bmatrix} \hat{\dot{\omega}}_s & \hat{\omega}_m \end{bmatrix}'$: the estimated state

$y = \omega_s$: the measured state

$L = \begin{bmatrix} l_1 & l_2 \end{bmatrix}'$: the observer gain

As a result of numerical simulations for the fin actuation servo system, $\Delta u(t)$, the additional control input to compensate the uncertainties, has a good stability in the presence of measurement noises. So, in this research, we eliminate the LPF Part to maximize the estimation performance of the proposed disturbance observer.

IV. Simulation and Experimental Results

To demonstrate the effectiveness of this designed controller, simulations and experiments are performed on the fin servo system for various conditions.

The simulation and experimental results of the designed controller are compared with those of a PID controller and a time delay controller(TDC). The gains of the PID controller are tuned as $K_P = 17$, $K_I = 300$ and $K_D = 0.15$. The gains of the TDC are obtained by choosing $\zeta = 0.78$, the damping coefficient of the reference model, and $\omega_n = 11.2$ [Hz], the natural frequency of the reference model.

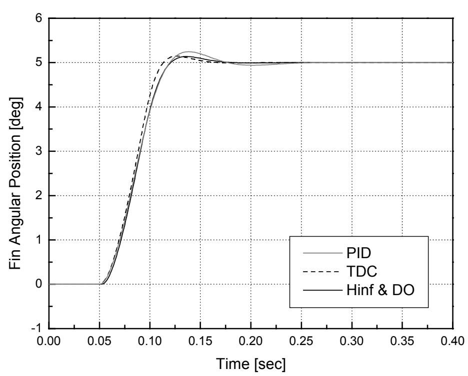
In the simulation model we include the influence of an actuator saturation, a motor inductance, anti-aliasing filters and mechanical couplings. Controllers for the fin actuator are transformed to discrete time forms with the sampling time of 2 [ms] in consideration of the practical use.

The simulation results in Fig. 5 show the transient responses of the three controllers to a step reference input. Fig. 5-(a) exhibits the step responses of each controller under no load condition. Three controllers show faster response. Fig. 5-(b) exhibits the step responses of each controller when a spring load of 1000/15 [lb-in/deg] is applied to the fin position servo system. The designed controller and the TDC show better performance than the PID controller.

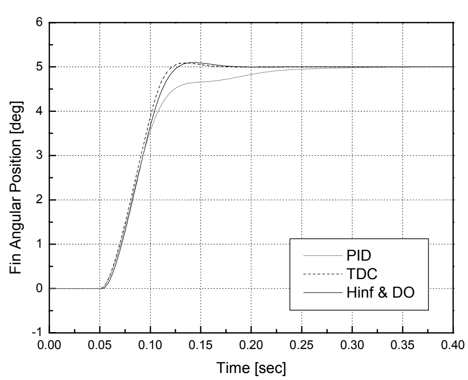
Fig. 6 shows the simulation results of the step responses in the presence of motor parameter variations. The designed controller shows the best robustness in the presence of parameter variations.

The simulation results in Fig. 7 show the transient responses of the three controllers to a step disturbance input. A step disturbance of 1000 [lb-in] is injected into the fin servo system at $t = 0.1$ [sec] after the start time. The designed controller shows the best disturbance rejection characteristics.

The experimental results in Fig. 8 show the transient responses of each controller to a step reference input. The experimental results in Fig. 8 are similar to the simulation results in Fig. 5.

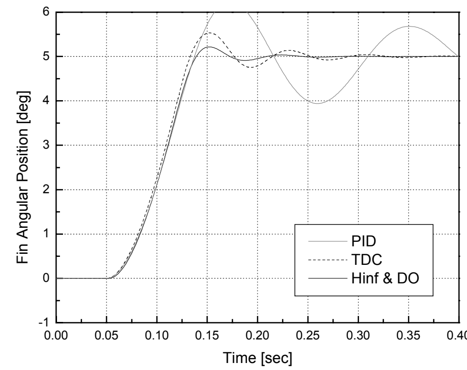


(a) No Load

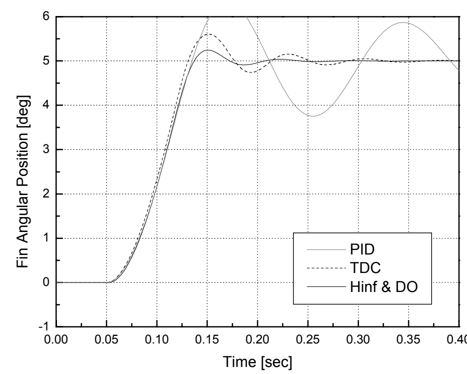


(b) Spring load (1000/15 [lb-in/deg])

Fig. 5. Step Responses (Simulation)



(a) variation of R_M ($R_M \rightarrow R_M \times 3$)



(b) variation of J_e ($J_e \rightarrow J_e \times 3$)

Fig. 6. Step responses in the presence of motor parameter variations (Simulation)

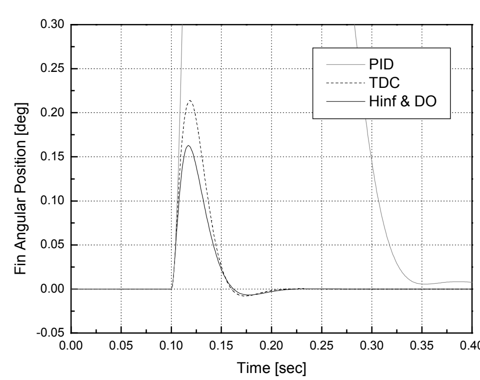
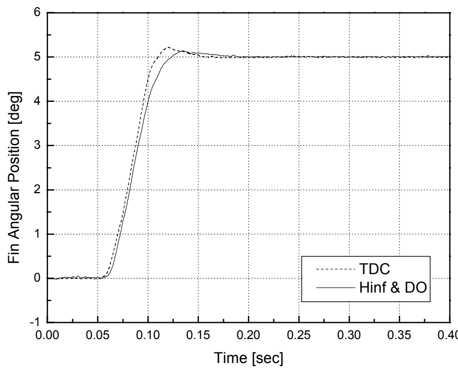
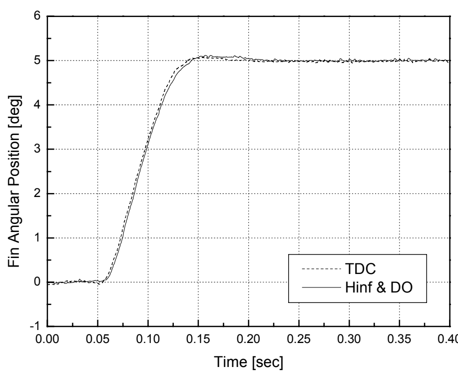


Fig. 7. Transient responses to a step disturbance input (Simulation)



(a) No Load



(b) Spring load (1000/15 [lb-in/deg])

Fig. 5. Step Responses (Experiment)

V. CONCLUSION

In this paper, a robust controller for a fin position servo system of a missile has been designed. The designed controller has a two-degree of freedom control structure and uses an H_∞ controller and a disturbance observer. Also a new disturbance observer scheme has been proposed and applied to the fin position servo system.

The performance of the designed controller has been examined through simulations and experiments. Its performance has been compared with a PID and a TDC controller. The simulation and experimental results show that the designed controller gives faster and more accurate responses especially in the presence of system parameter variations and external disturbances.

Currently extensive experimental tests are being performed under various conditions, and we will add the experimental results later on.

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