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A Noncausal Approach for the Improvement of PID Control Performances

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***Abstract*— A new approach for the improvement of the set-point following performances achieved by a PID controller is presented in this paper. Basically, the devised methodology consists of designing, by means of a stable dynamic inversion procedure, a suitable command signal to be applied to the closed-loop control system in order to achieve a desired transient response when the process output is required to assume a new value. A closed-form solution of the problem is obtained, making the technique suitable to be applied in the industrial context. Simulation and experimental results show that high performances are obtained despite the presence of model uncertainties and, above all, independently on the PID tuning. Thus, the PID gains can be selected in order to guarantee good load rejection performances without impairing the set-point transient responses.**

I. INTRODUCTION

Proportional-Integral-Derivative (PID) controllers are the most adopted controllers in industry, due to the good cost/benefit ratio they are able to provide. To help the operator to select the controller gains to address given control specifications, many tuning formulas have been devised in the past [1] and autotuning functionalities are almost always available in commercial products [2], [3]. However, it is well-known that good performances both in the set-point following and in the load disturbances rejection task are often difficult to achieve at the same time. To solve this problem, which is of concern in many applications, the typical approach is to adopt a two degrees-of-freedom controller, namely, to adopt a feedforward (linear) compensator [4]. The use of the well-known set-point weighting strategy [5] falls in this framework. The main disadvantage of this method is that the reduction of the overshoot is paid by a slower set-point response. To overcome this drawback, the use of a variable set-point weight [6], [7] or of a feedforward action [3], [8], [9], [10] has been proposed.

In this paper we propose to use a dynamic inversion based approach to determine a suitable command signal to be applied to the closed-loop (PID based) control system, instead of the typical step signal, in order to achieve high-performances (i.e. low rise time and low overshoot at the same time) when the process output is required to assume a new value. To better understand the framework of the proposed method and the differences with the usual approaches, assume that the process variable is required

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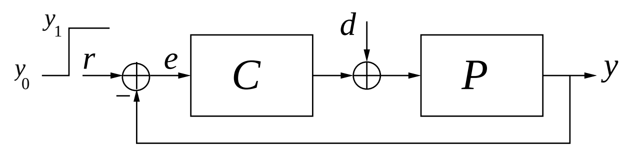

Fig. 1. The classic one degree-of-freedom PID based control scheme.

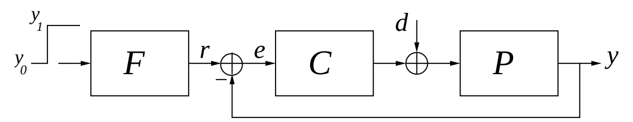

Fig. 2. The typical two degrees-of-freedom PID based control scheme.

to assume a steady-state value y_1 starting from a steady-state value y_0. The standard unity-feedback control scheme based on a one degree-of-freedom PID controller is shown in Figure 1, where P is the process, y is the process output and C is the PID controller. The typical two degrees-of-freedom PID based control scheme is reported in Figure 2, where F is a second order system [11]; note that the use of a set-point weight is equivalent to filtering the step signal to be applied to the closed-loop system. The control scheme related to the new technique is shown in Figure 3. Conversely to the other methods, here a step signal is not employed, but the knowledge of y_1 is adopted by a command signal generator block to calculate a command signal r to be applied to the closed-loop PID control system in order to guarantee a high performance transient response. Basically, the new method consists of choosing a desired function to achieve a process output transition from y_0 to y_1 and then determining the command function r that causes the desired transition by inverting the closed-loop dynamics by means of a stable inversion procedure. Note that the concept of dynamic inversion [12], [13] has been already proven to be effective in different areas of the automatic control field, such as motion control [14], [15], flight control [16], robust control [17], [18].

The paper is organized as follows. The overall methodology is explained in Section 2, while the stable dynamic inversion procedure is presented in Section 3. Simulation results are shown and discussed in Section 4 and experimental results are presented in Section 5. Conclusions are drawn in the last section.

II. METHODOLOGY

A. Modeling

As a first step of the devised method, the process to be controlled is modelled as a first-order plus dead-time

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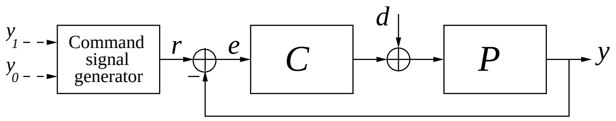

Fig. 3. The new dynamic inversion based control scheme.

(FOPDT) transfer function, i.e.:

$$P(s;K,T,L) = \frac{K}{Ts+1}e^{-Ls}. \tag{1}$$

This is a typical choice in industrial practice and a variety of methods, based on simple experiments, for the identification of the parameters K, T and L are available (e.g. the well-known area method [3] based on the open-loop step response). Then, in order to have a rational transfer function, the dead-time term is approximated by means of a second order Padé approximation. In this way, the approximated process transfer function results to be:

$$\tilde{P}(s;K,T,L) \cong \frac{K}{Ts+1}\frac{1-Ls/6+L^2s^2/12}{1+Ls/6+L^2s^2/12}. \tag{2}$$

B. Tuning the PID controller

In order to apply the dynamic inversion based methodology that will be presented in the next, the PID controller can be tuned according to any of the many methods proposed in the literature or even by hand. However, since the purpose of the dynamic inversion procedure is the attainment of high performances in the setpoint following task, disregarding of the controller gains, it is sensible to select the PID parameters aiming only at obtaining good load rejection performances.

The PID controller transfer function be denoted as follows:

$$C(s;K_p,T_i,T_d,T_f) = K_p\left(1+\frac{1}{T_is}+T_ds\right)\frac{1}{T_fs+1} \tag{3}$$

where K_p is the proportional gain, T_i is the integral time constant, T_d is the derivative time constant and T_f is the time constant of a first order filter that makes the transfer function proper.

C. Output function design

As a desired output function that defines the transition from a setpoint value y_0 to another y_1 (to be performed in the time interval $[0,\tau]$) we choose a "transition" polynomial [19], i.e a polynomial function that satisfies boundary conditions and that is parameterized by the transition time τ. In the following, without loss of generality we will assume $y_0 = 0$, Formally, define

$$y(t) = c_{2k+1}t^{2k+1} + c_{2pk}t^{2k} + \cdots + c_1t + c_0$$

The polynomial coefficients can be uniquely found by solving the following linear system, in which boundary conditions at the endpoints of interval $[0,\tau]$ are imposed:

$$\begin{cases} y(0) = 0; & y(\tau) = y_1 \\ y^{(1)}(0) = 0; & y^{(1)}(\tau) = 0 \\ \vdots & \\ y^{(k)}(0) = 0; & y^{(k)}(\tau) = 0 \end{cases}$$

The results can be expressed in closed-form as follows ($t \in [0,\tau]$):

$$y(t;\tau) = y_1\frac{(2k+1)!}{k!\tau^{2k+1}}\sum_{i=0}^{k}\frac{(-1)^{k-i}}{i!(k-i)!(2k-i+1)}\tau^it^{2k-i+1}. \tag{4}$$

The order of the polynomial can be selected by imposing the order of continuity of the command input that results from the dynamic inversion procedure [19]. Specifically, if the plant is modelled as a FOPTD transfer function (see (1)), its relative degree is equal to one. Taking into account that the relative degree of the PID controller is zero, the relative degree of the overall closed-loop system is one. Thus, a third order polynomial ($k = 1$) suffices if a continuous command input function is required, i.e.:

$$y(t;\tau) = y_1\left(-\frac{2}{\tau^3}t^3+\frac{3}{\tau^2}t^2\right). \tag{5}$$

Outside the interval $[0,\tau]$ the function $y(t;\tau)$ is equal to 0 for $t < 0$ and equal to y_1 for $t > \tau$.

Remark 1. The value of τ can be selected by solving an optimization problem where the transition time has to be minimized subject to actuator constraints [19]. Alternatively, from a more practical point of view, the choice of the transition time τ can be made by the user either directly or through a (possibly) more intuitive reasoning. For example, the user might select a ratio between the bandwidth of the open-loop system and that of the closed-loop one, from which the value of τ can be determined easily. In any case, parameter τ represent a very desirable feature from a user point of view, as it allows to handle the trade-off between performance, robustness and control activity [20], [21].

III. STABLE DYNAMIC INVERSION ALGORITHM

At this point we address the problem of finding the command signal $r(t;K,T,L,K_p,T_i,T_d,T_f,\tau)$ that provides the desired output function (5). For the sake of clarity of notation, in the following we will omit the dependence of the functions and of the resulting coefficients from the parameters K, T, L, K_p, T_i, T_d, T_f. The closed-loop transfer function be denoted as

$$H(s) := \frac{C(s)\tilde{P}(s)}{1+C(s)\tilde{P}(s)} = K_1\frac{b(s)}{a(s)} \tag{6}$$

where $b(s)$ and $a(s)$ are monic polynomials. As $H(s)$ is nonminimum phase, a stable dynamic inversion procedure is necessary, that is a bounded input function has to be found in order to produce the desired output. Denote the set of all cause/effect function pairs $(r(\cdot),y(\cdot))$ associated to $H(s)$ by

$\mathcal{B}$. Now, in order to perform the stable inversion, we rewrite the numerator of the transfer function (6) as follows:

$$b(s) = b_-(s)b_+(s)$$

where $b_-(s)$ and $b_+(s)$ denote the polynomials associated to the zeros with negative real part (i.e. those of the PID controller) and positive real part (i.e. those of the Padè approximation) respectively. Now, consider the inverse system of (6) whose transfer function can be written as:

$$H(s)^{-1} = \gamma_0 + \gamma_1 s + H_0(s)$$

where γ_0 and γ_1 are suitable constants and $H_0(s)$, a strictly proper rational function, represents the zero dynamics. This can be uniquely decomposed according to

$$H_0(s) = H_0^-(s) + H_0^+(s) = \frac{c(s)}{b_-(s)} + \frac{d(s)}{b_+(s)}$$

where $c(s)$ and $d(s)$ are suitable polynomials. The modes associated to $b^-(s)$ and $b^+(s)$ be denoted by $m_i^-(t)$, $i = 1, 2$, and by $m_i^+(t)$, $i = 1, 2$ respectively. Being $\mathcal{L}$ the Laplace transform operator, define:

$$\eta_0^-(t) := \mathcal{L}^{-1}[H_0^-(s)]$$
$$\eta_0^+(t) := \mathcal{L}^{-1}[H_0^+(s)].$$

The following propositions and the following theorem represent the solution to the stable dynamic inversion problem.

Proposition 1:

$$\int_0^t \eta_0^+(t-v)y(v;\tau)dv = H_0^+(0)y(t;\tau) + \frac{1}{\tau^3}\sum_{i=1}^2 p_i^+(\tau)m_i^+(t) + \frac{1}{\tau^3}T_0^+(t,\tau) \quad (7)$$

where

$$T_0^+(t,\tau) = \begin{cases} \sum_{i=0}^1 s_i^+(t)\tau^i & \text{if} \quad t \in [0,\tau] \\ \sum_{i=1}^2 q_i^+(\tau)m_i^+(t-\tau) & \text{if} \quad t > \tau \end{cases} \quad (8)$$

and $p_i^+(\tau)$, $q_i^+(\tau)$, $i = 1, 2$ are suitable τ-polynomials and $s_i^+(t)$, $i = 0, 1$ are suitable t-polynomials.

Proposition 2:

$$\int_0^t \eta_0^-(t-v)y(v;\tau)dv = H_0^-(0)y(t;\tau) + \frac{1}{\tau^3}\sum_{i=1}^2 p_i^-(\tau)m_i^-(t) + \frac{1}{\tau^3}T_0^-(t,\tau) \quad (9)$$

where

$$T_0^-(t,\tau) = \begin{cases} \sum_{i=0}^1 s_i^-(t)\tau^i & \text{if} \quad t \in [0,\tau] \\ \sum_{i=1}^2 q_i^-(\tau)m_i^-(t-\tau) & \text{if} \quad t > \tau \end{cases} \quad (10)$$

and $p_i^-(\tau)$, $q_i^-(\tau)$, $i = 1, 2$ are suitable τ-polynomials and $s_i^-(t)$, $i = 0, 1$ are suitable t-polynomials.

Theorem 1: The function $r(t;\tau)$ defined as

$$r(t;\tau) = -\frac{1}{\tau^3}\sum_{i=1}^2 p_i^+(\tau)m_i^+(t) - \frac{1}{\tau^3}\sum_{i=1}^2 q_i^+(\tau)m_i^+(t-\tau) \quad \text{if} \quad t < 0 \quad (11)$$

$$r(t;\tau) = \gamma_1 Dy(t;\tau) + \gamma_0 y(t;\tau) + H_0(0)y(t;\tau) + \frac{1}{\tau^3}\sum_{i=0}^1 (s_i^+(t) + s_i^-(t))\tau^i - \frac{1}{\tau^3}\sum_{i=1}^2 q_i^+(\tau)m_i^+(t-\tau) + \frac{1}{\tau^3}\sum_{i=1}^2 p_i^-(\tau)m_i^-(t) \quad \text{if} \quad t \in [0,\tau] \quad (12)$$

$$r(t;\tau) = \gamma_0 + H_0(0) + \frac{1}{\tau^3}\sum_{i=1}^2 p_i^-(\tau)m_i^-(t) + \frac{1}{\tau^3}\sum_{i=1}^2 q_i^-(\tau)m_i^-(t-\tau) \quad \text{if} \quad t > \tau. \quad (13)$$

is bounded over $(-\infty, +\infty)$ and $(r(t;\tau), y(t;\tau)) \in \mathcal{B}$.

Proofs of the above propositions and of the above theorem are not reported for brevity [22].

Summarizing, the determined function $r(t; K, T, L, K_p, T_i, T_d, T_f, \tau)$ exactly solves the stable inversion problem for FOPDT plants (in which the dead-time term has been approximated) controlled by a PID controller (3) and for a family of output functions, which depend on the free transition time τ.

Actually, from a practical point of view, in order to use the synthesized function (11)-(13), it is necessary to truncate it, resulting therefore in an approximate generation of the desired output $y(t;\tau)$. This can be done with arbitrarily precision given any couple of small parameters $\varepsilon_0 > 0$ and $\varepsilon_1 > 0$. Indeed, compute

$$t_0 := \max\{t' \in \mathbb{R} : |r(t;\tau)| \le \varepsilon_0 \quad \forall t \in (-\infty, t']\}$$

and define

$$t_s := \min\{0, t_0\}.$$

Similarly, compute

$$t_f := \min\{t' \in \mathbb{R} : |r(t;\tau) - y_1| \le \varepsilon_1 \quad \forall t \in [t', \infty)\}$$

Hence, the approximate reference signal to be actually used is

$$r_a(t;\tau) := \begin{cases} 0 & \text{for} \quad t < t_s \\ r(t;\tau) & \text{for} \quad t_s \le t \le t_f \\ y_1 & \text{for} \quad t > t_f. \end{cases}$$

Note that it normally occurs that $t_s < 0$, resulting in the so-called "preaction control" [23], and therefore the inverse input is noncausal.

Remark 2. It is worth stressing again that the overall stable dynamic inversion procedure can be performed by means of a symbolic computation, i.e. a closed-form expression of $r(t; K, T, L, K_p, T_i, T_d, T_f, \tau)$ results. Indeed, the actual command signal to be applied for a given plant and a given controller is determined by substituting the actual value of the parameters into the resulting closed-form expression.

Remark 3. The choice of using a second order Padè approximation is motivated by keeping the expression of $r(; K, T, L, K_p, T_i, T_d, T_f, \tau)$ as simple as possible while guaranteeing at same time high performances.

Remark 4. The presented stable dynamic inversion procedure is based on a general one [22], where $H(s)$ can be the rational transfer function of any (stable) system, provided that there are not purely imaginary zeros. Thus, the proposed approach can be straightforwardly applied also to integral and unstable processes, as it is based on the inversion of the dynamics of the closed-loop system $H(s)$. Analogously, the same method can be trivially extended to PI, P and PD control.

Remark 5. As said in Remark 4, the devised method can be extended also to high-order processes. Thus, a more

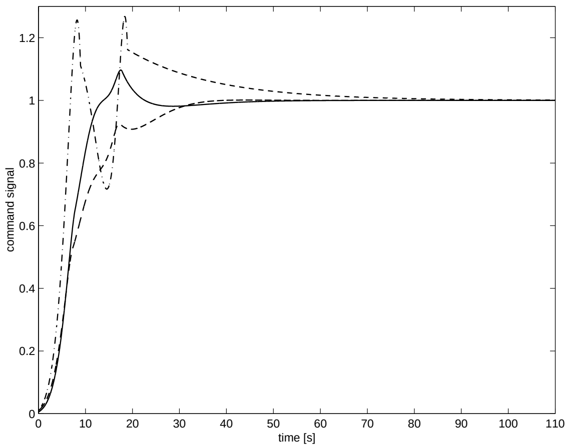

Fig. 4. The determined command signals for the three considered PID tuning for the FOPDT system.

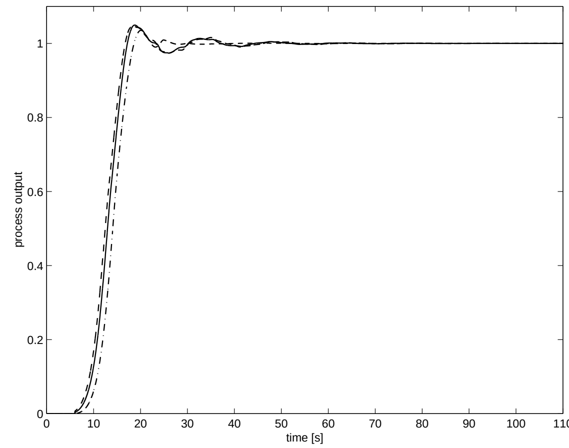

Fig. 5. The resulting process outputs with the new method for the three considered PID tuning for the FOPDT system.

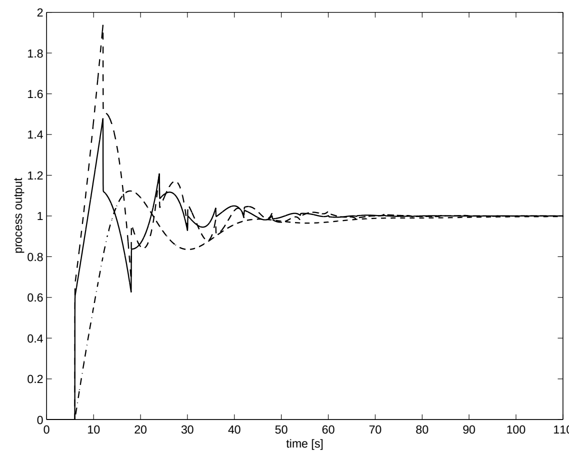

Fig. 6. The resulting process outputs with the classic method for the three considered PID tuning for the FOPDT system.

accurate model of the process, if available, can be fully exploited. However, in this case, the dynamic inversion procedure has to be performed on purpose and this might prevent the use of the method in single-station controllers. Conversely, if a FOPDT model is employed, the determined general expression of $r(t; K, T, L, K_p, T_i, T_d, T_f, \tau)$ can be used.

IV. SIMULATION RESULTS

In the following examples the process output has to perform a transition from 0 to $y_1 = 1$.

A. FOPDT system

Consider the following FOPDT process:

$$P_1(s) = \frac{1}{10s+1}e^{-6s}. \qquad (14)$$

To prove the effectiveness of the method with different PID tunings, three sets of PID parameters have been considered, namely, the Ziegler-Nichols step response PID formula ($K_p = 2$, $T_i = 12$, $T_d = 3$), the Ziegler-Nichols step response PI formula ($K_p = 1.5$, $T_i = 18$, $T_d = 0$), and the one that results from the minimization of the ISTE integral criterion for the load disturbance rejection [24] ($K_p = 2.41$, $T_i = 7.33$, $T_d = 2.74$). In each case it has been set $T_f = 0.01$. By setting $\varepsilon_0 = 0.01$, the resulting values of the preaction time in the three cases are $t_s = -7.8$, $t_s = -8.9$ and $t_s = -7.3$ respectively (note that, for convenience, the time axis has been properly shifted in order to have $t_s = 0$). The determined command functions are reported in Figure 4 (solid line: Ziegler-Nichols PID; dash-dot line: Ziegler-Nichols PI; dashed line: integral criterion minimization) and the corresponding process outputs are plotted in Figure 5. For the sake of comparison, the process outputs resulting with the classic method, i.e. by applying a step set-point signal are shown in Figure 6.

B. High-order system

In order to evaluate the robustness of the method to unstructured uncertainties, consider the following FOPDT process:

$$P_1(s) = \frac{1}{(s+1)^8}. \qquad (15)$$

By applying the area method, a FOPDT transfer function has been estimated, resulting in $K = 1$, $T = 3.04$ and $L = 4.97$. With respect to these parameters, the same tuning formula as before has been adopted, resulting in $K_p = 0.73$, $T_i = 9.93$, $T_d = 2.48$ for the Ziegler-Nichols PID tuning, $K_p = 0.55$, $T_i = 14.90$, $T_d = 0$ for the Ziegler-Nichols PI tuning, and $K_p = 1.06$, $T_i = 4.26$, $T_d = 2.48$ for the minimization of the ISTE integral criterion. In each case it has been set $T_f = 0.01$. By setting again $\varepsilon_0 = 0.01$, the resulting values of the preaction time in the three cases are $t_s = -5.19$, $t_s = -6.55$ and $t_s = -5.04$ respectively. The command functions determined by applying the dynamic inversion procedure are reported in Figure 7 and the corresponding process outputs are plotted

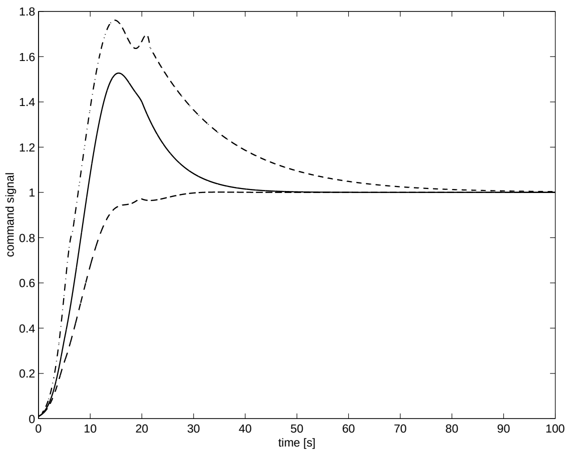

Fig. 7. The determined command signals for the three considered PID tuning for the high-order system.

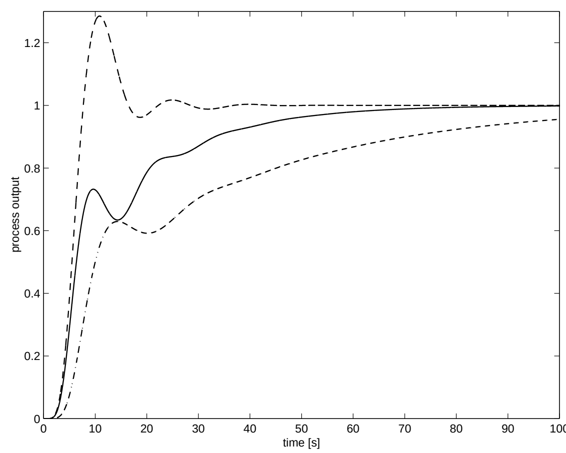

Fig. 9. The resulting process outputs with the classic method for the three considered PID tuning for the high-order system.

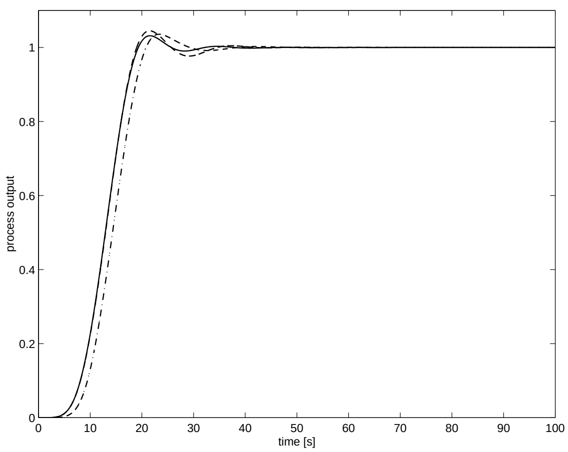

Fig. 8. The resulting process outputs with the new method for the three considered PID tuning for the high-order system.

in Figure 8. The process outputs resulting with the classic method, i.e. by applying a step set-point signal are shown in Figure 9. The same line types as before have been adopted also in this example.

C. Discussion

From the presented results, it emerges that the main feature of the proposed methodology is that it is able to provide predefined performances regardless of the values of the PID parameters. Actually, it appears that using a step input very different performances are obtained with different tuning formulas, whilst the use of the inversion-based command input provides very similar transient responses, with both low overshoots and settling times (note that this is a major difference with respect to other feedforward approaches). This fact can be exploited for an effective decoupling of the set-point following and load disturbances rejection tasks and to simplify the overall control system design as the determination of the command signal is performed automatically.
It has also to be noted that, despite the command signal is calculated by considering a FOPDT model of the process, the overall approach is robust to unstructured uncertainties, as the example related to the high-order system demonstrates. Thus, it appears that the devised method can be applied in general for a wide range of plants.

V. EXPERIMENTAL RESULTS

In order to prove the effectiveness of the devised technique in practical applications, a laboratory experimental setup (made by KentRidge Instruments) has been employed. Specifically, the apparatus consists of small perspex tower-type tank (whose area is 40 cm^2) in which a level control is implemented by means of a PC-based controller. The tank is filled with water by means of a pump whose speed is set by a DC voltage (the manipulated variable), in the range 0-5 V, through a PWM circuit. The tank is fitted with an outlet at the base in order for the water to return to a reservoir. The measure of the level of the water is given by a capacitive-type probe that provides an output signal between 0 (empty tank) and 5 V (full tank). Since the apparent dead-time of the system is very small with respect to its dominant time constant, in order to provide a significant result, a time delay of 10 s has been added via software at the plant input.
Despite the model is nonlinear (as the flow rate out of the tank depends on the square root of the level), a FOPDT model has been estimated by applying the area method to the open-loop response with a step from 2 V to 2.5 V at the process input. The FOPDT model obtained is

$$P_3(s) = \frac{1.98}{29s+1}e^{-11s}$$

Based on this model, it has been set $K_p = 1.24$, $T_i = 31$, $T_d = 0$). The derivative action has not been employed due to the excessive measurement noise. In order to obtain a process output transition from 2 V to 3 V (starting when the process is at the steady-state), which corresponds approximately to a level transition from 6 cm to 12 cm, it has been fixed $\tau = 10$ s and the inversion-based command

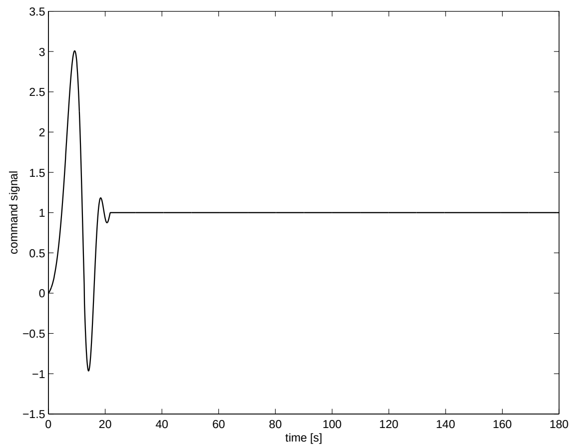

Fig. 10. The determined command signals for the level control experiment.

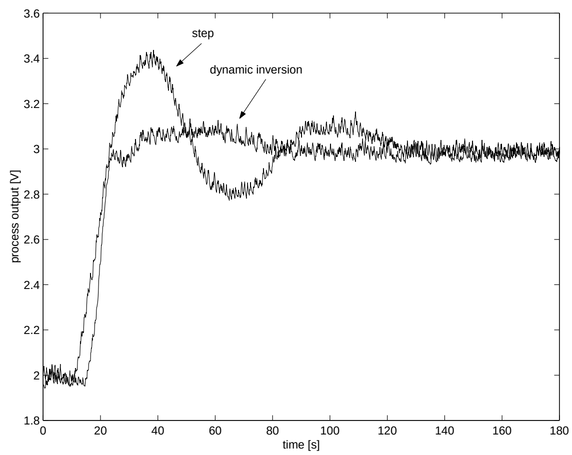

Fig. 11. The resulting process outputs with the new and the classic method for the level control experiment.

signal has been determined. It is shown in Figure 10 (the preaction time is $t_s = -1.7$s). The corresponding process output, together with the step response is plotted in Figure 11. A significant reduction of the overshoot and of the settling time appears. It turns out that these experimental results confirms the effectiveness of the methodology.

VI. CONCLUSIONS

The computational power available nowadays allows to implement more and more easily useful additional features in the context of industrial controllers. In this paper, we propose to use a noncausal (i.e. based on a stable dynamic inversion procedure) approach to determine an appropriate command signal that can be adopted instead of the typical step signal in order to achieved predefined high performances in the set-point following task. This is achieved regardless of the values of the PID parameters, so that the overall design is greatly simplified. It is believed that the devised technique can be implemented as a "one-shot" procedure in single-station controllers and in Distributed Control Systems. Further, the generality of the method makes it suitable to be extended also to more complex control architecture such as cascade control.

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