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Implementation of a SHM method on a numerical model of a cable-stayed bridge

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***Abstract*—This paper discusses the implementation of a structural health monitoring (SHM) technique on a numerical model of the Bill Emerson Memorial Bridge. The method uses the natural excitation technique (NExT) and the eigensystem realization algorithm (ERA) to identify the modal parameters of the structure. A least squares solution of the eigenvalue problem is used to detect elemental structural parameters from the identified modal parameters. Damage is identified by comparing the elemental parameters before and after damage. It is found that the singular values of the Hankel matrix are not a reliable approach to determine the number of modal parameters identified with the ERA for this type of structure. A variation of the method is used to differentiate between computational values due to noise and numerical errors and those representing the structural characteristics.**

I. Introduction

CURRENTLY visual inspection is the used to assess the condition of structures such as buildings and bridges. This method requires a considerable amount of manpower and special equipment, and is thus a slow, expensive and a subjective process. When several structures need to be inspected promptly after a severe natural event, (e.g. an earthquake, or hurricane) the process might take several days or weeks, impeding rescue efforts to affected areas and potentially disrupting the economy. This implies a need to develop faster, more reliable and less expensive strategies to improve the structural inspection process.

Structural Health Monitoring (SHM) uses innovative technologies to determine the existence, type, extent and location of damage in structures. The implementation of such technologies optimizes the resources for structural inspection, giving owners vital information for decision making. Large structures, such as cable-stayed bridges and skyscrapers, are the first structures in which SHM methodologies would be implemented because of their importance and cost. This type of structure has complex dynamic behavior (i.e. low frequencies, closely spaced modes, etc.), increasing the challenges to overcome by SHM methods based on the structure's dynamics.

This paper discusses the implementation of a SHM method to detect damage on a numerical model of a cable-stayed bridge. Accelerations are used to simulate measurement of the structural responses. The bridge used for this study is the Bill Emerson Memorial Bridge, located in Cape Girardeau, Missouri. Construction of the bridge was finished in January of 2004 and instrumentation will be permanently installed, opening the possibilities of implementing the described methodology on the real structure. A finite element model of the bridge, developed based on detailed drawings of the structure [4], [5], is used in simulations to generate response data. A simpler second identification model is used for damage detection. The SHM method is described, the finite element model is discussed, and the main results are presented. The method has been found to be effective for the detection of damage in the bridge structure. Additionally it is implementable on substructures, or portions of the bridge [3], making this technique attractive for such applications.

II. SHM Method

The SHM method used in this paper employs the eigenvalue problem of the undamped equations of motion to identify stiffness values. Changes in the identified stiffness values will indicate the existence, location and extent of damage in a structure. Techniques currently available in the literature are used to obtain the modal parameters of the structure from acceleration records, validating their use for this type of civil structure.

A. Modal Parameters Identification

The first step of the SHM method is the identification of modal parameters. The Natural Excitation Technique (NExT) and the Eigensystem Realization Algorithm (ERA) are used herein. NExT allows one to obtain data that can be treated as free responses from a structure when the input is not measured, or is actually unmeasurable. Here the excitation is assumed to be stationary with frequency content that spans the modes of vibration of the structure,

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and uncorrelated with prior responses of the structure. The technique was developed by James et al. [7], and shows that the matrix of correlation functions between the responses of the system and a response to be selected to be the reference response is a solution of the homogenous matrix equation of motion. The ERA was developed by Juang and Pappa in 1985 [8], and it has been shown to be an effective method for modal identification of flexible structures. This algorithm uses the principles of minimum realization to obtain a state space representation of the system using the free response data obtained from NExT. The natural frequencies and mode shapes of the structure are calculated from the state space representation. In the calculations of the realization of the system a singular value decomposition of the Hankel matrix is performed. These singular values can be normally used to determine the number of identified modal parameters. Higher singular values will correspond to information from the structure while low singular values are numerical modes.

B. *Elemental Structural Parameters*

Parameters of the structural elements are obtained in the second part of the method. For this step, a least squares solution of the eigenvalue problem is used [2], [3]. The problem is formulated using a finite element model of an identification model, for straightforward implementation. In this paper the structural parameter to be identified is Young's modulus of the element, although other parameters such as area or moment of inertia can be obtained using this technique. Consider the undamped eigenvalue problem

$$(\mathbf{K} - \lambda_i \mathbf{M})\phi_i = 0 \tag{1}$$

where **K** and **M** are the n by n stiffness and mass matrices, respectively, and λ_i and ϕ_i are the i-th eigenvalue and eigenvector. Equation (1) can be written as

$$\mathbf{K}\phi_i = \lambda_i \mathbf{M}\phi_i \tag{2}$$

Typically the number of identified modes is significantly smaller than the number of degrees of freedom in the identification model. Thus, from (2) it is not generally possible to determine **K** because there are $n(n+1)/2$ unknowns (due to symmetry), and there are only n x m equations, where m is the number of identified natural frequencies and mode shapes. Equation (2) can be rewritten as

$$\theta_i r = \lambda_i \mathbf{M}\phi_i \tag{3}$$

where θ_i is a matrix formed using the elements of the i-th eigenvector ϕ_i and r is a vector of the unknown parameters. The vector of unknown parameters r can be obtained using

$$r = \Theta^{-P}\Gamma \tag{4}$$

where $(\ )^{-P}$ denotes the pseudoinverse and

$$\Theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_m \end{bmatrix} \quad \Gamma = \begin{bmatrix} \lambda_1 M\phi_1 \\ \lambda_2 M\phi_2 \\ \vdots \\ \lambda_m M\phi_m \end{bmatrix} \tag{5}$$

If more equations are available than unknowns, the solution is equivalent to a least squares solution for the unknown parameters.

As shown by Caicedo [2] [3], the procedure to obtain the matrix θ_i in (3) can be automated in a way similar to the development of the finite element method. First, identification matrices for each element of the finite element identification model are obtained. Then, a coordinate transformation is performed to transform local coordinates to global coordinates. The next step is to assemble a global identification matrix with all the elemental matrices. Finally, constraint equations and boundary conditions are applied. One of the advantages of this methodology is the ability to include known information of structural parameters to reduce the number of unknowns and the possibility to apply the methodology to a portion of the structure reducing the number of sensors needed for implementation.

Figure 1. Construction of the Bill Emerson Memorial Bridge
Source: http://www.modot.state.mo.us/local/d10/emersonbridge

III. Bridge Description

The cable-stayed bridge considered in this paper is the Bill Emerson Memorial Bridge. The bridge is located in Cape Girardeau, Missouri spanning the Mississippi River on Missouri 74–Illinois 146. The bridge was designed by the

HNTB Corporation [6] and construction was completed in January of 2004. Instrumentation will be installed on the Emerson Bridge and surrounding soil to evaluate the structural behavior and seismic risk.

The bridge is composed of two towers, 128 cables, and 12 additional piers in the approach bridge from the Illinois side. It has a total length of 1205.8 m (3956 ft.) with a main span of 350.6 m (1150 ft.) and side spans of 142.7 m (468 ft.) in length. The approach on the Illinois side is 570 m (1870 ft.). The bridge has four lanes plus two narrower bicycle lanes, for a total width of 29.3 m (96 ft.). The deck is composed of steel beams and prestressed concrete slabs. Steel, ASTM A709 grade 50W, is used, with an f_y of 344 MPa (50 ksi). The concrete slabs are made of prestressed concrete with a f'_c of 41.36 MPa (6 ksi). Additionally, a concrete barrier is located in the center of the bridge, and a railing is located along the edges of the deck.

A. *Finite Element Model*

Based on detailed drawings of the Emerson Bridge, a three-dimensional finite element model was developed in Matlab [10]. The model was originally developed for the benchmark problem in structural control of cable-stayed bridges under seismic excitation [4], [5] and has been modified to study the advantages and disadvantages of different SHM techniques. A linear model is used in this paper. However, the stiffness matrices used in this linear model are those of the structure determined through a nonlinear static analysis corresponding to the deformed state of the bridge with dead loads [13]. Cable-stayed bridges behave nonlinearly under dead loads even when the material is in the linear range. These nonlinearities are due to three major factors: i) nonlinear behavior in the cables; ii) beam-column effects due to large compression forces; and iii) large displacements producing changes in geometry [11]-[13]. All these factors were considered in the development of this finite element model of the Emerson Bridge. Additionally, the bridge is assumed to be attached to bedrock, and the effects of soil–structure interaction are neglected.

The finite element model employs beam elements, cable elements, rigid links and lumped masses. The nonlinear static analysis is performed in Matlab using a finite element toolbox developed at Washington University. The finite element model, shown in Fig. 2, has a total of 572 nodes, 418 rigid links, 156 beam elements, 198 nodal masses and 128 cable elements. The towers are modeled using 224 nodes, 80 beam elements and 144 rigid links. Constraint equations are applied to restrict the deck from moving in the lateral and vertical directions and rotate with respect to the *x* axis at all supports. Because the attachment points of the cables to the deck are above the neutral axis of the deck, and the attachment points of the cables to the tower are outside the neutral axis of the tower, rigid links are used to connect the cables to the tower and to the deck. The use of the rigid links ensures that the length and inclination angles of the cables in the model agree with the drawings. Additionally, the moment induced in the towers by the movement of the cables is taken into consideration in this approach. In the case of variable sections, the average of the section is used for the finite element.

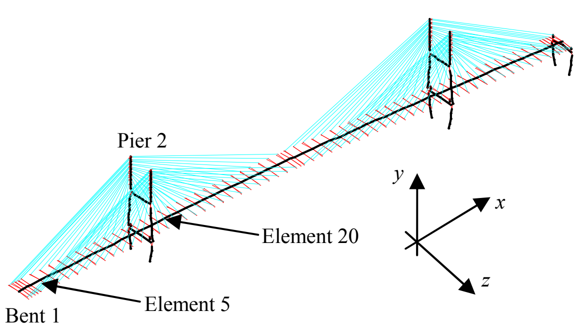


Figure 2. Finite Element Model

The deck was modeled using the method described by Wilson and Gravelle [13]. In this approach the deck is modeled as a central beam (the spine) which has no translational mass. Lumped masses are employed to model the mass of the deck, and are connected to the spine using rigid links. The masses are included to more realistically model the torsional response of the deck to lateral loads, and have been shown to be important in the modeling of this structure [1].

This paper focuses on damage of the deck, although the applied methodology could readily be applied to other locations. Damage in the structure is simulated by reducing Young's modulus of certain structural elements. Two case scenarios are studied herein. In the first case Young's modulus of element 5 (see Fig 2) is reduced by 30% and in the second case Young's modulus of element 20 is reduced by 20%.

B. *Identification Model*

To develop the most effective identification model several candidates were constructed for testing. Each candidate was tested using the theoretical natural frequencies and mode shapes and the least square solution of the eigenvalue problem. The first identification model was constructed using the same degrees of freedom as are in the finite element model. Subsequently, degrees of freedom in the identification model were restrained to determine detect the effects of modeling errors in the structural parameter identification process.

The results of this comparative study indicated that when either rotational degrees of freedom with respect to the *z* axis, or displacement with respect to the *y* axis are restrained, the modeling errors in the structure are large and damage can not be identified.

The identification model selected was also modified to study the possibility of dividing the deck into several substructures. This model representing a substructure, or portion of the deck, considers only the degrees of freedom of the deck between Bent 1 and Pier 2. Damage between elements 1 and 16 may be identified by this model, and damage outside these elements should not affect the identified change in stiffness between Bent 1 and Pier 2. The successful development of techniques that are applicable to substructures would allow for methods that can focus on critical parts of the structure or locations which are difficult access. This identification model has a total of 62 DOF.

IV. NUMERICAL RESULTS

A. Modal Identification

NExT and ERA are applied using 30 minutes of acceleration records for the identification of natural frequencies, mode shapes and damping ratios. A sampling rate of 3Hz was used for these records, which is capable of measuring the first 11 vertical modes of the structure. When calculating the cross spectral density function, a boxcar window of 512 points and 75% overlapping between frames is used. In the ERA, 77 points of the cross spectral density functions are used to form a 40 by 1200 Hankel matrix. In selecting the size of the Hankel matrix, special care was taken to select only the part of the cross correlation function that clearly shows a decay, avoiding the "noisy" data at the end of the cross-correlation function as shown in Figure 3.

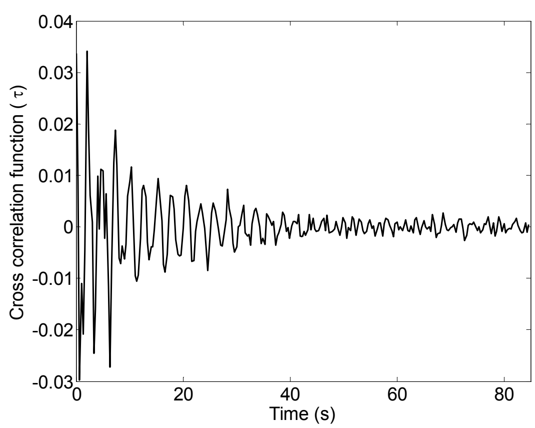


Figure 3. Typical cross correlation function

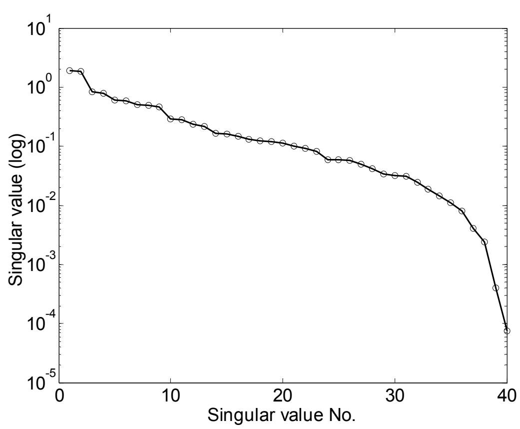


Figure 4. Singular values of the Hankel matrix

Figure 4 provides a typical plot of the singular values of the Hankel matrix for the ERA calculations. Note that the singular values decrease gradually in contrast to the sharp decreases observed in other structures. Theoretically, 11 vertical modes would be identified in this exercise producing a jump after the 22nd singular value. This effect is not observed in this example. By examining the distribution of the singular values alone, it is not possible to determine the number of frequencies to be identified.

NExT is applied by selecting a single reference channel. This reference channel should be selected at a point on the structure that is far from a node of vibration. In buildings the obvious solution would be a sensor located on the roof of the structure, but for this implementation the selection of the reference channel is not obvious. Only one column of the full cross correlation function matrix will be used for one reference channel, but the same method can be applied to all the columns of the cross correlation function matrix by selecting each channel as the reference. This approach will result in l sets of identified natural frequencies and mode shapes, where l is the number of reference channels.

Figure 5.a shows the identified natural frequencies using each channel of data as a reference channel. Correct natural frequencies appear at the same frequency for several reference channels while numerical errors appear as isolated frequencies in few reference channels. The natural frequencies of the structure can be obtained using the histogram of the identified natural frequencies shown in Fig. 5.b

Table 1 shows the natural frequencies identified using NExT and ERA. Different sets of natural frequencies are identified for the different damage cases. The 6th, 9th, 10th and 11th modes are not identified for the healthy structure; the 4th, 6th, 10th, and 11th modes are not identified for the first damage case; and for the second damage case the 1st, 8th, and 11th modes are not identified. Note that the least squares solution of the eigenvalue problem can be applied, even though the same modes are not obtained in each case.

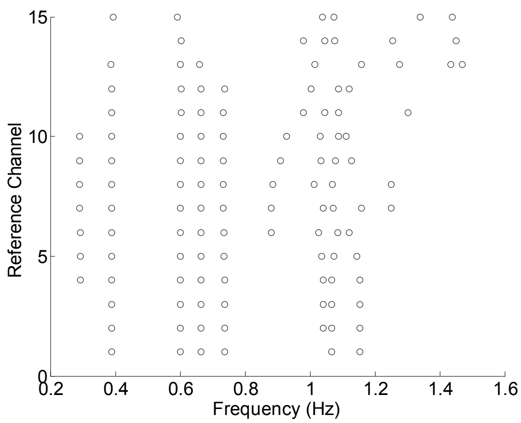
TABLE I
IDENTIFIED NATURAL FREQUENCIES FOR DAMAGE SCENARIOS

Freq. No	Undamaged (Hz)	30% at 5 (Hz)	20% at 20 (Hz)
1	0.2922	0.2937	-
2	0.3892	0.3882	0.3936
3	0.6024	0.5967	0.6064
4	0.6639	-	0.6594
5	0.7353	0.7300	0.7242
6	-	-	0.8925
7	1.0326	1.0378	1.0255
8	1.0762	1.0737	-
9	-	1.0899	1.0969
10	-	-	1.2703

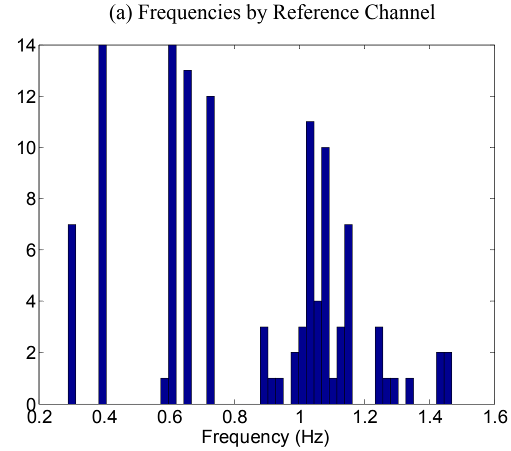
Similarly, the identified mode shapes are slightly different for various reference channels. This variation can be used as a measure of the accuracy of the identified mode shape. Figure 6 shows the average, upper bound, and lower bound of the 1st and 6th identified modes for the undamaged case. To compute the average, the identified modes were each normalized to have a maximum value equal to one. After the average is computer, each mode is then scaled with respect to the maximum point of the average mode to compute the deviation. The maximum difference between the upper bound and the lower bound for the 1st mode shape is 0.028, and for the 6th mode shape is 0.254 indicating that the 1st mode shape is more precisely identified. This error between the upper and lower bounds can be used as a measure to accept or reject an identified mode. It was found that closely spaced modes present higher errors in the estimation of the mode shapes than modes that are separate

B. Structural Parameters

Figure 7 shows the identified loss in stiffness in the damage scenarios. Damage can be clearly located and quantified through the loss in the stiffness of element number five for the first damage case, as shown in Fig. 7.a. The second damage case did not significantly affect the identified

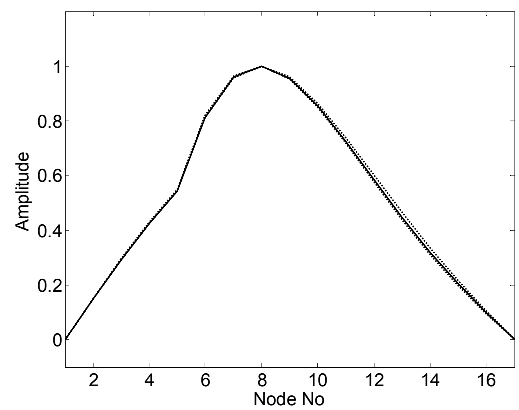


(a) Frequencies by Reference Channel

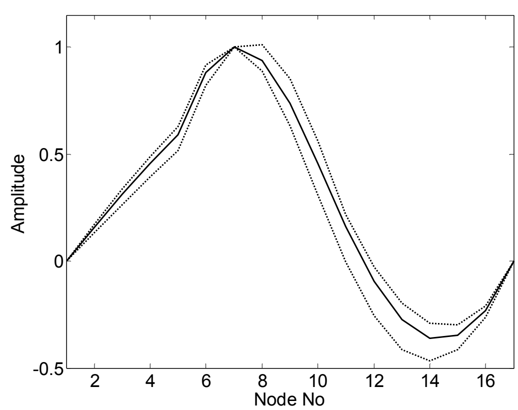


(b) Histrogram

Figure 5. Identified natural frequencies with different reference channels



(a) 1st mode shape



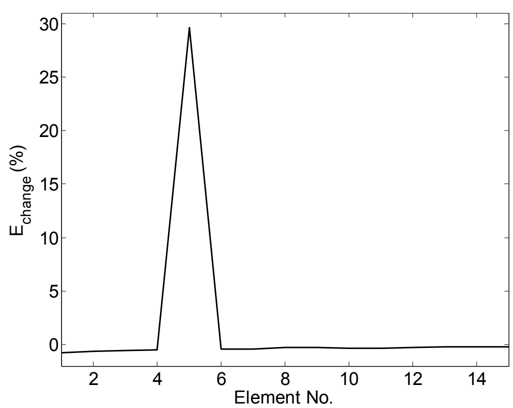
(b) 6th mode shape

Figure 6. Identified mode shapes

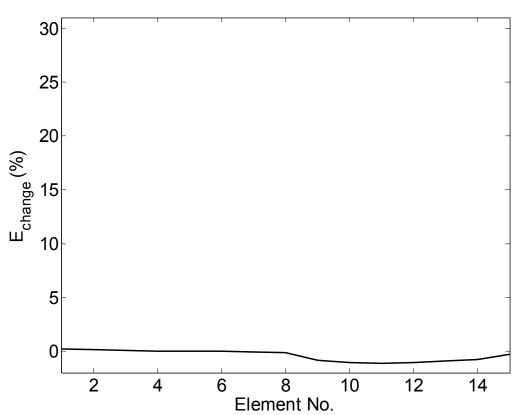
change in stiffness between Bent 1 and Pier 2, indicating that the methodology can be applied to a substructure (see Fig. 7.b).

V. CONCLUSIONS

This study demonstrates the efficacy for structural health monitoring techniques as applied to cable-stayed bridges. The results demonstrate that this technique can be applied to detect damage in the bridge when an appropriate identification model is employed. The NExT and ERA correctly identified natural frequencies and mode shapes of the structure using 30 minute records with a sampling frequency of 3Hz. Long records are needed in this type of structure because of their low frequency behavior, although a high sampling rate is not necessary. Seven of the eleven existing modes in the frequency range considered were identified for the undamaged structure. Even though not all of the available modes were identified, the identification of damage was accurate. The identified mode shapes using two different reference channels vary somewhat. A set of natural frequencies and mode shapes for a particular mode can be obtained by applying the methodology using different reference channels. A lower bound and upper bound for the estimated mode shapes can be obtained from this set, indicating the variability in the identification process.



(a) 30% reduction in element 5



(b) 20% reduction in element 20

Figure 7. Stiffness Change

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