Design of A Novel Simply Structured Mixed-Sensitivity Loop-shaping Robust Controller for a Gas-Turbine

Amin Nobakhti Neil Munro*
Control Systems Centre
University of Manchester Institute of Science and Technology
Manchester M60 1QD, UK
multivariable@IEEE.org

***Abstract*— A previously developed technique for the design of simply structured robust controllers; that is robust in the sense of meeting some given singular value constraint; is extended to deal with the mixed-sensitivity case. The method presented uses the Gershgorin disks of the system and can result in relatively simple controllers. The method is illustrated by application to the control of an aircraft gas-turbine engine.**

I. INTRODUCTION

In [1], it was shown that if $A \in \mathbf{C}^{m\times m}$ is any real or complex matrix, the following inclusion region would always hold true for its singular values $\forall x \geq 0$,

$$\{\min(a_{ii}) - D_{max}(A)\} \leq x \leq \{\max(a_{ii}) + D_{max}(A)\} \quad (1)$$

where,

$$D_{max}(A) = \max\{C_{max}(A) \quad , \quad R_{max}(A)\} \quad (2)$$
$$C_{max}(A) = \max_j \sum_{\substack{i=1 \\ i\neq j}}^{m} |\ a_{ij}\ | \quad , \quad R_{max}(A) = C_{max}(A^T)$$

Remark 1: Observe that $D_{max}(A)$ equals the radius of the largest column or row Gershgorin disk of A. Thus, within the band covered from $\{\min(a_{ii}) - D_{max}(A)\}$ to $\{\max(a_{ii}) + D_{max}(A)\}$ lies all the singular values of A.

This interesting result was combined with the technique of Diagonal Dominance [2] to propose a novel technique for design of simply structured robust controllers. The technique was then used to design a controller for a 2 input 2 output model of an automotive gas-turbine which was able to meet some criteria on the complementary sensitivity function such as a constraint on its H_∞ norm. In this work, the technique is extended, and is shown how it could be used with equal effectiveness to design controllers able to meet *mixed-sensitivity* constraints. The example used here, to demonstrate this, is a 3 input 3 output model of a jet engine.

*The authors thank EPSRC, UMIST and the IEE for supporting this research.

II. THE COMPOSITE BODE PLOT

In connection with this technique, the Composite Bode Plot [1] was proposed as an effective tool for the design stage; called composite, because it can be used both for the first stage of the design process which is maximizing dominance, and also for the second part which is the shaping of the singular values.

To check for dominance in the Nyquist array, it is required that the Gershgorin disks do not include the origin for dominance. In other words, for $Q(s) = G(s)K(s)$ one has to show that

$$|\ q_{ii}(jw)\ | - \sum_{\substack{j=1 \\ j\neq i}}^{n} |\ q_{ji}(jw)\ | \geq 0, \quad \forall w \in w_b, \quad (3)$$

which is equivalent to showing that

$$|\ q_{ii}(jw)\ | > \sum_{\substack{j=1 \\ j\neq i}}^{n} |\ q_{ji}(jw)\ |, \quad \forall w \in w_b. \quad (4)$$

It should be noted that the last inequity just requires that, for a given column and at a given frequency, the sum of the modulus of the off-diagonal responses is less than the diagonal one. Further, unlike the Nyquist array, in a Bode framework, one is able to exactly 'read off' the frequency at which dominance is lost and has a much better idea of the system's interactions across the entire frequency band. Thus, using the Composite Bode Plot, one can effectively assess and measure the amount of system diagonal dominance even more accurately than from the Nyquist array. Hence, the two elements always contained in the Composite Bode Plots are the magnitude of the response of the diagonal elements and of the Gershgorin bands (which at each frequency equals the sum of the magnitude of the off-diagonal responses)

In addition to these two features, the upper and lower singular value bounds may also be plotted. However, typically only the bounds that the given specifications concern are plotted. For example, if the specifications are only on the maximum singular value (typical H_∞ scenario) then only the upper Gershgorin singular value bound is plotted. In addition, it should be noted that the Composite Bode Plot may be plotted for any configuration of the system. For example, for a system $G(s)$ and controller $C(s)$, one may wish to plot the CBP of $Q(s)$, $I(I+Q(s))^{-1}$ or $Q(s)(I+Q(s))^{-1}$; the latter two being the the complementary sensitivity and the sensitivity functions, respectively.

III. The Rolls Royce Spey Gas-turbine engine

In this example, the proposed design technique will be applied to a complex and highly interacting multivariable system; namely, the twin spool Rolls-Royce RB.168 Spey Mk. 202 Gas-Turbine engine. The state space composite model (engine + actuators) contains 21 states. It has three inputs; Fuel Flow (FF), Inlet Guide Vanes (IGVs) and Nozzle Area (NA); and three outputs; Low-pressure spool speed (%NL), High-pressure spool speed (%NH) and Surge margin (SM). The control exercise is to control the three outputs with the three inputs in the order that they are given as this -through dynamic RGA analysis- was found to be the optimal I/O pairing. The engine model is highly non-linear and thus has been linearised at several operating points. %NH has been chosen as the output with respect to which these operating points are defined. All models are detectable and stabilizable. However, they are non-minimum phase. The particular model which is the subject of this example is linearized at the 87% NH operating point. This engine has been subject to various design studies, including a dominance based Linear Parameter Varying controller [3], an Evolutionary Computing design multivariable PI controller [4], and a two-degree of freedom H_∞ based controller [5]. In this example, it is aimed to demonstrate the potential of this new technique by attempting a *mixed-sensitivity* design for the engine. That is to say, criteria will be imposed not only on the singular values of the complementary sensitivity function $T(s)$, but also on the singular values of the sensitivity function and the proposed design technique will be used to design a multivariable controller which meets the constraints on the singular values of both of these functions. Here, the standard definition for $T(s)$ and $S(s)$ are used; namely,

$$\begin{aligned} T(s) &= (I+G(s)C(s))^{-1}G(s)C(s) \\ S(s) &= I - T(s) \end{aligned} \tag{5}$$

The following are the design constraints set on $T(s)$ and $S(s)$,

- $\bar{\sigma}(S(j\omega)) < 1.5\ \forall\omega$.
- bandwidth on $\bar{\sigma}(T(s))$ of 2 rads/sec
- bandwidth on $\bar{\sigma}(S(s))$ of 1 rads/sec
- minimum roll off of 20dBs/decade for both $S(s)$ and $T(s)$.
- $\bar{\sigma}(T(j\omega))$-$\underline{\sigma}(T(j\omega)) < 50dB, \quad \forall\omega < 100rads/sec$

The first criterion ensures that the peak over shoot of the system is limited and, in particular, if the plant has resonant poles, that they are well damped. Note, however, since $T(s)+S(s)=I$, it follows that $\mid 1-\bar{\sigma}(S(s)) \mid \leq \bar{\sigma}(T(s)) \leq 1+\bar{\sigma}(S(s))$ and $\mid 1-\bar{\sigma}(T(s)) \mid \leq \bar{\sigma}(S(s)) \leq 1+\bar{\sigma}(T(s))$, which shows that $\bar{\sigma}(S(s))$ is large only if $\bar{\sigma}(T(s))$ is large and vice versa. Therefore, the constraint imposed here on $\bar{\sigma}(S(s))$ will also constrain $\bar{\sigma}(T(s))$. The second and third criteria are to ensure that the system responses will have a minimum guaranteed speed, and the fourth criterion ensures zero steady-state error to a step command. Finally, the last criterion ensures that the gain spread of the systems responses at all frequencies is limited. In turn, this will limit the difference between the slowest and fastest closed-loop system time constant.

First, consider the open loops step responses of the Spey engine, and its open loop DNA shown respectively in Figures 1 and 2.

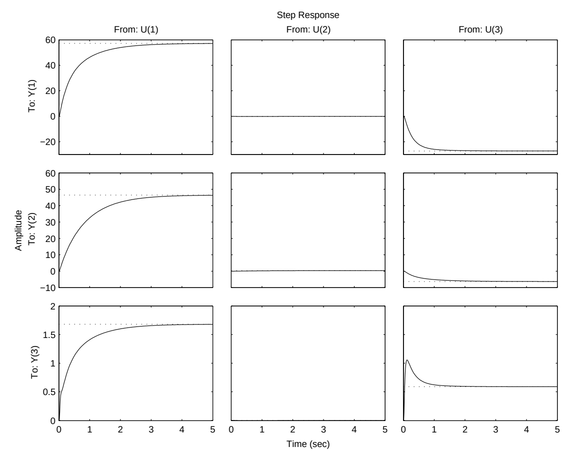

Fig. 1. Open-loop step response of the Spey engine

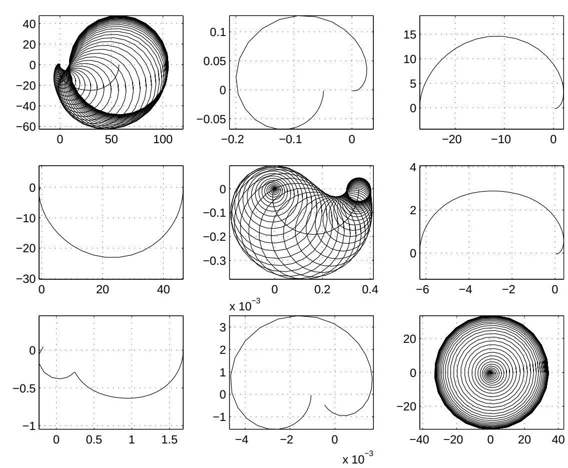

Fig. 2. Open-loop DNA of the Spey engine (0-100 rads/sec)

Clearly the system is heavily interacting and, in its initial condition, it is violating all the criteria on performance and robustness. However, one is *not* at this point concerned with *any* of the other performance criteria and the first step of the design process only has one aim; namely, to design a pre-compensator to maximise the amount of open-loop system dominance. Indeed, this is one of the advantages of using dominance for the design of controllers; namely, its ability to break down an otherwise demanding task, into several manageable and easy to understand and follow tasks. In this context, this advantage has also been harnessed to allow us to shape the singular values of the system. In this case, a technique proposed in [6] was used to design the following pre-compensator

$$K(s) = \begin{pmatrix} \frac{0.000724(s+9)}{s} & \frac{0.0185(s+0.35)}{s(s+3)} & \frac{0.00459(s+70)}{s} \\ -\frac{0.24626(s+5)}{s} & \frac{0,1338(s+19)}{s} & -\frac{1.0497(s+28)}{s} \\ -\frac{0.00052(s+40)}{s} & -\frac{0.0118(s+0.5)}{s(s+8)} & \frac{0.020221(s+36)}{s} \end{pmatrix}.$$

This pre-compensator, although of much lower order relative to the plant, achieves very high levels of dominance and reduces the interactions significantly. This can be seen clearly from the DNA of $G(s)K(s)$ and its Composite Bode Plot, shown in Figures 3 and 4.

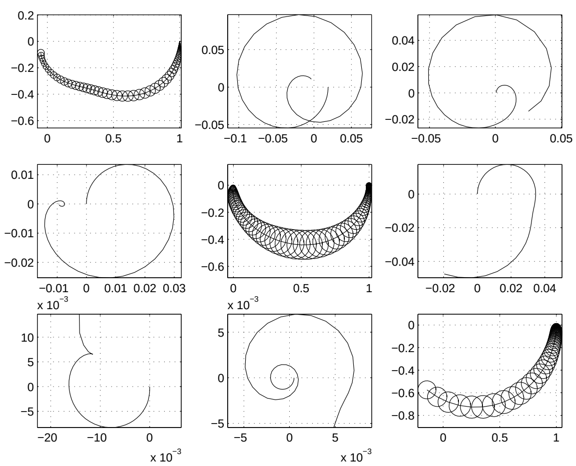

Fig. 3. NA of $G(s)K(s)$

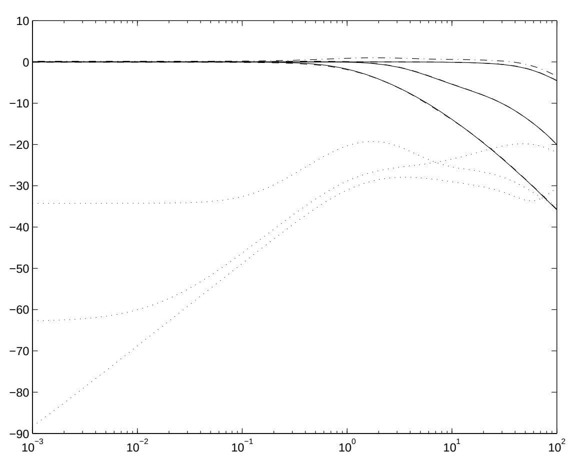

Fig. 4. CBP of $G(s)K(s)$ (where in this and all following Composite Bode Plots, the solid lines represent the diagonal elements, the dotted lines are the Gershgorin bands and the dashed line is the singular value upper bound)

It is clear that the dominance levels are very high. However, the resulting open-loop gains are varying by a large amount and currently only the fifth criterion is met. In the second stage, three loop-shaping SISO controllers are designed to meet all the performance and robustness criteria. In proceeding with the next section of the design process, it is noted that as mentioned previously $T(s)$ and $S(s)$ are constrained such that their sum must always be equal to one. This means there is a limitation on how much these can be *independently* manipulated to meet some given criteria [5], and that simultaneous independent control over both of these is not possible. This can be viewed both as a hinderance and also as an aid in the design process. In this case, one can use this to advantage by using it to break down the task of designing the SISO controllers for the mixed-sensitivity design. Namely, the SISO controllers will first only be designed to meet the criteria on the complementary sensitivity function, $T(s)$. The sensitivity function, $S(s)$, will then be checked to see if any of the criteria are violated. If so, the SISO controller will be modified to meet the criteria on $S(s)$ and, after the modification, $T(s)$ will be checked with the new SISO controllers so see if the system still satisfies this criteria. This process of iteration will be repeated until the SISO controllers satisfy both the criteria on (the singular values of) $T(s)$ and $S(s)$. Therefore, for the next part, initially the criteria which are imposed on $T(s)$ are to be achieved.

First, to meet the roll off criterion, three integrators will simply be added to each loop. Thus, initially the SISO controllers are[1],

$$Kc^1(s) \quad = \quad diag\left\{\frac{1}{s}, \frac{1}{s}, \frac{1}{s}\right\}. \qquad (6)$$

The composite Bode plot of $G(s)K(s)Kc^1(s)$ is shown in Figure 5, and the Composite Bode Plot of $T(s)$ with this controller is shown in Figure 6. It is clear that the

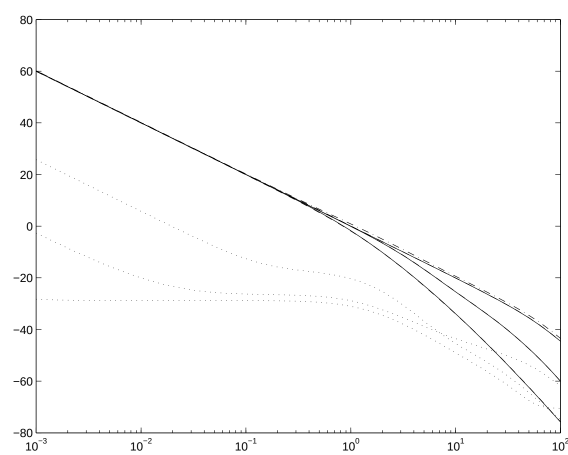

Fig. 5. CBP of $G(s)K(s)Kc^1(s)$

second and the third criteria have been met more than satisfactorily. However, the bandwidth is less than 1 rads/sec and needs to be increased. This can be achieved very simply by increasing the gain of the SISO controllers. In this case,

[1]the superscript [1] denotes the controller at iteration 1

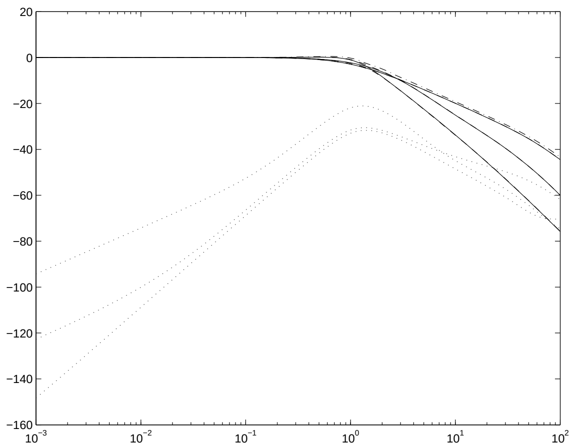

Fig. 6. CBP of $T(s)$ with $Kc^1(s)$

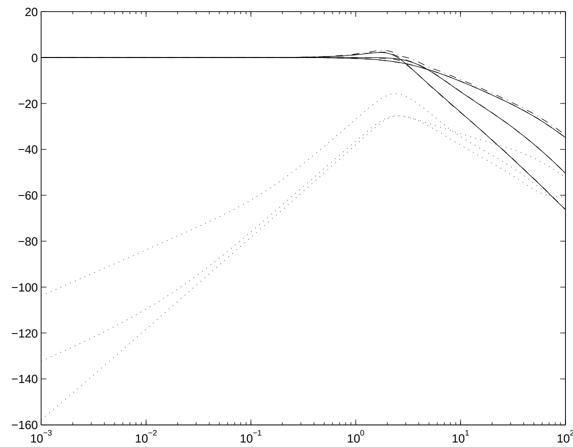

Fig. 7. CBP of $T(s)$ with $Kc^2(s)$

for example, all the loop gains were increased to three. Hence, the set of SISO controllers becomes,

$$Kc^2(s) \quad = \quad diag\left\{\frac{3}{s},\frac{3}{s},\frac{3}{s}\right\}. \tag{7}$$

Before proceeding with the remainder of this section, an important point is emphasized here. Essentially, one is addressing a compromise in this design procedure; namely, designing the SISO controllers for $T(s)$, looking at $S(s)$ and 'compromising' (or modifying) the SISO controllers if $S(s)$ violated some constraint. However, the critical aspect of this iteration process is for the designer not to be oblivious to the effect that choices made on the values of the SISO controller, with regards to, $T(s)$ will have on $S(s)$, and vice versa. An example of this was the choice of the gains above. The design requirements call for a bandwidth of at least 2 rads/sec on the largest singular value of $T(s)$. Obviously, one can achieve high bandwidth by just using higher gain values, thus not only meeting the criterion on the bandwidth, but exceeding it many fold. However, the price for this will surely be very high resonant peaks in both $T(s)$ and $S(s)$. Since the peak overshoot is *not* a criterion on $T(s)$, evidently the SISO controllers with high gain will not violate any of the criteria on $T(s)$ and indeed better it. However, once $S(s)$ is checked, where the peak overshoot *is* one of the criteria, then the designer will almost certainly be forced to reduce the gain of the SISO controllers. Hence, as mentioned, the designer should be aware of the effect the choices made with regards to $T(s)$ will have on $S(s)$ and vice versa.

The Composite Bode Plot of $T(s)$ with $Kc^2(s)$ is shown in Figure 7. The very simple controller $Kc^2(s)$ achieves a closed-loop bandwidth of 4 rads/sec, which is twice the required value. Further, there is minimum roll off of 20dBs/dec and the largest gain spread is about 35dBs which is well within the specifications. Hence, all the objectives on $T(s)$ have been met, and now one has to ensure the same will be true for $S(s)$.

Consider now Figure 8, which shows the composite Bode plot of $S(s)$ with the controller $Kc^2(s)$.

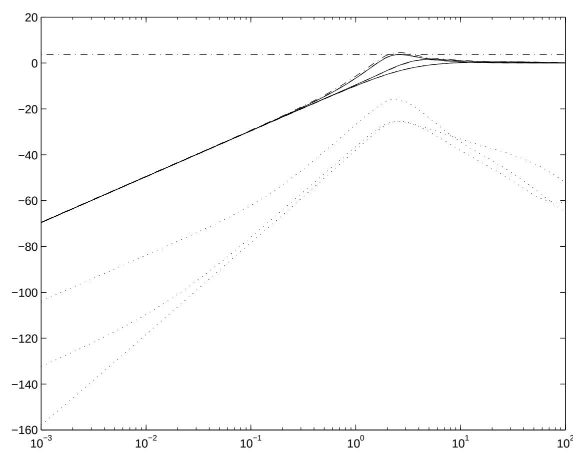

Fig. 8. CBP of $S(s)$ with the controller $Kc^2(s)$

From this figure, it can be seen that the amounts of dominance on $S(s)$ are so high that for most of the frequency range the singular value bound is sitting right on top of the largest diagonal element and this means there will be minimal conservatism involved in later choices. From this figure, it can also be seen that the bandwidth is roughly 1 rads/sec. However the singular value upper bound has the maximum value of 4.568dBs. This value is the guaranteed upper bound on the largest singular value of $S(s)$. Hence, if one is to ensure that $\bar{\sigma}(S(s))$ never goes more than 1.5 (3.5dBs), the upper bound must accordingly not rise above this value. Evidently, this means that $Kc^2(s)$ does not meet the specifications on $S(s)$, and must be modified accordingly. Here, the advantages that this technique offers becomes apparent. Using the Composite Bode Plot, it is only a matter of course to see that at the peak, the closest diagonal element response is that of the second loop; i.e. at the frequency of the peak on the singular value upper bound, the second diagonal element has the highest gain.

Since the upper bound is plotted on the largest element at each frequency, by simply reducing the gain on the second loop, the peak caused by the diagonal element of the second loop will be reduced, and the guaranteed upper bound will also be lowered. The technique thus gives the designer clear information on how each element of the SISO controllers will effect the singular values of $T(s)$ and $S(s)$.

Having identified that the singular value peak is caused by a large gain in the second loop, we simply reduce the gain of the SISO controller for the second loop from 3 to 2 to meet the specifications on $S(s)$. Hence, $Kc^3(s)$ becomes,

$$Kc^3(s) \quad = \quad diag\left\{\frac{3}{s}, \frac{2}{s}, \frac{3}{s}\right\}. \qquad (8)$$

The Composite Bode Plot of $S(s)$ with $Kc^3(s)$ is shown in Figure 9.

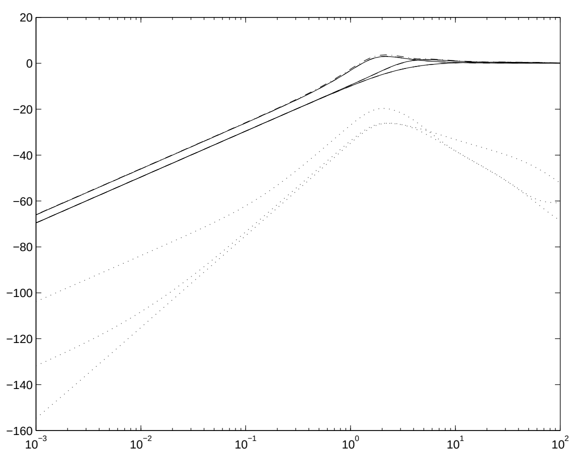

Fig. 9. CBP of $S(s)$ with $Kc^3(s)$

The maximum value the singular value upper bound takes is now 3.5dBs, which is only 0.1 dB larger than the specification and deemed satisfactory. Finally, one must check $T(s)$ with the new controller $Kc^3(s)$ to ensure than the gain reduction has not violated any specifications on $T(s)$; specifically to have reduced the bandwidth on the largest singular value to less than 2 rads/sec. The Composite Bode plot of $T(s)$ with $Kc^3(s)$ is shown in Figure 10, where it can be confirmed that this has not been the case.

Hence, in the end, all of the design specifications have been met with *only* a first order pre-compensator and three integrators. Indeed, had the specifications been more demanding, or the required exactness been higher one might have needed to resort to more complex compensators such as PIs, lead or lags. Nonetheless, these represented a real set of specifications on a real system and served to highlight one of the advantages of using this technique; namely, that the resulting controllers are no more complex than necessary as dictated by the specifications. The final step responses of the closed loop system with the pre-compensator $K(s)$ and the controller $Kc^3(s)$, are shown in Figure 11.

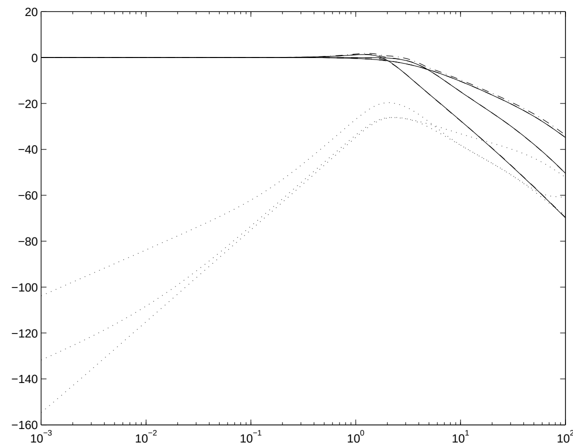

Fig. 10. CBP $T(s)$ with $Kc^3(s)$)

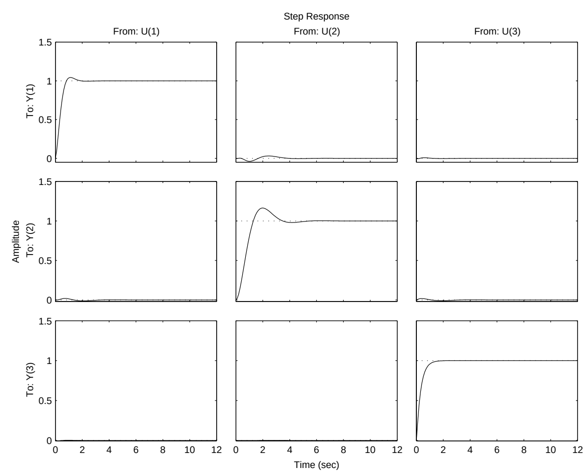

Fig. 11. Final closed loop step responses of the Spey

It should be noted from the step responses that *all* the conclusions about the system, which were made based on the Composite Bode Plot have been correct and may be confirmed from the step responses. First, notice how there are virtually no interactions. It was possible to see this from the Composite Bode Plot, because the singular value upper bound was very close to the maximum diagonal element and the Gershgorin bands were very low in magnitude. Secondly, notice that the three diagonal step responses have roughly the same time constant. Again, this could be predicted from the Composite Bode Plot because, in the specification, the gain spread of $T(s)$ and $S(s)$ was limited. In addition, it can be seen that for the majority of the bandwidth, the three diagonal elements are approximately of the same size. Thirdly, and perhaps most importantly, notice that the largest overshoot occurs in the second loop. Again, it was possible to readily 'predict' this from the composite Bode plot and the Gershgorin upper bound on the singular values.

IV. Final Remarks

In this work, it has been successfully demonstrated how the technique proposed in [1] can be used for the design of *mixed-sensitivity simply structured multivariable robust controllers* for complex and difficult systems. In particular, the advantages of this technique may be summarised by saying that,

- The complexity of the resulting controller is entirely dependant on the designer and the given specification. That is to say, the controller will only contain sufficient dynamics to meet the specifications.
- The technique breaks down the complex and tedious task of shaping the singular values, into two stages; namely, the first stage of designing the pre-compensator for which the designer can choose from a rich variety of techniques available, and the second stage of loop-shaping, for which simple knowledge of SISO loop-shaping is required. Hence, the technique gives the designer much greater insight and control over the system, without actually involving the use of a new tool.

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