# Variable parameter EW-RLS algorithm with dead zone for the trajectory tracking of the joints of the manipulator

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*Abstract*—A variable parameter RLS algorithm with dead zone is presented in this paper. The concept of the error level is proposed. Selection criteria of the error level are given according to the min-max principle. Good performance of the improved algorithm is verified by the experiments carried out on the robot. With it, a self-tuning PID controller is used for the trajectory tracking of the joints of the manipulator. Good tracking performance shows the effectiveness of the improved RLS algorithm.

# I. INTRODUCTION

PARAMETER tracking of time-invariant & time-varying system is still a vory active system is still a very active research field in the recent years. Although there are many new identification algorithms such as identification using genetic algorithms [1,2], fuzzy logic and wavelet theory, it take a lot time to compute estimation parameters and this makes them useless in the motion control. The most effective identification algorithms in the motion control are still RLS and gradient algorithms, which search along the direction of gradient [3,4]. These procedures are feasible when system parameters change slowly. If the parameters change fast, these algorithms will result in too small a data set that will lead to numerical problems. In this situation, the selection of the gain and/or forgetting factor becomes crucial. In general, a large gain and/or a small forgetting factor make the RLS and gradient algorithms to have a better ability for tracking the variation of parameters, but also make them sensitive to noise. On the other hand, a small gain and/or a big forgetting factor make the algorithms less sensitive to noise, but at the same time result in a poor tracking ability for slowly time-varying systems, for instance, the system with some infrequent but abrupt parameter changes. To solve this problem, a lot of papers have been published along the direction of using variable gain and variable forgetting factor [5-9].

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However, it is crucial to select the error levels corresponding to the variable algorithm parameters. Error selection of the error levels will lead the variable parameter algorithm to be useless, i.e, it cannot meet the fast tracking requirement and noise overcoming requirement simultaneously. In this paper, we present a new method which adopts min-max estimation algorithm to estimate the error levels, this solves the problems better. Our paper is organized as follows. An improved EW-RLS algorithm with variable parameters and dead zone is presented in section 2. A brief introduction of the experimental platform is given in section 3. Identification experiments and the results are given in section 4. Two experiments of the trajectory tracking of the joint of the manipulator and their results are given in section 5. The paper is concluded in section 6.

# II. VARIABLE PARAMETER EW-RLS ALGORITHM WITH DEAD ZONE

# A. Model description

A stochastic autoregressive model with exogenous input, or ARX model, is given by

$$A(q^{-1})y(k) = B(q^{-1})u(k) + \xi(k)$$
(1)

Where  $q^{-1}$  is the backward shift operator, y(k) is the output signal, u(k) is the input signal,  $\xi(k)$  is noise with variance,  $\sigma^2$  independent of u(k), and

$$A(q^{-1}) = 1 + a_1(k)q^{-1} + \dots + a_n(k)q^{-n}$$
<sup>(2)</sup>

$$B(q^{-1}) = b_1(k)q^{-1} + \dots + b_m(k)q^{-m}$$
(3)

are polynomials with time varying coefficients. Assume the true parameter vector is:

$$\theta = (a_1(k), \dots, a_n(k), b_1(k), \dots, b_m(k))$$
(4)

And the parameter vector to be identified is:

$$\theta = (\hat{a}_1(k), \dots, \hat{a}_n(k), b_1(k), \dots, b_m(k))$$
(5)

## B. Variable parameter EW-RLS algorithm with dead zone

Consider system (1), according to the previous discussion and Canudas de Wit's improved EW-RLS[10] algorithm, a

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variable parameter EW-RLS algorithm with dead zone is presented. Let:

$$0 < \varepsilon(1) < \varepsilon(2) < \dots < \varepsilon(s) \tag{6}$$

$$0 < \lambda(1) < \lambda(2) < \dots < \lambda(s) = 1 \tag{7}$$

$$0 = \alpha(1) < \alpha(2) < \dots < \alpha(s) \tag{8}$$

the proposed algorithm can be concluded as follows:

$$e_k = y(k) - \phi^T(k-1)\hat{\theta}(k-1)$$
 (9)

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{a_k P_{k-1} \phi_k}{1 + \phi_k^T P_{k-1} \phi_k}$$
(10)

$$P_{k} = \frac{1}{\lambda_{k}} \left[ P_{k-1} - \frac{\alpha_{k} P_{k-1} \phi(k) \phi^{T}(k) P_{k-1}}{1 + \phi^{T}(k) P_{k-1} \phi^{T}(k)} \right]$$
(11)

Where,  $\alpha_k$  denotes gain and  $\lambda_k$  denotes forgetting factor. And we have:

$$\alpha_{k} = \begin{cases}
\alpha(s) & \text{if } |e_{k}| > \varepsilon(s) \\
\dots & \dots \\
\alpha(2) & \text{if } \varepsilon(1) < |e_{k}| \le \varepsilon(2) \\
\alpha(1) & \text{if } |e_{k}| \le \varepsilon(1)
\end{cases}$$

$$\lambda_{k} = \begin{cases}
\lambda(s) & \text{if } |e_{k}| > \varepsilon(s) \\
\dots & \dots \\
\lambda(2) & \text{if } \varepsilon(1) < |e_{k}| \le \varepsilon(2) \\
\lambda(1) & \text{if } |e_{k}| \le \varepsilon(1)
\end{cases}$$
(12)
(13)

#### C. Parameter selection

In the variable parameter EW-RLS algorithm with dead zone, the selection of error levels is very important. If  $\varepsilon(i)$ , i=1,...,s, is to be selected too large, then the algorithm will be very sensitive to the noise. On the other hand, if  $\varepsilon(i)$ , i=1,...,s, is selected to be too small, then the algorithm will track very slowly. Generally,  $\varepsilon$  can be defined by min-max theory offline[11]. The error  $\varepsilon(k, \theta)$  is defined as

$$\varepsilon(k,\boldsymbol{\theta}) = y(k) - \boldsymbol{\varphi}^{\mathrm{T}}(k)\boldsymbol{\theta} \quad k=1,\dots,N$$
(14)

In the bounded-error context, the error should satisfy

$$\left|\varepsilon(k,\boldsymbol{\theta})\right| \le \varepsilon_{\max}(k) \tag{15}$$

To be consistent with the hypotheses,  $\theta$  must belong to the solution set S of the following set of the inequalities

$$|y(k) - \boldsymbol{\varphi}^{\mathrm{T}}(k)\boldsymbol{\theta}| \le \varepsilon_{\max}(k) \ \mathrm{k=1, ..., N}$$
 (16)

Assuming that all  $\varepsilon_{\max}(k)$  are equal to  $\varepsilon_{\max}$ , which is unknown, the set **S** as the solution of problem is a min-max estimate given by

$$S_{mm} = \left\{ \hat{\boldsymbol{\theta}} \middle| \hat{\boldsymbol{\theta}} = \operatorname{Arg\,min}_{\boldsymbol{\theta}} \max_{k} \left| y(k) - \boldsymbol{\varphi}^{\mathrm{T}}(k) \boldsymbol{\theta} \right| \right\} \quad (17)$$

The min-max estimation of Eq.(16) can be transformed into a differentiable problem under constraints by introducing an additional variable x and determining  $\theta$  so that

$$\hat{\boldsymbol{\theta}} = \operatorname{Arg\,min}_{\boldsymbol{\theta}} x \tag{18}$$

subject to the constraints

$$x - \varphi^{-1}(k)\theta + y(k) \ge 0, k=1,...,N,$$
 (19a)

$$x + \varphi^{-1}(k)\theta - y(k) \ge 0$$
 k=1,...,N, (19b)

The problem defined by Eqs.(18), (19a) and (19b) can be viewed as a linear programming problem and thus could be solved by classical techniques such as the simplex or projection algorithms. Let ,  $\varepsilon(i) = 2^{i-1}\varepsilon_{\text{max}}$  i=1,2,...,s.

S is the number of error level. The bigger the value of s, the finer the error is divided. This means that the more reasonable algorithm parameters can be selected and the integrated algorithm performance will be much better. But this will increase its logic computation time. Furthermore, too large s is unnecessary. Generally, s takes 4-6.

The bigger the gains  $\alpha$  and the smaller the  $\lambda$ , the faster the response and the more sensitive the algorithm is to the noise. Generally  $\alpha$  take (0-1) and  $\alpha(1) = 0$  to ensure the robustness of the improved algorithm and  $\lambda$  take [0.95,1] and  $\lambda(1) = 1$ .

#### **III. EXPERIMENTAL PLATFORM**

The experimental platform consists of a four-axis SCARA robot as shown in Figure (4). Each axis of the manipulator is driven by a Panasonic MINAS A series servo-motor, which is controlled by an Advantech industrial personal computer (IPC) 610 within Pentium III-550 processor. A Googoltech motion card GO-400 mounted on motherboard of IPC through ISA bus interface is selected as an I/O interface card, on which does not have any processor, and it is primarily used for I/O purposes. Motion computation is performed with an industrial computer. Its communication with I/O card is managed through an on-board FPGA chip on the GO-400. The encoder interface, D/A conversion, and optoisolated input/output are carried out also by GO-400 card. The pulses from optical encoder are transmitted into the card, and a counter driven by a 8MHZ clock generator effectively provides the measured position from the motor shaft. The control signal is generated through the D/A converter on the card and is amplified by an external power amplifier module.

With the development of computer hardware and software technology, high computational rate of hardware and corresponding fine performance of software make possible the implementation of controllers in the mode of single processor architecture. In order to make up the existed limitation and deficiency of Windows NT in dealing with real-time tasks[12], we adopt VenturCom's real-time extension module (RTX) as a subsystem of Windows NT, thus the whole operating systems can serve for multitasks concurrently, which is critical for mechatronic servo systems.

The control algorithms are implemented using the C++ programming language under Visual C++ 6.0 software development environment. The sampling rate is fixed at 250HZ (which is much above the required Nyquist rate for any frequency in the system).



Figure 1 Four-axis SCARA robot experimental platform

### IV. IDENTIFICATION EXPERIMENTS

To illustrate the good performance of the improved EW-RLS algorithm with dead zone, experiments have been carried out on the second motor joint of SCARA robot.

It is well known that the relationship between the applied voltage, u, and the output the angle of the motor shaft, y is:

$$G_o(s) = \frac{1}{Js^2 + Bs} \tag{20}$$

We use the improved EW-RLS algorithm and the RLS algorithm to carried out parameter estimation. The input voltages are a serial of Pseudo-Random binary Signals (PRBS) with the maximum length M=8. The output is position readout from the optical encoder.

The corresponding z-transfer function with zero-order holder is:

$$G(z) = \frac{\theta(z)}{u(z)} = \frac{c_1 z^{-1} + c_2^{-2}}{(1 - z^{-1})(1 - p z^{-1})}$$
(21)

where:

$$p = \exp(-\frac{T_s B}{J}) \tag{22}$$

$$c_1 = \frac{1}{B} (T_s - \frac{J}{B} (1 - p))$$
(23)

$$c_2 = -\frac{1}{B}(T_s p - \frac{J}{B}(1-p))$$
(24)

Simulation time is 1000 sampling cycles. Fig.2 shows the

response of the RLS algorithm. And Fig.3 shows the response of the variable parameter RLS algorithm with dead zone. Comparing these two figures, it is obvious that the parameters of the RLS algorithm varies greatly than that of the improved EW-RLS algorithm with dead zone. This can be solved by selecting small gain and big forgetting factor. But this will reduce the response of the RLS algorithm.

Fig.4 shows the experiment measurement data (black curve) and the estimates of the variable parameter EW-RLS algorithm with dead zone (red curve). It is obvious that the estimation algorithm tracks the measurement data better. To



Fig. 2 Parameter tracking of the RLS algorithm



Fig. 3 Parameter tracking of the variable parameter RLS algorithm with dead zone

show its good track ability, the estimation errors (green curve) of the improved RLS algorithm are also given. It can be seen that the tracking errors is very small with comparison to the measurement data.

# V. TRAJECTORY TRACKING

After identifying the discrete parameter model of the second motor joint, we experiment tracking of desired trajectories using the online identification algorithm presented in this paper. In order to avoid exciting the high frequency dynamic character of the motor joint and saturating the amplifier, a fifth order polynomial curve which is smooth up to the third order derivative both at the start and end points is used as the desired trajectory of the second joint. It accomplishes a sigmoid shaped transition with the amplitude in a transition time of seconds:



Fig. 4 Estimation error of the variable parameter RLS algorithm with dead zone



Fig. 5 Trajectory tracking of the control system with the improved RLS identifier

$$\theta_r = A \left[ 6 \left( \frac{t}{T_r} \right)^5 - 15 \left( \frac{t}{T_r} \right)^4 + 10 \left( \frac{t}{T_r} \right)^3 \right]$$
(25)

Transition time is defined as the time required for the joint to complete a rad. A and Tr are set to 1 rad and 0.25 sec respectively, and the corresponding trajectory is shown in Fig.5.

It is well known that there exists friction between mechanical components with relative motion. To overcome the friction, a nonlinear friction compensation method is used according to the Southward's compensator[13]. In the experiment, a self tuning PID controller is used [14,15].

From Fig.5, it can be seen that the system outputs track the desired trajectory very well. This demonstrates that the variable parameter EW-RLS algorithm with dead zone is very effective.

# VI. CONCLUTION

Because the EW-RLS algorithm cannot meet the requirement of fast track and the requirement of being not sensitive to the noise at the same time, a variable parameter EW-RLS algorithm with dead zone is presented. The concept of the error level is proposed and the selection criteria of the error level are given according to the min-max principle. Good performance of the improved algorithm is verified by the experiments carried out on the robot. Control experiments with self-tuning PID controller have been carried out for the trajectory tracking of the joints of the manipulator based on the improved RLS algorithm. Good tracking performance shows the effectiveness of the improved RLS algorithm.

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