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Reference Shaping of Periodic Trajectory for Systems Having Constraints

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***Abstract*— This paper is concerned with off-line reference shaping for closed-loop systems with state/input constraints. We propose a method to generate reference signals which achieve better tracking property for given periodic trajectories subject to system constraints. In order to take account of both transient and steady state tracking performance, the new reference signals consist of two parts. This plays a key role to deal with periodic trajectory tracking. The steady state reference signals are produced first so as to minimize tracking errors in the steady state subject to given system constraints. Then, the transient reference signals are obtained in a similar way with the additional constraint that the transient states are connected to the steady states smoothly. By combining these signals, we obtain the shaped references. Its effectiveness is demonstrated through detailed simulations. Furthermore, an experimental validation is performed.**

I. INTRODUCTION

Most real plants have some constraints on their state and/or input such as actuator saturation and amplitude limitation of certain state. Without taking these constraints into account, we may have wind-up phenomena and/or serious performance degradation. Therefore, much study has been done to overcome these problems. One method is to modify the reference signal so that the constraints imposed on pre-designed closed-loop system will be fulfilled, known as reference governor or reference management [2], [3], [4]. This method observes system state at each sampling instant to modify the reference signal in real-time.

However, in many practical cases, it is not restrictive to assume that the reference signals are given in advance. In such cases, there is no necessity for modifying the reference signal in real-time. In addition, it requires a heavy computational burden. This problem is avoided by adopting an off-line method. This also means that an embedded computer for the burden is not necessary in the system any longer. In addition, off-line method can be applied to mechanical systems which usually demand fast sampling time, since it belongs to feed-forward type strategies in substance.

Based on the above observation, some works have been done concerning off-line reference shaping. Sugie and Yamamoto have proposed a pure feed-forward approach, which generates designed reference signals through off-line computation for constrained linear systems[1]. This can be regarded as nonlinear two-degree-of-freedom control. This work has been followed by Hirata and Kogiso, where the constraint fulfillment is guaranteed in the infinite horizon[5]. Extensions to the case of presence of model uncertainties has been proposed in [6] and [7]. The reference[6] discusses the case of non-parametrized uncertainties, and the reference[7] is related to parametrized uncertainties.

These existing works, however, mainly focus on tracking the step reference signals. One natural and useful extension is to cope with periodic trajectories in off-line reference shaping. Therefore the paper considers infinite horizon reference shaping in case where the given references are periodic. Our idea is to separate the infinite period tracking problem into two finite ones to give a suboptimal solution. One is an initial transition tracking problem and the other is a steady-state periodic one. This does not only make the problem tractable but also has some additional advantages: that is, it does not require that the pre-designed closed-loop system has tracking ability to the given periodic (including step) trajectory, and it ensures constraint fulfillment over infinite horizon without calculating invariant sets.

This paper is organized as follows. In Section 2, the problem is described. Section 3 shows how to reduce the tracking problem for infinite horizon into finite horizon ones, and gives a suboptimal solution by the separation technique. Finally, the effectiveness of the proposed method is evaluated by the numerical examples and also by the experiment of the two mass-spring system in section 4.

II. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

Consider the linear discrete-time closed-loop system Σ which consists of a plant and its stabilizing controller. The system Σ is described as follows.

$$\Sigma : \begin{cases} x(t+1) &= Ax(t) + Br(t) \\ y(t) &= C_y x(t) + D_y r(t) \\ z(t) &= C_z x(t) + D_z r(t) \end{cases} \tag{1}$$

where $x(t) \in \boldsymbol{R}^n$ is the state vector of Σ, and $y(t) \in \boldsymbol{R}^r$ is the controlled output. $r(t) \in \boldsymbol{R}^m$ is the external input to Σ, which we have to find. The vector $z(t) \in \boldsymbol{R}^l$ denotes the variable to express the constraints imposed in Σ such as input saturation. The constrains are described as

$$\underline{z} \leq z(t) \leq \overline{z}. \tag{2}$$

Note that inequalities on vectors imply component-wise.

Now, we make the following assumptions.

Assumption 1: The initial state $x(0)$ is set to be 0.

Assumption 2: The matrix A is stable.

Assumption 3: (A, B) is controllable.

Assumption 4: The trajectory $r_0(t)$ (to be tracked) is given in advance, and has a periodic property, i.e. $r_0(N+t) = r_0(t)(\text{for } {}^\forall t \geq 0)$ holds.

Remark 1: Assumption 1 is assumed for simplicity. It is easy to cope with non-zero initial conditions.

The goal is to find a modified reference signal $r(t)$ so that the output $y(t)$ follows the given trajectory $r_0(t)$ as precisely as possible subject to the constraints(see Fig.1). This corresponds the design of nonlinear feed-forward compensators in the context of 2DOF control.

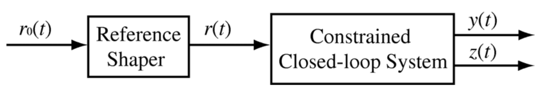


Fig. 1. Feed-forward type reference shaping

To this aim, we now introduce an objective function J which explicitly evaluates the tracking property to trajectory $r_0(t)$ [1].

$$J = \sum_{t=0}^{t_f-1} \|r_0(t) - y(t)\|^2 + \sum_{t=0}^{t_f-1} w(t)\|r_0(t) - r(t)\|^2 \tag{3}$$

where t_f is a sufficiently large number[1], and t_f/N is an integer. $w(t)(>0)$ is the weighting coefficient to be chosen by the designer. The first summation explicitly evaluates the tracking property to the given trajectory $r_0(t)$. The second summation is a regularization term.

Our goal is to find the reference signals $r(t)$ which minimize the objective function subject to constraints (2). However, this problem induces a computationally intractable problem in a case of sufficiently large t_f. Therefore in the following section, we develop how to give a suboptimal solution, by separating the above problem into two problems which evaluate the objective function for only one period .

III. Reference Shaping of a Periodic Trajectory

In this section, we give the main result which yields a suboptimal shaped reference signals ensuring the constraints fulfillment. The obtained reference signals consist of an initial transient part and a steady-state periodic part, the former is firstly applied to the closed-loop system and the latter is iteratively applied during infinite periods.

First, we show how to generate the steady-state periodic part. Second, the initial transient part is given in a similar way. Then we summarize the whole procedure of shaping the reference signals.

A. Steady-state Part

We show that the state converges to the periodic steady-state when the periodic signals are injected. Exploiting this property, we formulate the trajectory tracking problem for the periodic steady-state. The result is given in terms of LMIs.

1) periodicity: Define

$$R := \begin{bmatrix} r^{\mathrm{T}}(0) & r^{\mathrm{T}}(1) & \cdots & r^{\mathrm{T}}(N-1) \end{bmatrix}^{\mathrm{T}} \tag{4}$$

which is the reference signal sequence consists of N steps. Applying the reference signals $r(0), \cdots, r(N-1)$ iteratively from an initial condition x_0, we get the system state at the k-th iteration as

$$x(N(k-1)+j) = A^j x(N(k-1)) + \sum_{i=0}^{j-1} A^{j-1-i} B r(i) \tag{5}$$

where $j = 0, 1, \cdots, N-1$ and the state can be separated into two portions, one depends on k and the other does not.

Define G by

$$G := \begin{bmatrix} A^{N-1}B & A^{N-2}B & \cdots & AB & B \end{bmatrix}$$

then the following equation holds.

$$\begin{aligned} x(Nk) &= A^N x(N(k-1)) + GR \\ &= A^{Nk} x_0 + \sum_{i=0}^{k-1} \left(A^N\right)^i GR \end{aligned} \tag{6}$$

Here we suppose $k \to \infty$, then it follows

$$x(Nk) \to \sum_{i=0}^{\infty} \left(A^N\right)^i GR \qquad (k \to \infty). \tag{7}$$

from Assumption 2. This quantity does not depend on the initial state x_0 any longer. From this and (5) we see that $x(t)$ shows periodic property. Note that here it is supposed that the constraints are satisfied.

We denote this periodic steady state with $x_s(t)(:= \lim_{k\to\infty} x(Nk+t),\ t = 0, \cdots, N-1)$, and corresponding variables with $y_s(t), z_s(t)$ as well.

2) Formulation of Steady-state Reference Signals: Exploiting the periodicity and restricting our attention to just one period, we can derive the reference signals

$$R_s := \begin{bmatrix} r_s^{\mathrm{T}}(0) & r_s^{\mathrm{T}}(1) & \cdots & r_s^{\mathrm{T}}(N-1) \end{bmatrix}^{\mathrm{T}} \tag{8}$$

which become optimal in the sense of the objective function at periodic steady-state. Now we show such signals can be given by solving an LMI optimization problem.

First, we show the dependence of variables y_s, z_s on R_s explicitly. From (5) and (7), the following equation holds

$$x_s(0) = (I - A^N)^{-1} G R_s \tag{9}$$

for any initial condition x_0.

[1] $t_f = \infty$ admits J to remain a finite value if the equation $\lim_{t\to\infty} y(t) - r_0(t) = 0$ holds subject to $r(t) = r_0(t)$, however it does not hold generally so we suppose t_f is a finite number.

Remark 2: The existence of $(I - A^N)^{-1}$ is ensured by Assumption 2.

Then $y_s(t)$ is given by

$$y_s(t) := C_y A^t (I - A^N)^{-1} G R_s + \begin{bmatrix} C_y A^{t-1} B & \cdots & C_y B & D_y & 0 & \cdots & 0 \end{bmatrix} R_s \tag{10}$$

Define the vector Y_s by

$$\begin{aligned} Y_s &:= \begin{bmatrix} y_s^{\mathrm{T}}(0) & y_s^{\mathrm{T}}(1) & \cdots & y_s^{\mathrm{T}}(N-1) \end{bmatrix}^{\mathrm{T}} \\ &= M_{y_s} R_s \end{aligned} \tag{11}$$

where M_{y_s} is defined as follows.

$$M_{y_s} := M_y + \begin{bmatrix} C_y \\ C_y A \\ \vdots \\ C_y A^{N-1} \end{bmatrix} (I - A^N)^{-1} G \tag{12}$$

$$M_y := \begin{bmatrix} D_y & 0 & \cdots & 0 \\ C_y B & D_y & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ C_y A^{N-2} B & \cdots & C_y B & D_y \end{bmatrix} \tag{13}$$

z_s, Z_s, M_{z_s} and M_z are defined in a similar way.

In the steady-state, the property for one period horizon represents the whole steady-state property. Therefore we consider next objective function for a period.

$$J_s = \sum_{t=0}^{N-1} \left(\|r_0(t) - y_s(t)\|^2 + w(t) \|r_0(t) - r_s(t)\|^2 \right) \tag{14}$$

Let γ_s be an upper bound of J_s, then it is shown that $J_s > \gamma_s$ is equivalent to the following LMI [1];

$$\begin{bmatrix} \Theta_{s_1} & R_s \\ R_s^{\mathrm{T}} & \gamma_s - \Theta_{s_2} + R_s^{\mathrm{T}} \Theta_{s_3}^{\mathrm{T}} + \Theta_{s_3} R_s \end{bmatrix} > 0 \tag{15}$$

where coefficient matrices are defined by

$$\begin{aligned} \Theta_{s_1} &:= (M_{y_s}^T M_{y_s} + W)^{-1} \\ \Theta_{s_2} &:= R_0^T (I_{mN} + W) R_0 \\ \Theta_{s_3} &:= R_0^T M_{y_s} + R_0^T W. \\ R_0 &:= [r_0^{\mathrm{T}}(0) \quad r_0^{T}(1) \quad \cdots \quad r_0^{\mathrm{T}}(N-1)]^{\mathrm{T}} \\ W &:= \mathbf{diag}\,(w(0), w(1), \cdots, w(N-1)) \otimes I_m \end{aligned}$$

with Kronecker product $\otimes$ and $n \times n$ identity matrix I_n.

Next, we consider the constraints satisfaction. We impose

$$\underline{Z} < M_{z_s} R_s < \overline{Z} \tag{16}$$

as constraints, where the vectors $\underline{Z}, \overline{Z}$ are obtained by

$$\underline{Z} := \begin{bmatrix} \underline{z}^{\mathrm{T}} & \cdots & \underline{z}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \; \overline{Z} := \begin{bmatrix} \overline{z}^{\mathrm{T}} & \cdots & \overline{z}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}.$$

These inequalities for one period long ensure the constraints over the whole steady-state.

To summarize, the modified reference signals for the steady-state periodic part is given by the following LMI optimization problem:

$$\min_{R_s} \gamma_s \quad \text{subject to} \quad (15), (16)$$

B. Initial Transient Part

Here we consider how to modify the reference signals for initial transient part to be combined with the steady-state signals. For simplicity, we assume that the horizon of the initial transient reference is the same as just one period long(i.e. the length N).

Remark 3: We can arbitrary choose the length of the initial transient part, however long horizons lead to computationally intractable problems and short horizons sometimes make the problems infeasible.

We define the initial transient reference signals by

$$R_t := \begin{bmatrix} r_t^{\mathrm{T}}(0) & r_t^{\mathrm{T}}(1) & \cdots & r_t(N-1)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}. \tag{17}$$

Under Assumption 1, we can express the state reached by R_t as $x(N) = G R_t$. Next equation is introduced to hold the equivalence between the two states i.e. $x(N)$ and $x_s(0)$.

$$G R_t = (I - A^N)^{-1} G R_s \tag{18}$$

This joint condition (18) is inevitable to ensure the tracking property and constraints fulfillment in the steady state part. Using the Moore-Penrose inverse matrix(denoting with $(\cdot)^+$), this equation is replaced by [2]

$$R_t = G^+ (I - A^N)^{-1} G R_s + \left(I - G^+ G \right) \zeta_t \tag{19}$$

where whole R_t that satisfies (18) is parametrized by ζ_t which has the same length as R_t. (19) can be rewritten as

$$R_t = G^+ (I - A^N)^{-1} G R_s + T \zeta \tag{20}$$

where ζ, an optimization vector, is parameterizing whole of R_t and has the same length as the number of columns of T. T consists of the independent columns of $I - G^+ G$, fulfilling the condition $\mathbf{Im}(I - G^+ G) = \mathbf{Im}(T)$. This additional manipulation reduces the degree of the optimization vector so that the numerical burden is decreased to some extent.

To generate R_t, we introduce the objective function:

$$J_t = \sum_{t=0}^{N-1} \left(\|r_0(t) - y(t)\|^2 + w(t) \|r_0(t) - r_t(t)\|^2 \right), \tag{21}$$

which evaluates the initial transient part only. The result is given by the LMIs below, defining γ_t: an upper bound of the objective function J_t.

$$\min_{\zeta} \gamma_t \quad \text{subject to}$$

$$\begin{bmatrix} \Theta_1 & R_t \\ R_t^{\mathrm{T}} & \gamma_t - \Theta_2 + R_t^{\mathrm{T}} \Theta_3^{\mathrm{T}} + \Theta_3 R_t \end{bmatrix} > 0, \tag{22}$$

$$\underline{Z} < M_z R_t < \overline{Z} \tag{23}$$

where $\Theta_i, i = 1, 2, 3$ are matrices given by

$$\begin{aligned} \Theta_1 &:= (M_y^T M_y + W)^{-1} \\ \Theta_2 &:= R_0^T (I_{mN} + W) R_0 \\ \Theta_3 &:= R_0^T M_y + R_0^T W. \end{aligned}$$

[2] If the length of R_t is longer than n, then G becomes row-fullrank because of **Assumption 3**. This is sufficient to ensure the existence of G^+.

(22),(23) are described as LMIs with respect to R_t, these are also LMIs with respect to ζ by substituting (20).

C. Procedure of Generating References

Here we summarize the above two and give the whole procedure to generate reference signals as follows.

step 1 Solve the LMI optimization problem to obtain R_s.

$$\min_{R_s} \gamma_s \quad \text{subject to} \quad (15), (16)$$

step 2 Solve the LMI optimization problem to obtain ζ.

$$\min_{\zeta} \gamma_t \quad \text{subject to} \quad (20), (22), (23)$$

step 3 Substitute ζ gained in step 2 for (20) to obtain R_t.

step 4 Align the signals like $\left[\begin{array}{cccc} R_t^{\mathrm{T}} & R_s^{\mathrm{T}} & R_s^{\mathrm{T}} & \cdots \end{array} \right]^{\mathrm{T}}$.

This aligned vector $\left[\begin{array}{cccc} R_t^{\mathrm{T}} & R_s^{\mathrm{T}} & R_s^{\mathrm{T}} & \cdots \end{array} \right]^{\mathrm{T}}$ obtained in step 4 is a suboptimal solution in the sense of the objective function which appears in (3) because the longer initial transient part is, the more closely modified signal converges to the optimal solution. Applying this, a periodic steady-state is achieved after the initial transient part so that it ensures the tracking for infinite horizon fulfilling constraints.

This method is developed to track periodic trajectories. However, this can be applied to the set point tracking as well in the presence of off-set property. The new references compensate it. In this sense, the proposed method give an improved version of the one proposed in [1].

IV. Examples

In this section, we evaluate the effectiveness of the proposed method by simulating numerical examples and implementing this to the two mass-spring system.

A. Numerical Examples

Here we show numerical examples implementing the reference signals obtained by the proposed method. For each example, the periodic trajectory tracking problem is given in the closed-loop system which contains an input saturation factor(see Fig.2).

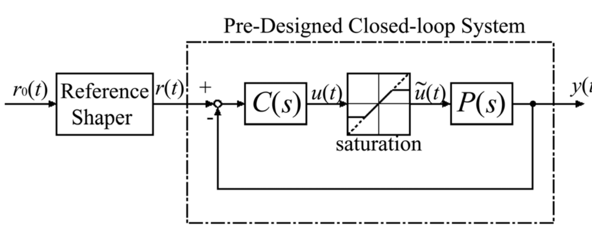


Fig. 2. Input-saturating system with reference shaping

Example 1: Now we consider a tracking problem of a sinusoidal trajectory for a closed-loop system which contains a non-minimal phase plant. The transfer functions of the plant and the controller are given as

$$C(s) = -\frac{1.8}{s} \quad , \quad P(s) = \frac{s-2}{(s+1)(s+3)}$$

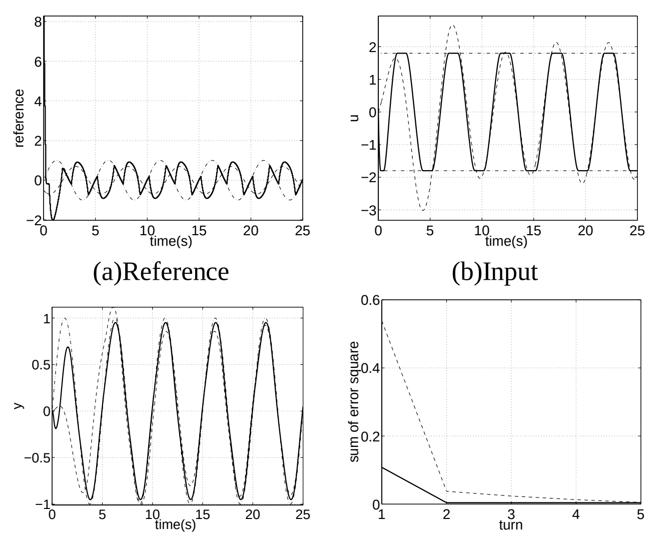
and each transfer function is discretized at the sampling time of 50[ms] using zero-order hold. The saturation factor, that exists between controller and plant(see Fig.(2)), is given by

$$|\tilde{u}(t)| \leq 1.8 \ .$$

The sinusoidal reference trajectory is given as $r_0(t) = \sin{(2/5)\pi t}$ so the period is 5[s]. To track this signal, we set R_t as first one period long(i.e. as $0, \cdots, 99$th steps in the discretized system), and a weighting coefficient as $w(t) = 0.01$. With this specification we computed the reference signal with the proposed method. It took 18[s] to calculate $r(t)$ using the PC whose CPU is a PentiumIV 3.0GHz.

The results are shown in Fig.3(a)∼(d). In Fig.3, (a),(b) and (c) show the obtained reference signal $r(t)$, the controller output $u(t)$ and the plant output $y(t)$, respectively. In Fig.3(d), the sum of the squared tracking error(per one step) in each period is described, which corresponds to the term J_1 in (3). In each figure, the solid line describes the value of proposed method, and the dashed line shows the value obtained by applying the reference signal $r(t) = 1.44\sin{((2/5)\pi t + 2.31)}$ whose amplitude and phase are reformed to track $r_0(t)$ by considering its output property of frequency. The dash-dot lines express the original reference $r_0(t)$ in Fig.3(a),(c) and the saturation limits in Fig.3(b), respectively.

In Fig.3(b), the proposed method succeeded to satisfy the constraints, whereas the other violates it. Concerning the output responses in Fig.3(c), the dashed line is ahead in the phase and takes two periods to converge. As opposed to this, though the undershoot response appears, we see the output of proposed method converges to the trajectory more quickly. In Fig.3(d), moreover, its tracking property is clear.



(a)Reference (b)Input

(c)Output (d)Error

Fig. 3. Simulation results (Example 1)

Example 2: Next, we consider the tracking problem of a periodic square-wave trajectory for a given unstable plant

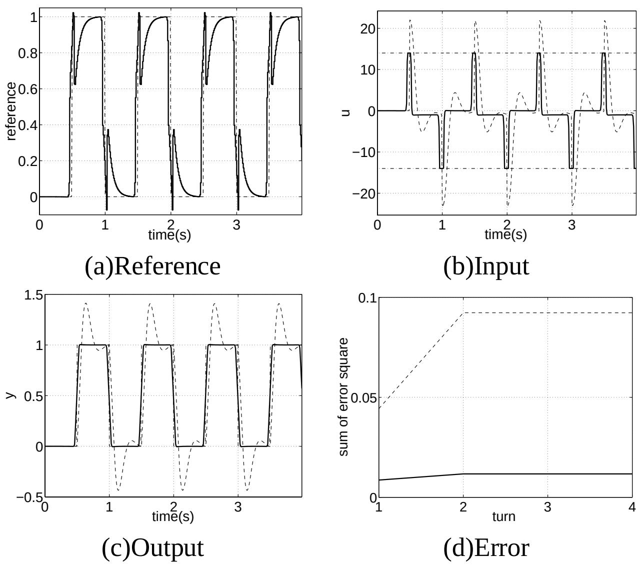


Fig. 4. Simulation results(Example 2)

and its stabilizing controller. The transfer functions are given by

$$C(s) = \frac{22s + 300}{s} \quad , \quad P(s) = \frac{1}{s - 1} \ .$$

These transfer functions are discretized at the sampling time of 10[ms] using zero-order hold. The saturation is given by

$$|\tilde{u}(t)| \leq 14 \ .$$

The reference trajectory is given as a square wave whose amplitude is 1 and one period takes 1[s]. We set R_t as the reference signal for the first period, and the weight as $w(t) = 0.01$. It took 31[s] to modify the reference signal using the same PC as in Example 1.

We show the results in Fig.4(a)∼(d). All figures(a)∼(d) exactly correspond to those of Fig.3 except for the dashed line which show the values obtained by $r_0(t)$ here.

In Fig.4(c), the output response by $r_0(t)$ overshoots in every period because of input saturation, while the proposed method does not. In addition, the output by the proposed method responds the considerably quick variation of $r_0(t)$ and tracks more quickly.

From these two examples, we see that the proposed method not only ensures the fulfillment of constraints, but also improves tracking properties in both transient and steady periodic parts.

This technique corrects the phase lag as well as output overshoot in particular. The phase lag is easily corrected by considering the frequency property in the cases of sinusoid trajectories. However this frequency method is not applicable for general trajectories like the one in Example 2. As opposed to this, the proposed method can correct the phase lag in the case of more general trajectories as we have seen in Example 2.

B. Experiment in two mass-spring system

Here we evaluate the effectiveness of the proposed method by experiment.

1) Description of the control system:

(A) Plant description

In Fig.5 we show a sketch of the two mass-spring system. The plant consists of the motor and the disc connected with

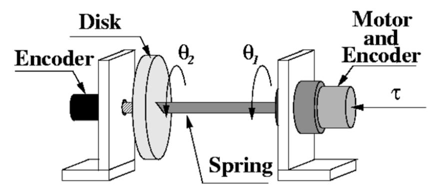


Fig. 5. Two mass-spring system

the flexible joint.

The dynamics of the plant is given by following differential equations:

$$\begin{cases} \dot{x}(t) & = & Ax(t) + Bu(t) \\ y(t) & = & Cx(t) \end{cases} \tag{24}$$

$$x(t) = \begin{bmatrix} \theta_1 & \dot{\theta}_1 & \theta_2 & \dot{\theta}_2 \end{bmatrix} \ , \ u(t) = \tau \ , \ y(t) = \theta_2$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k/j_2 & -d_2/j_2 & k/j_2 & 0 \\ 0 & 0 & 0 & 1 \\ k/j_1 & 0 & -k/j_1 & -d_1/j_1 \end{bmatrix}, \tag{25}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 1/j_1 \end{bmatrix} \ , \ C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \tag{26}$$

where θ_1[rad] and θ_2[rad] denote the rotation angles, and τ[Nm] is the input torque. The subscripts 1 and 2 denote the motor and the disc, respectively. The plant parameters are given as follows. The moments of inertia are given by $j_1 = 2.50^{-3}$[kgm^2] and $j_2 = 2.76^{-3}$[kgm^2]. The viscous coefficients are given by $d_1 = 1.83^{-1}$[Nms/rad] and $d_2 = 2.76^{-3}$[Nms/rad]. The spring constant of the flexible joint is $k = 6.16$[Nm/rad].

The input torque τ has the saturation constraint given by

$$|\tau| \leq 1.5[\mathrm{Nm}].$$

(B) Controller design

First, we obtain the discrete-time plant model

$$\begin{cases} x(t+1) & = & A_p x(t) + B_p u(t) \\ y(t+1) & = & C_p x(t) \end{cases} \tag{27}$$

by discretizing the system (24) with the sampling time $T_s = 0.001$[s], using zero-order hold.

Next, we construct the servo system by state feedback with an integrator. We give the input torque command with

$$u(t) = Fx(t) + G\sum_{k=0}^{t}(r(k) - y(k)) \,,$$
$$F = \begin{bmatrix} -4.85 & -9.76\times10^{-2} & 1.61 & 3.82\times10^{-2} \end{bmatrix},$$
$$G = 1.45\times10^{1}.$$

These state feedback gains F and G are chosen based on the LQ optimal control.

(C) Computation of the reference signal $r(t)$

We generate the modified reference signal $r(t)$ for the periodic trajectory $r_0(t) = 90\sin(2/3)\pi t + 90\sin(4/3)\pi t$ (whose length of one period is 3[s])

For better implementation, here we introduce several additional techniques in order to improve the robustness and to reduce the computational burden.

First, sampling time T_s is so short that the optimization vector has too large dimension to compute the modified signal. Therefore, we reduce the degree of the optimization vector by re-sampling this closed-loop system with the new sampling time $T_s = 0.020$[s]. Second, the system description (27) ignores the effect of the plant uncertainty. So, in order to compensate this effect, we restrict the input torque by

$$|\tau(t)| \leq 1.3[\text{Nm}]$$

at the stage of the reference modification. Third, to avoid high frequency vibrations caused by the sudden variation of input torque, we introduce the following term:

$$\sum_{t=0}^{N-1} \omega(t)\|r(t+1) - r(t)\|^2 \tag{28}$$

and we use this instead of $\sum_{t=0}^{N-1} \omega(t)\|r_0(t) - r(t)\|^2$ as a portion of the objective function.

In addition to these techniques, we set R_t as the first one period, and the weight as $w(t) = 2$ for R_t and $w(t) = 1$ for R_s.

Generated signal is shown in Fig.6(a) as solid line, where the dashed line describes $r_0(t)$.

2) Experiment results and discussions: The experimental results are shown in Fig.6(b)∼(d), each figure shows the input, the output and the sum of the squared error(per one step) for each period, respectively. The solid line shows the results of proposed method, the dashed line describes the results of the non-shaped, and the dash-dot lines describe the saturation values of the input in Fig.6(b), and also describes $r_0(t)$ in Fig.6(c).

In Fig.6(b), we see the result of the proposed method satisfies the constraints, though that of non-shaped one does not. Correspondingly, the proposed method improves the tracking properties in Fig.6(c). In Fig.6(d), the properties for both the transient and the steady-state parts are shown explicitly. The deviation of phase and the output overshoot are especially improved.

Applying the proposed method to the real experimental system, we have improved the trajectory tracking property. These results demonstrate the validity of our method.

V. Conclusion

In this paper, we have addressed the periodic trajectory tracking problem for systems having constraints.

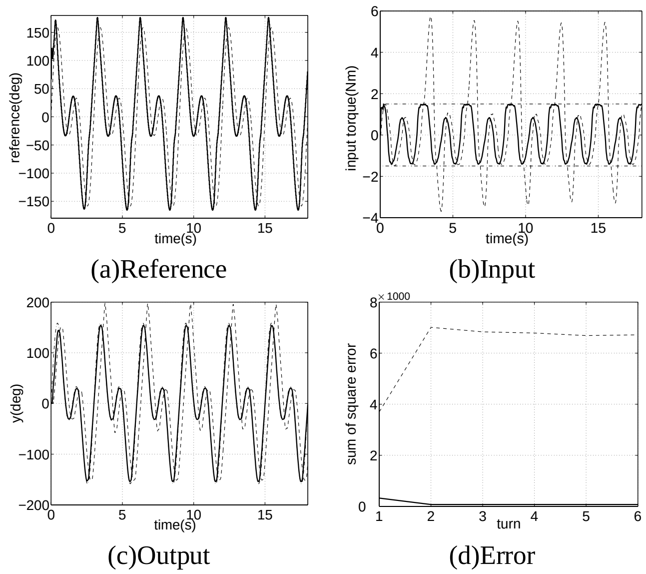


Fig. 6. Results of experiment

The key idea is to produce the steady state part and the transient part separately subject to the smooth connecting condition of the system state. Its effectiveness has been demonstrated through simulations. Furthermore, the experimental validation has been performed.

Since the proposed method is purely feed-forward type, it would be necessary to combine with feedback type methods such as anti-windup compensation in order to cope with model uncertainty and/or disturbances. It is one of the interesting open problems how to combine feed-forward and feedback for tracking performance improvement in the presence of system constraints.

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