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Inverted Pendulum Stabilization through the Ethernet Network, Performance Analysis

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***Abstract*—Networked control systems are becoming more and more attractive due to lower costs of installation, maintenance and scalability compared to traditional ones. However, problems due to the transmission delay cannot be neglected. Referring to a single subnet network, the induced delay affects control performances depending both on the Medium Access Control (MAC) method and on the network load; in this work the performances loss due to the Ethernet MAC method on a pole placement controller, an optimal controller, and an H_∞ controller are analyzed by mean of simulations while stabilizing an inverted pendulum model. Issues concerning the choice of the controller sampling period and the available network bandwidth are considered.**

***Index Terms*—Networked control systems, Ethernet network, inverted pendulum, robust control, optimal control.**

I. INTRODUCTION

DESIGNING control laws for a networked control system (NCS) involves a deep insight toward implementation issues since the strategy conceiving. Discrete time varying models of NCS and design issues were early reported in [1] and [2], while in [3] the characteristics of network induced delay are explored for three common Medium Access Control (MAC) methods from a control point of view; a survey on NCS stability analysis issues are studied in [4]; in [5] a MAC method specific for control applications is proposed and its induced stability characteristics are there studied.

In this work performances loss analysis is conducted by simulations: three types of controller facing an Ethernet network are designed to stabilize an inverted pendulum model. The controllers are designed without having in mind the presence of the network; a single subnet network is considered, where this implies that the transmission delay is only due to the MAC method, and the signal propagation time is neglectable. A random releasing policy is assigned to all the processes sharing the network medium, that is, each of the processes starts its periodic transmission after waiting a random time smaller than its period.

This paper is organized as follows: in section II the considered network is reviewed, section III describes the inverted pendulum model, while in section IV the designed controllers are presented; finally, in section V simulation results are discussed, and in section VI conclusions and our future work guidelines follow.

II. ETHERNET MAC METHOD

History of MAC methods development and their characteristics are discussed in [6], while three of them (CSMA/CD, Token Bus, CAN) are analyzed and compared from a control point of view in [3]; here the Ethernet MAC is briefly reviewed.

Ethernet is a MAC policy definition based on the IEEE standard 802.3; the word Ethernet itself commonly is related to the entire Ethernet/Tcp-Ip local area network definition up to the fourth level of the ISO-OSI model (see [7]), but technically it defines only the first two levels of it. The acronym CSMA/CD stands for Carrier Sense Multiple Access with Collision Detection which shortly describes how this MAC policy works:

- When a station linked to the network has a message ready to be transmitted, it listens at the network medium, and if the medium is free, the station begins the transmission.
- While transmitting, the station listens at the network to detect collisions[1] if any; if there is no collision the transmission ends successfully; at this level, however, there is no guarantee that the message has been correctly received.
- If a collision is detected, all the stations responsible of the problem stop their own transmission attempts. Before retrying the transmission, each station waits a random period of time between between 0 and $2\tau(2^i - 1)$, where i is the number of consecutive collisions, and τ is the signal propagation time along the medium. After 10 consecutive collisions, the period to wait before retrying is fixed to 1023τ, and after 16 consecutive collisions the transmission is aborted and the problem is notified to the upper layers of the OSI model. This algorithm is usually called *BEB*, which stands for Binary Exponential Backoff algorithm.

The Ethernet datagram is depicted in fig. 1. The useful data transmitted with each packet can be as large as

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[1] A collision happens if at least two stations start to transmit over the medium almost simultaneously.

1500 bytes, and so it is well suited and normally used for aperiodic communications in plant supervision oriented applications.

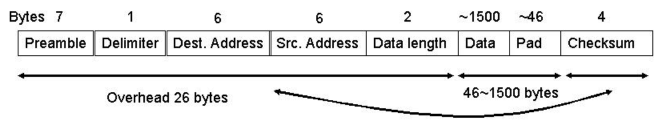

Fig. 1. The fields of the Ethernet datagram.

The main advantages of this MAC policy are that at low traffic conditions only a little amount of the bandwidth is wasted on arbitration so that the transmissions attempts experience almost no delay, and that the large dimension of the Data field in the Ethernet datagram normally allows to transmit control data within only one packet. This policy however is non deterministic and at high network loads the delay can easiliy grow unbounded.

III. Inverted pendulum description

The system considered is depicted in fig. 2: a vertical rod can rotate around a fixed point on the testbed ground, and the corresponding angle from the vertical upward position is $\vartheta(t)$, measured positive counterclockwise; an horizontal rod can slide on the top of the vertical one, and the displacement from the central position is $z(t)$, positive to the left; a linear force $u(t)$ can be applied to the horizontal rod using a d.c. motor placed at the bottom of the vertical rod and coupled to the horizontal one through transmission mechanics; thanks to two encoders, $z(t)$ and $\vartheta(t)$ can be both measured. The position of the vertical rod center of gravity can be changed along the rod itself, so that l_c is the distance of the center of gravity measured from the pivot and positive toward the top.

The non linear model is described by

$$\begin{bmatrix} m_1 & m_1 l_0 \\ m_1 l_0 & \bar{J} + m_1 z^2 \end{bmatrix} \begin{bmatrix} \ddot{z} \\ \ddot{\vartheta} \end{bmatrix} + \begin{bmatrix} 0 & -m_1 z \dot{\vartheta} \\ 2 m_1 z \dot{\vartheta} & 0 \end{bmatrix} \begin{bmatrix} \dot{z} \\ \dot{\vartheta} \end{bmatrix} + \begin{bmatrix} -m_1 \sin\vartheta \\ -(m_1 l_0 + m_2 l_c)\sin\vartheta - m_1 z \cos\vartheta \end{bmatrix} g = \begin{bmatrix} 1 \\ 0 \end{bmatrix} u, \quad (1)$$

where time dependance has been not explicitly indicated. Parameters meaning and values are listed in table I; $\bar{J}$ is the nominal momentum of inertia calculated when the pendulum is in the upward position with the sliding rod centered, and for the given value of l_c. The fact that l_c is negative does not imply the stability of the system (see the following section); in this study, friction is not modeled.

Letting the state vector be $x = [z, \dot{z}, \vartheta, \dot{\vartheta}]^\top$, (1), the linearization around the equilibrium point $\bar{x} = \underline{0}$ under $u(t) = 0$, gives a state space representation in the standard form $\dot{x}(t) = A\,x(t) + B\,u(t),\ y(t) = C\,x(t)$, where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -21.54 & 0 & 14.96 & 0 \\ 0 & 0 & 0 & 1 \\ 65.28 & 0 & -15.59 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 8.10 \\ 0 \\ -10.31 \end{bmatrix}, \ C = I. \quad (2)$$

Matrix A eigenvalues are $\lambda_{1,2} = \pm 7.07\jmath$, $\lambda_3 = 3.58$ and $\lambda_4 = -3.58$; the two pure complex eigenvalues depend on the vertical rod dynamics without friction, while the positive real one depends on the sliding rod dynamics.

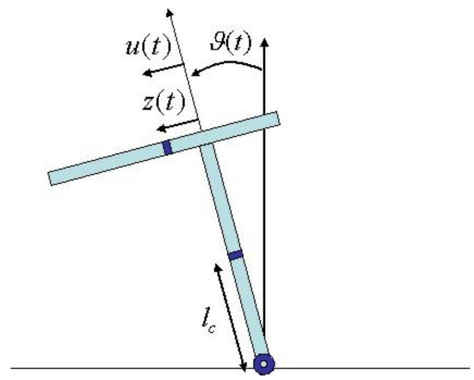

Fig. 2. The inverted pendulum considered.

Parameter name	Value	Meaning
m_1	0.213 kg	Mass of the horizontal rod.
m_2	1.785 kg	Mass of the vertical rod.
l_0	0.33 m	Length of the vertical rod.
l_c	-0.029 m	Vertical rod c.g. position.
g	$9.807\ \frac{m}{s^2}$	Gravity acceleration.
$\bar{J}$	$0.055\ Nm^2$	Nominal momentum of inertia.

TABLE I

Meaning of the parameters appearing in (1).

IV. The control strategies

The inverted pendulum stabilization problem can be difficult due to the relatively fast dynamics of the unstable mode and to the presence of RHP (right half plane) zeroes on each of the four output channels. Output feedback is intrinsically not robust because each of the SISO open loop transfer function presents a RHP zero relatively close to the unstable pole (in the $\vartheta(t)$ feedback case, the ratio between the RHP zero and the RHP pole is $\frac{z_{RHP}}{p_{RHP}} = 1.5$), which in turn causes high nominal sensitivity peak lower bounds [8]. Full state feedback avoids the problem of RHP zeroes thanks to the lack of transmission zeroes, but requires a large closed loop bandwidth to stabilize the plant, and in turn fast sampling periods when it comes to the discretized controller implementation (around $1 \sim 6$ms depending on the controller design).

Here, we compare the loss of performances of two static state feedback control laws and of an H_∞ two degrees of freedom dynamic control law. The chosen controller sampling time is $T_s = 1$ms for all the strategies.

A. Static state feedback with pole placement controller

A pole placement controller designed with the aim to obtain the closed loop poles $[-5 + 0.5\jmath, -5 - 0.5\jmath, -10, -20]$, leads to a feedback gain $K_{pp} = -[94.94, 18.55, 27.22, 10.69]$; fig. 4a and fig. 4b show the simulations of the non linear plant model without the network in the loop.

B. Optimal state feedback controller

A static feedback optimal control on an infinite time horizon has been designed to minimize the following integral index [9]:

$$J = \int_0^\infty \left(x^\top Q x + u^2 r\right) \, \mathrm{d}t, \tag{3}$$

with $Q = diag[100, 20, 50, 10]$, and $r = 100$. The feedback gain is $K_{lq} = -[15.59, 3.87, 5.72, 1.82]$ that causes the closed loop poles to be $[-2.69 + 6.75\jmath, -2.69 - 6.75\jmath, -4.00, -3.20]$; fig. 4c and fig. 4d show the simulations of the non linear plant model without the network in the loop.

C. H_∞ two degrees of freedom controller

A two degrees of freedom controller has been designed solving the mixed sensitivity problem referring to the standard control problem configuration of fig. 3; letting $G(s) = \left[G_z(s), G_{\dot{z}}(s), G_\vartheta(s), G_{\dot{\vartheta}}(s)\right]^\top$ be the vector transfer function corresponding to (2), and given the definitions

$$S_u(s) = (1 + K_2(s)G(s))^{-1}, \tag{4}$$

$$T_u(s) = I - S_u(s), \tag{5}$$

$$S_e(s) = 1 - S_u(s)K_1(s)G_\vartheta(s), \tag{6}$$

a controller $K(s) = [K_1(s)\ K_2(s)]$ has been designed to satisfy the following with the smallest achievable γ [10].

$$\left\| \begin{matrix} W_e S_e & -W_e S_u G_\vartheta \\ W_u S_u K_1 & W_u T_u \end{matrix} \right\|_\infty < \gamma, \tag{7}$$

where the weighting functions are

$$W_e(s) = \frac{s + 0.35}{0.5s + 35}, \quad W_u(s) = \frac{0.01s + 75}{s + 75}. \tag{8}$$

The obtained controller structure is discussed in the appendix.

Fig. 4e and fig. 4f show the simulations of the non linear plant model without the network in the loop. In fig. 4 it can be seen that the H_∞ controller has a faster response compared to pole placement one using a comparable control signal, even if at a slightly larger overshoot. The optimal controller shows instead a slower response but with a considerable smaller amplitude signal, as it should be expected. Note that, in all cases, the effects of the RHP zeroes on the output channels (i.e. the undershoots in both the state variables) cannot be avoided.

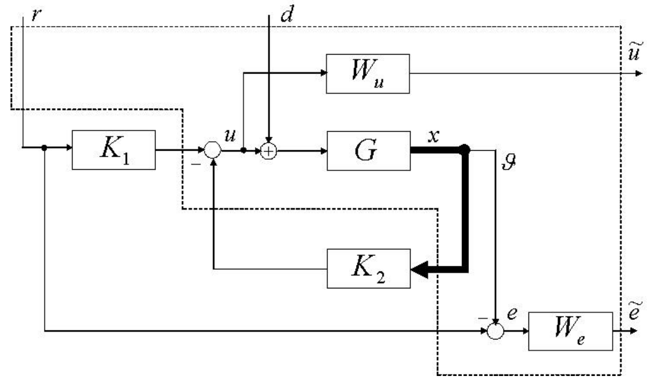

Fig. 3. Standard plant configuration.

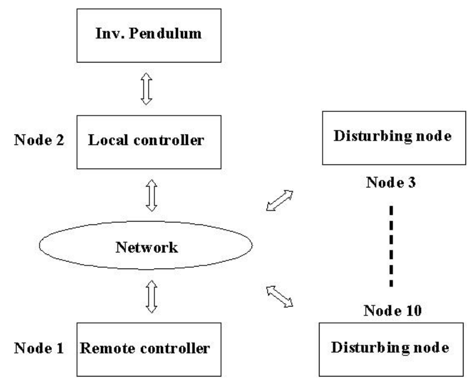

Fig. 5. The network configuration considered.

V. Simulated Network in the Loop

Here some performance criteria are proposed and used to compare the previous described controllers in a NCS simulated framework.

A. Framework description

The considered network configuration is illustrated in fig. 5. The protocol coordinating the local and remote CPU is explained in the timing diagram in fig. 6, where t_{pre_i} is the partial delay due to computation at the source nodes, t_{wait_i} is the time elapsed waiting the network availability, t_{x_i} is the actual transmission time, and t_{post_i} is post processing delay at the destination nodes [3].

To simulate the network loads due to pre-existing traffic the following sets of processes are considered to share the network:

- n dummy periodic control processes characterized by an execution period T_i^p, by a release time[2] $T_i^{p_r} \leq T_i^p$, and by a message complexity expressed as the number of sent bytes C_i^p.

[2]The release time defines the first execution instant of a periodic process after the simulation has started.

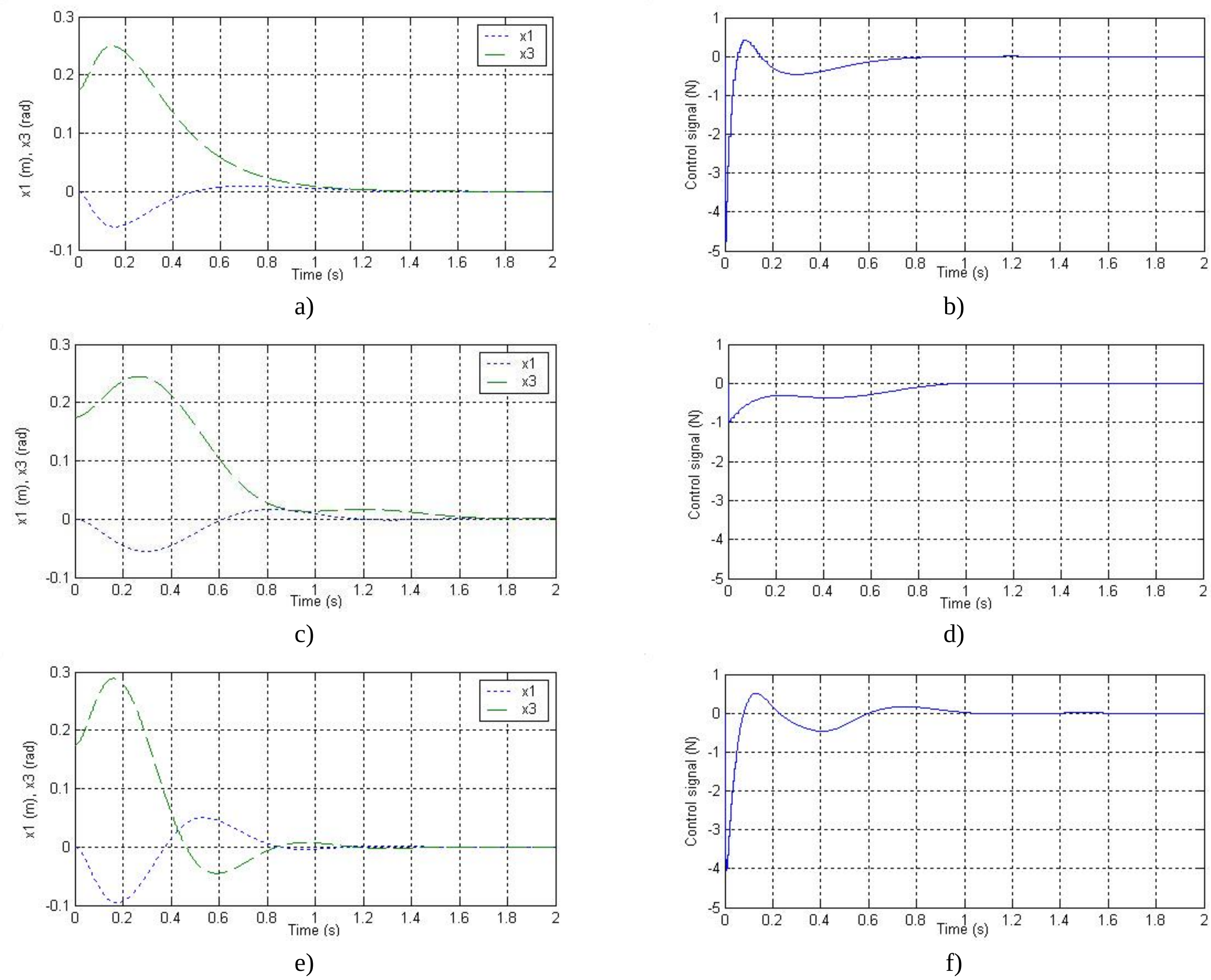

Fig. 4. Inverted pendulum model stabilization starting from the initial condition $x_0 = [0,\ 0,\ 0.17,\ 0]$: a) - b) pole placement control strategy; c) - d) optimal control strategy; e) - f) H_∞ control strategy. Network is not in the loop.

n	8	m	8
T_i^p	6 ms	T_j^s	6 ms
$T_i^{p_r}$	$[0\dots6]$ ms	$T_j^{s_r}$	$[0\dots6]$ ms
C_i^p	32 bytes	C_j^s	1.5 Kbytes
		S_j	0.01

TABLE II

EXPERIMENT PARAMETERS; $T_i^{p_r}$ AND $T_j^{s_r}$ ASSUME RANDOM UNIFORMLY DISTRIBUTED VALUES IN THE SHOWN INTERVALS.

- m dummy periodic supervision processes characterized by an execution period T_j^s, by a release time $T_j^{s_r} \leq T_j^s$, a probability S_j to send a message at each activation, and by a message complexity C_j^s.

The messages generated by the supervision processes are such that $\forall j \in 1\dots m,\ C_j^s \geq \max_{i \in 1\dots n} C_i^p$. All dummy messages are not sent neither to the local CPU neither to the remote one; those parameters are set in table II.

The simulations are carried out using the TrueTime toolbox developed at University of Lund [11].

B. Performance indexes

- $J_{tr} = \int_0^{t_f} t\,\vartheta^2(t)\,\mathrm{d}t$ provides a measure of stabilization performances giving less importance to the transient phase; it is evaluated on a time horizon of 2 seconds, starting from each of the initial conditions belonging to the set $X_0 = \left\{\left[0,\ 0,\ i\frac{\pi}{180},\ 0\right]\right\}_{i=1\dots10}$.
- Measure of the network traffic conditions that leads the closed loop to limit of unstability.
- The operating point of a network can be characterized by its efficiency level, N_{eff} and its utilization level N_{ut} [3], defined as follows:

$$N_{eff} = \frac{\sum_{i=1\dots n+m}\sum_{k=1\dots l_i} t_x^{(i,k)}}{\Delta_{Tot}}, \qquad (9)$$

$$N_{ut} = \frac{\sum_{i=1\dots n+m}\sum_{k=1\dots l_i} t_x^{(i,k)}}{T_{Tot}}, \qquad (10)$$

where $t_x^{i,k}$ is the time needed to send the kth message once the ith node gains access to the network medium, and l_i is the total number of sent messages; Δ_{Tot}

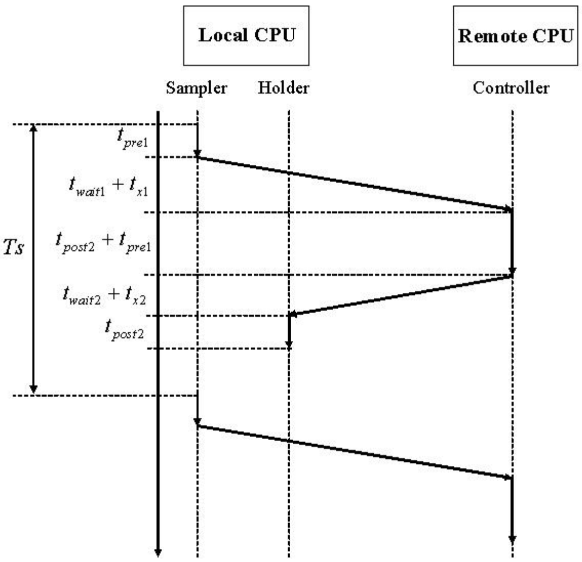

Fig. 6. Timing diagram for the protocol coordinating the local and the remote controller; T_s is the generic sampling period. Due to the network traffic, the growth of t_{wait_i} can cause the protocol cycle time to become greater than T_s.

is the total delay experienced by all the transmission attempts; T_{Tot} is the total elapsed time; to simplify the expression of (10) with respect to that reported in [3], the time needed to eventually retransmit the same message is considered as the time needed to send a new one. $N_{eff} \to 1$ expresses that few time due to network arbitration is wasted, while $N_{ut} \to 1$ denotes a full network bandwidth usage; note that an N_{ut} value close to 1 does not imply an efficient use of the network i.e. without delay.

C. Simulation results

With a full network bandwidth[3] of 10Mbps, simulations show that the performance of the three controllers are substantially equivalent to the ones obtained via direct controller-plant coupling.

To find the limits of each strategy, the network bandwidth has been gradually reduced[4] to find the lower limit after which the controller cannot anymore stabilize the plant. Table III shows the network traffic conditions in the four cases considered; parameters shown therein have been evaluated on a time basis of ten times the time horizon used to estimate the index J_{tr}. Fig. 7 shows the comparisons of the performances index for each strategy; the H_∞ strategy shows better performance in terms of the index values

[3]The network bandwidth expresses the transmission rate usually as Mbps; 1Mbps=$\frac{1Mbit}{s}$; $1Mbit = 2^{20}\, bits = 1'048'576\, bits$.

[4]Reducing the network bandwidth has two tied effects: it increases the probability of collisions, because it takes more time transmit a message (t_x increases). However, because t_x is small (for instance, 57.6μs at 10Mbps, 169.4μs at 3.4Mbps) compared to the BEB induced delay (t_{wait} can grow up to ∞), only the latter can be considered responsible of the performances loss.

Bandwidth	N_{eff}	N_{ut}	Av. delay	Max delay
10Mbps	82.7%	21.0%	81μs	3.9ms
4Mbps	49.1%	52.3%	0.3ms	27ms
3.6Mbps	34.7%	57.6%	0.4ms	40ms
3.4Mbps	13.3%	61.7 %	2.1ms	280ms

TABLE III

AVERAGE NETWORK TRAFFIC CONDITIONS WITH THE SAMPLING PERIOD $T_s = 1ms$.

Bandwidth	N_{eff}	N_{ut}	Av. delay	Max delay
10Mbps	85.9%	11.5%	75μs	1.2ms
2.5Mbps	38.5%	45.0%	0.7ms	45ms

TABLE IV

AVERAGE NETWORK TRAFFIC CONDITIONS WITH THE SAMPLING PERIOD $T_s = 6ms$.

achieved, of the transmission delay endurable, and of the needed network bandwidth.

However the two static controllers can be implemented also at a lower sampling frequency ($T_s = 6$ms) still guaranteeing closed loop stability and without significantly losing in performance as fig. 8 shows; in this case the minimum needed bandwidth is smaller than the minimum achieved formerly. Table IV shows the network traffic conditions with $T_s = 6$ms. The H_∞ controller cannot be sampled at this frequency, because of its large bandwidth. Then a tradeoff should be made between the best performance achievable (H_∞ with high sampling frequency) and the lower network bandwidth occupation with acceptable performance (static state feedback at a lower sampling frequency).

VI. CONCLUSIONS AND FUTURE WORK

In this work an analysis of performance loss for three control strategies has been conducted by simulations when each controller face the plant trough an Ethernet network in the same load conditions. Results suggest that, given an Ethernet network with assigned bandwidth and traffic, the most important question is how much the network efficiency will be lowered adding a new periodic process. Unless it is strictly imposed by design specifications not related to the presence of the network, a control strategy requiring a larger sampling period will generate less collisions, keeping low the induced delay, and so needing in turn less design complexity; adding a further periodic process on a network can also influence the formerly present periodic processes.

Ongoing work concerns the validation on an experimental setup; future work will deal with the characterization of a structured network traffic, such that it would be possible to predict the delay characteristics induced by a set of processes sharing the communication medium. With such a tool, the available bandwidth could be one of the control design specifications.

VII. ACKNOWLEDGMENT

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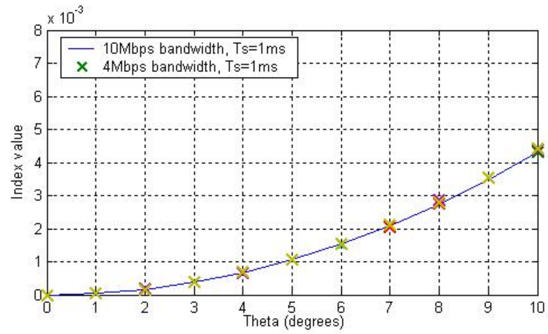

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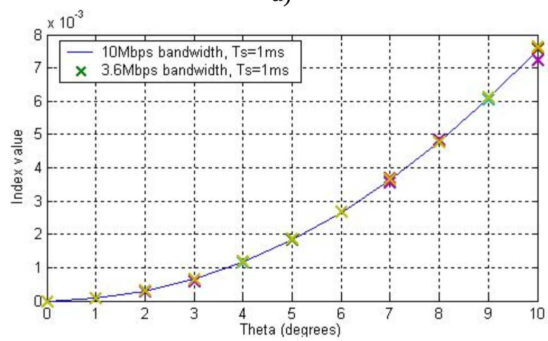

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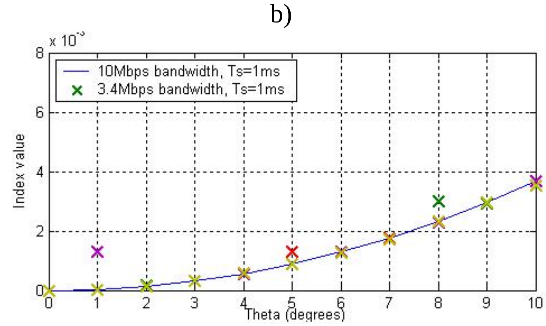

c)

Fig. 7. Performance index for the designed controllers: a) pole placement control strategy; b) optimal control strategy; c) H_∞ control strategy.

(reference:HPRN-CT-1999-00046), and the NECS projet (http://www.lag.ensieg.inpg.fr/canudas/necs.htm) founded by the CNRS.

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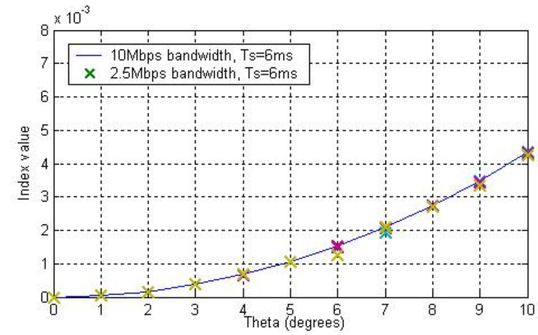

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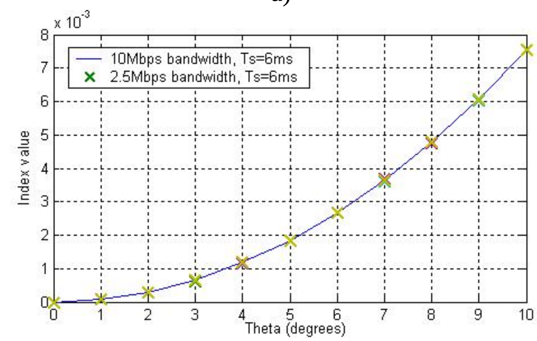

b)

Fig. 8. Performance index for the designed controllers: a) pole placement control strategy; b) optimal control strategy.

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VIII. APPENDIX - H_∞ CONTROLLER STRUCTURE

The H_∞ continuous controller has been designed solving the problem defined by (7); the order of the controller has then been reduced by residualization of the faster modes, such that its behavior could be preserved at low frequencies. Then the controller has been discretized with the Tustin's method with a sampling time $T_s = 1$ms. The resulting discrete controller structure $K(z) = [K_1(z)\ K_2(z)]$ is a five elements column vector, whose each element is a transfer function of the form:

$$K_i(z) = \frac{\sum_{k=0}^{n} a_{i,k} z^k}{\sum_{k=0}^{n} b_k z^k}, \tag{11}$$

where $n = 2$, $b = [1,\, 1.68,\, 0.68]$, $a_1 = [0.11,\, 0,\, 0.02]$, $a_2 = [-15.57,\, 15.3,\, -0.24]$, $a_3 = [-3.42,\, 3.18,\, 0.15]$, $a_4 = [-7.98,\, 7.85,\, -0.04]$, $a_5 = [-1.57,\, 1.74,\, -0.22]$.