Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004



# Analysis of Persistent Bounded Disturbance Rejection for Lurie Systems of The Neutral Type

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*Abstract*— **This paper mainly deals with the problem of persistent bounded disturbance rejection performance and stability for Lurie systems of the neutral delay type. Using Lyapunov-Krasovskii functional method, we simultaneously develop suf£cient conditions on persistent bounded disturbance rejection performance and stability (delay-dependent and delay-independent) in terms of linear matrix inequalities (LMIs). Similarly, we study the corresponding problem for Lurie systems of the neutral type with uncertainties. Finally a numerical example is given to illustrate the ef£ciency of the proposed result.**

## I. INTRODUCTION

Neutral type delay systems have received much attention in recent years, see, e.g., [12], [16], and the references therein. The systems that can be described by neutral type systems include steam or water pipes, lumped parameter networks interconnected by transmission lines, systems of turbojet engine, etc. The effect of small delays on the stability properties of some closed-loop neutral systems has been considered in [13] and the references therein. Recently, [16], [17] have developed suf£cient conditions on delay-independent stability of neutral delay systems; delay-dependent results have developed in [9], [12]. Furthermore, $H_\infty$ control has been considered in [15], [19]. However, what most papers concern focuses on stability or $H_\infty$ control, there are few papers simultaneously deal with persistent bounded disturbance rejection performance and absolute stability for neutral type systems.

On the other hand, the absolute stable problem, formulated by Lurie and coworkers in 40's, has been a well studied and fruitful area of research as presented in [2]. Many results in the theory of stability and control, such as Popov's criterion, the circle criterion, the positive-real lemma [6] are all closely related to the problem. Some of these tests, however, involve graphical constructions which induce dif£culties. The problem of absolute stability for time-delay Lurie systems has received attention see, e.g., [5]. In our paper, some algebra criteria are obtained by using the direct Lyapunov-Krasovskii functional method to absolute stability and performance problem for Lurie systems of the neutral type.

This work is supported by the National Natural Science Foundation of China under grants (No. 10372002, No. 60304014 and No. 60274001), the National Key Basic Research and Development Program (No. 2002CB312200), China Postdoctoral Program Foundation (No. 2003033076).

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Disturbance rejection problem induced by signal input (energy-bounded or peak-bounded) [3], [4] is one major issue in control systems. Since many objectives in control engineering practice involve signal peak and the disturbance signals of the plants are persistent bounded in most cases, many papers have dealt with the problem of persistent bounded disturbance rejection without delay (see, e.g., [6], [10], [14], and the references therein). [7], [8] have researched the optimal $L_1$ and $l_1$ control problem for continuous and discrete linear systems, respectively. Moreover [14] has discussed this problem for nonlinear systems. [10], [11] have studied disturbance rejection problem for Lurie system, but little attention has been drawn to the problem of persistent bounded disturbance rejection for delay systems.

Based on above researches, this paper considers the persistent bounded disturbance rejection problem for Lurie systems of the neutral type. organization of the paper is as follows. The preliminary results are given in Section 2. The main work is in Section 3: for both of delay-dependent and delay-independent cases, we give suf£cient conditions on guaranteeing stability and achieving $\rho$-performance for Lurie systems of the neutral type. Then similar analysis has been developed for the system with uncertainties. An example is given in Section 4 to illustrate the ef£ciency and feasibility of our proposed approach. The last section gives conclusion of this paper.

In this paper, $\mathbf{R}$ is the set of all real numbers, $\mathbf{R}^n$ is the set of all $n$-tuples of real numbers, and $\mathbf{R}^{m\times n}$ is the set of all real matrices with $m$ rows and $n$ columns. Denote by $A^T$ the transpose of a matrix $A$. $I$ denotes the unit matrix of appropriate dimension. $\mathcal{C}_{n,\tau} = \mathcal{C}([-\tau, 0], \mathbf{R^n})$ denotes Banach space composed of continuous vector-valued functions from $[-\tau, 0]$ to $R^n$. Given a linear operator $H : L_\infty \to L_\infty$, we de£ne the induced $L_\infty$ norm of $H$ to be

$$\|H\|_{i\infty} := \sup_{\|w\|_\infty \le 1} \|Hw\|_\infty$$

see [1] for more details.

## II. PRELIMINARIES

Consider Lurie systems of the neutral delay type with adding exogenous disturbance:

$$\begin{cases} \dot{x} - E\dot{x}(t-\tau) = Ax(t) + A_d x(t-\tau) + bu + B\omega, \\ x(t_0+s) = \psi(s), \qquad s \in [-\tau, 0], \\ y = c^T x, \\ z = Cx + D\omega, \end{cases} \tag{1}$$

where $x \in \mathbf{R}^n, \ \omega \in \mathbf{R}^p$ are the state and exogenous disturbance input vectors, $y \in \mathbf{R}$, $z \in \mathbf{R}^m$ are measured output and controlled output vectors, respectively. $u$ belongs to a class of sector nonlinearities, that is, $u = \phi(y)$ satis£es $\mu \geq \phi(y)/y \geq 0$ for $y \in \mathbf{R}, \ \mu > 0$ (denoted $u \in \mathcal{F}[0, \mu]$). $\tau > 0$ is a given constant scalar. $(t_0, \psi) \in \mathbf{R}^+ \times \mathcal{C}_{n,\tau}$, $\psi(\cdot)$ is a given continuous differentiable initial function on $[-\tau, \ 0]$. $A, A_d, E, B, C, D$ are all constant matrices with compatible dimensions. $b, c$ are $n$-dimensional vectors. The $L_\infty$ norm is de£ned by $\|w\|_\infty =: \sup_t \|w(t)\|_2$. Assume that the admissible disturbance set is $\mathcal{W} =: \{w : \mathbf{R} \to \mathbf{BR}^p, \ w$ is measurable $\}$, where $\mathbf{BR}^p = \{\omega \in \mathbf{R}^p : \ \|\omega\|_2 \leq 1\}$.

The *origin-reachable set* ($R_\infty(0)$) of system (1) is a set that the state of the system can reach from the origin for all admissible disturbances. A set $\mathcal{S}$ is said to be *a positive invariant set* for a dynamical system, if $x(t_0) \in \mathcal{S}$ implies the trajectory $x(t)$ of system (1) remains in $\mathcal{S}$ for all $t > t_0$. *An attractor* $\Omega$ of system (1) is a set that satis£es for any $u \in \mathcal{F}[0, \mu]$ and $\omega \in \mathcal{W}$, the state trajectories of system (1) initiating from any point outside of $\Omega$ eventually enter and remain in it. Obviously, an attractor is positively invariant.

For system (1), de£ne *performance* set by:

$$\Omega(\rho) =: \{x : \|z\|_\infty = \|Cx + D\omega\| \leq \rho, \ \forall \omega \in \mathcal{W}\}.$$

System (1) is said to have $\rho$-*performance* if $\|z\|_\infty \leq \rho$ for all $w \in \mathcal{W}$. By the de£nition of performance set, in order to show that system (1) has $\rho$-*performance*, we only need to prove that $\Omega(\rho)$ contains $R_\infty(0)$.

**Lemma 1.** [18] For any positive scalar $\alpha$ and symmetric positive de£nite matrix $Q \in \mathbf{R}^{n\times n}$, the following inequalities hold.

$$\begin{array}{c} 2x^T y \leq \frac{1}{\alpha} x^T x + \alpha y^T y, \\ 2x^T y \leq x^T Q^{-1} x + y^T Q y, \end{array}$$

where $x \in \mathbf{R}^n, \ y \in \mathbf{R}^n$.

To guarantee that the difference operator $\Re$: $C[-\tau, 0] \to \mathbf{R}^n$ given by $\Re(x_t) = x(t) - Ex(t-\tau) + \int_{t-\tau}^t A_d x(\upsilon) d\upsilon$ is stable, we assume [12]

$$\tau |A_d| + |E| < 1,$$

where $|\cdot|$ is any matrix norm.

## III. MAIN RESULTS

### *A. Analysis of Persistent Bounded Disturbance Rejection for Lurie Systems of The Neutral Type*

In this section, suf£cient conditions are given on guaranteeing the absolute stability (delay-dependent & delay-independent) and achieving persistent bounded disturbance rejection performance.

We £rst consider the delay-independent case.

**Theorem 1.** If there exist symmetric positive de£nite matrices $P, \ Q \in \mathbf{R}^{n\times n}$, and positive scalars $\alpha, \ \beta, \ \gamma$ satisfying the following matrix inequality:

$$\begin{bmatrix} \begin{pmatrix} A^T P + PA + \alpha P + Q \\ +\beta\mu^2 cc^T + \gamma\mu^2 cc^T \end{pmatrix} & Pb & PB & PA_d - A^T PE & 0 \\ b^T P & -\beta I & 0 & 0 & 0 \\ B^T P & 0 & -\alpha I & -B^T PE & 0 \\ A_d^T P - E^T PA & 0 & -E^T PB & -A_d^T PE - E^T PA_d - Q & E^T Pb \\ 0 & 0 & 0 & b^T PE & -\gamma I \end{bmatrix} < 0, \tag{2}$$

$$\begin{bmatrix} \alpha P & 0 & C^T \\ 0 & (\rho^2 - \alpha)I & D^T \\ C & D & I \end{bmatrix} > 0, \tag{3}$$

then for any $\tau > 0$ the ellipsoid $\Omega_P = \{x : x^T P x \leq 1\}$ is an attractor of system (1), $\Omega(\rho) \subset \Omega_P$ and system (1) has absolute $\rho$-performance. Furthermore, inequality (2) guarantees absolute stability of the system.

*Proof:* Let us consider the following Lyapunov-Krasovskii functional:

$$\begin{aligned} V(x_t) = \ & (x(t) - Ex(t-\tau))^T P (x(t) - Ex(t-\tau)) \\ & + \int_0^\tau x^T(t-s) Q x(t-s) ds. \end{aligned}$$

The time derivative of $V(x_t)$ along the trajectory of system (1) is given by

$$\begin{aligned} \dot{V}(x_t) = \ & 2(Ax + A_d x_\tau + bu + B\omega)^T P (x - Ex_\tau) \\ & + x^T Q x - x_\tau^T Q x_\tau \\ = \ & x^T (A^T P + PA + Q) x + 2x^T PB\omega + 2x^T Pbu \\ & + 2x^T PA_d x_\tau + x_\tau^T (-A_d^T PE - E^T PA_d^T - Q) x_\tau \\ & - 2\omega^T B^T PEx_\tau - 2x_\tau^T E^T Pbu - 2x^T A^T PEx_\tau. \end{aligned}$$

By Lemma 1 and the property of $\phi(y)$, the following inequalities hold.

$$\begin{aligned} 2x^T Pbu \leq \ & \tfrac{1}{\beta} x^T Pbb^T Px + \beta u^T u \\ \leq \ & \tfrac{1}{\beta} x^T Pbb^T Px + \beta\mu^2 x^T cc^T x, \\ -2x_\tau^T E^T Pbu \leq \ & \tfrac{1}{\gamma} x_\tau^T E^T Pbb^T PEx_\tau + \gamma\mu^2 x^T cc^T x. \end{aligned}$$

It follows that

$$\dot{V}(x) \leq \quad X^T \Omega X - \alpha x^T P x + \alpha \omega^T \omega,$$

where

$$\begin{aligned} X = \ & \begin{bmatrix} x^T & \omega^T & x_\tau^T \end{bmatrix}^T, \\ \Omega = \ & \begin{bmatrix} \nu & PB & PA_d - A^T PE \\ B^T P & -\alpha I & -B^T PE \\ {A_d}^T P - E^T PA & -E^T PB & \Sigma \end{bmatrix}, \\ \nu = \ & A^T P + PA + \tfrac{1}{\beta} Pbb^T P + \alpha P + Q \\ & + \beta\mu^2 cc^T + \gamma\mu^2 cc^T, \\ \Sigma = \ & -A_d^T PE - E^T PA_d^T - Q + \tfrac{1}{\gamma} E^T Pbb^T PE. \end{aligned}$$

Because $x^T P x > 1$ outside of $\Omega_P$, we obtain $\dot{V}(x) < 0$, if the following matrix inequality holds,

$$\begin{bmatrix} \nu & PB & PA_d - A^T PE \\ B^T P & -\alpha I & -B^T PE \\ {A_d}^T P - E^T PA & -E^T PB & \Sigma \end{bmatrix} < 0.$$

By Schur complement, it is equivalent to $(2)$. By the de£nition of attractor, the ellipsoid $\Omega_P = \{x : x^T P x \le 1\}$ is an attractor of system $(1)$.

Furthermore, for any $u \in \mathcal{F}[0, \mu]$, the negativity of the Lyapunov functional derivation does not use any information about the delay size and in conclusion, the absolute stability property holds for any positive delay.

On the other hand, by Schur complement, inequality $(3)$ is equivalent to the following inequality

$$\begin{bmatrix} \alpha P - C^T C & -C^T D \\ -D^T C & (\rho^2 - \alpha) I - D^T D \end{bmatrix} > 0.$$

From it we obtain

$$\begin{gathered} 0 < \alpha < \rho^2, \\ \alpha x^T P x + (\rho^2 - \alpha)\omega^T \omega - \|Cx + D\omega\|^2 > 0. \end{gathered}$$

It's obvious that if $x^T P x \le 1$ and $\omega^T \omega \le 1$, then we have $\|Cx + D\omega\|^2 \le \rho$. It follows that $\Omega_P \subset \Omega(\rho)$. Because $\Omega_P$ is a closed attractor which contains origin, it is a positive invariant set. While origin reachable set is the smallest positive invariant closed set that contains origin, we have $R_\infty(0, \mu) \subset \Omega_P \subset \Omega_\rho$. Thereby when the controller $u$ in system $(1)$ takes values from the nonlinear sector $\mathcal{F}[0, \mu]$, the closed system has $\rho$-performance. Because for any $u \in \mathcal{F}[0, \mu]$, $\dot{V}/_{(1)}(x) < 0$ is guaranteed, $\Omega_P$ is attractable for any $u \in \mathcal{F}[0, \mu]$, and thus we have $R_\infty(0) \subset \Omega_P \subset \Omega_\rho$. That is to say system $(1)$ has absolute $\rho$-performance. ■

Now, we consider the delay-dependent case for the system under consideration.

**Theorem 2.** For Lurie system of the neutral type $(1)$, given a positive scalar $\Gamma$, if there exist symmetric positive de£nite matrices $P$, $Q_1$, $Q_2 \in \mathbf{R}^{n \times n}$, positive scalars $\alpha$, $\beta$ satisfying $(3)$ and the following matrix inequality:

$$\begin{bmatrix} \Psi & Pb & \Gamma(A + A_d)^T P & \Gamma\mu c b^T P & \mu c b^T P & -(A + A_d)^T PE & PB & 0 \\ b^T P & -\beta & 0 & 0 & 0 & 0 & 0 & 0 \\ \Gamma P(A + A_d) & 0 & -\Gamma Q_1 & 0 & 0 & 0 & 0 & 0 \\ \Gamma\mu P b c^T & 0 & 0 & -\Gamma Q_1 & 0 & 0 & 0 & 0 \\ \mu P b c^T & 0 & 0 & 0 & -Q_2 & 0 & 0 & 0 \\ -E^T P(A + A_d) & 0 & 0 & 0 & 0 & -E^T Q_2 E & -E^T PB & 0 \\ B^T P & 0 & 0 & 0 & 0 & -B^T PE & -\alpha I & \Gamma B^T P \\ 0 & 0 & 0 & 0 & 0 & 0 & \Gamma PB & -\Gamma Q_1 \end{bmatrix} < 0, \quad (4)$$

where $\Psi = (A + A_d)^T P + P(A + A_d) + 3\Gamma A_d^T Q_1 A_d + \alpha P + 2E^T Q_2 E + \beta\mu^2 c c^T$, then for any $\tau : 0 \le \tau \le \Gamma$ the ellipsoid $\Omega_P = \{x : x^T P x \le 1\}$ is an attractor of system $(1)$, $\Omega_P \subset \Omega(\rho)$ and system $(1)$ has absolute $\rho$-performance. Moreover inequality $(4)$ guarantees that it is absolutly stable.

*Proof:* Let

$$\begin{aligned} &z(x_t) = x(t) - Ex(t - \tau) + \textstyle\int_{t-\tau}^{t} A_d x(\upsilon) d\upsilon, \\ &\dot{z}(x_t) = (A + A_d)x(t) + B\omega + bu. \end{aligned}$$

Take the following Lypunov-Krasovskii functional:

$$\begin{aligned} &V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t), \\ &V_1(x_t) = z^T(x_t) P z(x_t), \\ &V_2(x_t) = 3 \textstyle\int_{t-\tau}^{t} \int_{s}^{t} x^T(\upsilon) A_d^T Q_1 A_d x(\upsilon) d\upsilon ds, \\ &V_3(x_t) = 2 \textstyle\int_{t-\tau}^{t} x^T(\upsilon) E^T Q_2 E x(\upsilon) d\upsilon. \end{aligned}$$

The time derivative of $V(x)$ along the trajectory of system $(1)$ is given by

$$\begin{aligned} \dot{V}(x) = \; & \dot{V}_1(x) + \dot{V}_2(x) + \dot{V}_3(x) \\ = \; & 2((A + A_d)x + B\omega + bu)^T P(x - Ex_\tau + \\ & \textstyle\int_{t-\tau}^{t} A_d x(\upsilon) d\upsilon) + 3\tau x^T A_d^T Q_1 A_d x + 2x^T E^T Q_2 E x \\ & -3 \textstyle\int_{t-\tau}^{t} x^T(\upsilon) A_d^T Q_1 A_d x(\upsilon) d\upsilon - 2x_\tau^T E^T Q_2 E x_\tau \\ = \; & x^T((A + A_d)^T P + P(A + A_d) + 3\tau A_d^T Q_1 A_d + \\ & 2E^T Q_2 E)x + 2x^T PB\omega + 2x^T Pbu - 2u^T b^T PEx_\tau \\ & -2x^T(A + A_d)^T PEx_\tau - 2\omega^T B^T PEx_\tau + \\ & 2x^T(A + A_d)^T P \textstyle\int_{t-\tau}^{t} A_d x(\upsilon) d\upsilon - 2x_\tau^T E^T Q_2 E x_\tau \\ & +2\omega^T B^T P \textstyle\int_{t-\tau}^{t} A_d x(\upsilon) d\upsilon + 2u^T b^T P \int_{t-\tau}^{t} A_d x(\upsilon) d\upsilon \\ & -3 \textstyle\int_{t-\tau}^{t} x^T(\upsilon) A_d^T Q_1 A_d x(\upsilon) d\upsilon. \end{aligned}$$

By Lemma 1, the following inequalities hold.

$$\begin{aligned} &2x^T Pbu \le \tfrac{1}{\beta} x^T Pbb^T Px + \beta\mu^2 x^T cc^T x, \\ &-2ub^T PEx_\tau \le x^T \mu^2 cb^T PQ_2^{-1} Pbc^T x + x_\tau^T E^T Q_2 E x_\tau, \\ &2\omega^T B^T P \textstyle\int_{t-\tau}^{t} A_d x(\upsilon) d\upsilon \le \tau\omega^T B^T PQ_1^{-1} PB\omega + \tilde{Q}, \\ &2u^T b^T P \textstyle\int_{t-\tau}^{t} A_d x(\upsilon) d\upsilon \le \tau\mu^2 x^T cb^T PQ_1^{-1} Pbc^T x + \tilde{Q}, \\ &2x^T(A + A_d)^T P \textstyle\int_{t-\tau}^{t} A_d x(\upsilon) d\upsilon \\ &\le \tau x^T(A + A_d)^T PQ_1^{-1} P(A + A_d)x + \tilde{Q}, \end{aligned}$$

where

$$\tilde{Q} = \int_{t-\tau}^{t} x^T(\upsilon) A_d^T Q_1 A_d x(\upsilon) d\upsilon.$$

Hence we have

$$\dot{V}(x) \leq \quad X_1^T \Omega_1 X_1 - \alpha x^T P x + \alpha \omega^T \omega,$$

where

$$X_1 = \begin{bmatrix} x^T & x_\tau^T & \omega^T \end{bmatrix}^T,$$

$$\Omega_1 = \begin{bmatrix} \Lambda & -(A+A_d)^T PE & PB \\ -E^T P(A+A_d) & -E^T Q_2 E & -E^T PB \\ B^T P & -B^T PE & \Upsilon \end{bmatrix},$$

with

$$\begin{aligned} \Lambda = \quad & (A+A_d)^T P + P(A+A_d) + 3\tau A_d^T Q_1 A_d \\ & +2E^T Q_2 E + \alpha P + \tfrac{1}{\beta} P b b^T P + \beta \mu^2 c c^T \\ & +\mu^2 c b^T P Q_2^{-1} P b c^T + \tau \mu^2 c b^T P Q_1^{-1} P b c^T \\ & +\tau (A+A_d)^T P Q_1^{-1} P (A+A_d), \\ \Upsilon = \quad & \tau B^T P Q_1^{-1} P B - \alpha I. \end{aligned}$$

Since $x^T P x > 1$ for $x \notin \Omega_P$, we obtain $\dot{V}(x) < 0$, if $\Omega_1 < 0$. By Schur complement, it is equivalent to $(4)$ where $\Gamma$ is replaced by $\tau$. Thus the ellipsoid $\Omega_P$ is an attractor of system $(1)$ for given $\tau : 0 \leq \tau \leq \Gamma$.

As the proof of Theorem 1, for given $\tau : 0 \leq \tau \leq \Gamma$, system $(1)$ has absolute $\rho$-performance and systems $(1)$ is absolutely stable. ■

*B. Analysis of Persistent Bounded Disturbance Rejection for Lurie System of The Neutral Type with Structured Uncertainty*

Let us consider Lurie system of the neutral type with structured uncertainty:

$$\begin{cases} \dot{x} - E\dot{x}(t-\tau) = Ax(t) + A_d x(t-\tau) + B_1 p + B\omega + bu, \\ x(t_0 + s) = \psi(s), \qquad s \in [-\tau, 0], \\ y = c^T x, \\ z = Cx + D\omega, \\ q = C_1 x + D_{11} p, \\ p = \Delta q, \end{cases} \tag{5}$$

where $C_1$ and $D_{11}$ are matrices with compatible dimensions, $p$ and $q$ are uncertain input and output of the plant. $\Delta$ is the structured uncertainty between $p$ and $q$, i.e., $\Delta$ has the structural property:

$$\Delta \in \mathbf{\Delta} =: \{\Delta : \ \Delta = diag(\Delta_1, \cdots, \Delta_k, \delta_1 I, \cdots, \delta_l I)\},$$

where $k, l \in N, \ \delta_j \in R, \ \Delta_i$ is full block and $|\delta_j| \leq 1, \ \|\Delta_i\| \leq 1, \ $ for $i = 1, \cdots, k, \ j = 1, \cdots, l$. Other variables and matrices are de£ned as that of system $(1)$ and the performance set is de£ned as before. Such description of uncertainty can be £nd in [6].

The set of symmetric matrix corresponding to arbitrary blocked diagonal structure $\Delta$ can be described by:

$$S_\Delta =: \{S : S = S^T, \ S\Delta = \Delta S, \ \forall \ \Delta \in \mathbf{\Delta}\}.$$

Similarly, the set of antisymmetric matrix corresponding to arbitrary blocked diagonal structure $\Delta$ can be described by: (see [10] for more details)

$$T_\Delta =: \{T : T^T = -T, \ T\Delta = \Delta^T T, \ \forall \ \Delta \in \mathbf{\Delta}\}.$$

To study the performance problem, we give other de£nitions. *A robust attractor* $\Omega$ of system $(5)$ with respect to $\mathcal{W}$ and $\Delta$ is a set that satis£es for any $\|\Delta\| \leq 1$ and $\omega \in \mathcal{W}$, the state trajectory of system $(5)$ initiating from any point outside of $\Omega$ eventually enters and remains in it. Similarly we can de£ne *robust positive invariant set*. Obviously, a robust attractor is robust positively invariant.

**Lemma 2** [10]. If $S, \ \hat{S} \in S_\Delta$ and $T \in T_\Delta$, then $Y = ST\hat{S} \in T_\Delta$. Furthermore, if $S^{-1}$ exists, then $S^{-1} \in S_\Delta$.

From the property mentioned above, it's easy to say that for all $S \in S_\Delta, \ S > 0, \ T \in T_\Delta$, and for any blocked diagonal structure $\Delta$, if $\Delta \in \mathbf{\Delta}$ and $p = \Delta q$, then the following inequality holds.

$$\begin{bmatrix} q \\ p \end{bmatrix}^T \begin{bmatrix} S & T^T \\ T & -S \end{bmatrix} \begin{bmatrix} q \\ p \end{bmatrix} \geq 0. \tag{6}$$

Just as before, we £rst present a suf£cient condition on guaranteeing delay-independent stability and achieving $\rho$-performance.

**Theorem 3.** If there exist a symmetric positive de£nite matrices $P \in \mathbf{R}^{n \times n}$, a symmetric matrix $S \in S_\Delta$, an antisymmetric matrix $T \in T_\Delta$ and positive scalars $\alpha, \ \beta$ satisfying $(3)$ and the following matrix inequality:

$$\left[\begin{array}{cccc|} \Phi & Pb & PA_d - A^T PE & 0 \\ b^T P & -\beta & 0 & 0 \\ {A_d}^T P - E^T PA & 0 & -A_d^T PE - E^T PA_d & E^T Pb^T \\ 0 & 0 & bPE & -\gamma \\ B^T P & 0 & -B^T PE & 0 \\ B_1^T P + TC_1 & 0 & -B_1^T PE & 0 \\ SC_1 & 0 & 0 & 0 \end{array}\right.$$

$$\left.\begin{array}{ccc} PB & PB_1 + C_1^T T^T & C_1^T S \\ 0 & 0 & 0 \\ -E^T PB & -E^T PB_1 & 0 \\ 0 & 0 & 0 \\ -\alpha I & 0 & 0 \\ 0 & -S + TD_{11} + D_{11}^T T^T & D_{11}^T S \\ 0 & SD_{11} & -S \end{array}\right] < 0, \tag{7}$$

where $\Phi = A^TP + PA + \alpha P + \beta\mu^2 cc^T + \gamma\mu^2 cc^T$, then for any $\tau > 0$ the ellipsoid $\Omega_P = \{x : x^TPx \le 1\}$ is a robust attractor of system (5), $\Omega_P \subset \Omega(\rho)$ and system (5) has robust absolute $\rho$-performance. Moreover (7) guarantees that system (5) is robustly absolutely stable.

*Proof:* Let us consider the following Lyapunov functional:

$$\bar V(x_t) = V(x_t) + \int_0^t \begin{bmatrix} q(s) \\ p(s) \end{bmatrix}^T \begin{bmatrix} S & T^T \\ T & -S \end{bmatrix} \begin{bmatrix} q(s) \\ p(s) \end{bmatrix} ds,$$

where

$$V(x_t) = (x(t) - Ex(t-\tau))^T P (x(t) - Ex(t-\tau)).$$

By (6), we have $\dot V(x_t) < 0$ if $\dot{\bar V}(x_t) < 0$. The time derivative of $\bar V(x)$ along the trajectory of system (5) is given by

$$\begin{aligned} \dot{\bar V}(x) = \ & 2(Ax + A_dx_\tau + B\omega + B_1p + bu)^T P(x - Ex_\tau) \\ & + \begin{bmatrix} q \\ p \end{bmatrix}^T \begin{bmatrix} S & T^T \\ T & -S \end{bmatrix} \begin{bmatrix} q \\ p \end{bmatrix} \\ = \ & x^T(A^TP + PA)x + 2x^TPB\omega + 2x^TPA_dx_\tau \\ & + 2x^TPB_1p + 2x^TPbu - 2x^TA^TPEx_\tau \\ & - 2x_\tau^TA_d^TPEx_\tau - 2\omega^TB^TPEx_\tau - 2p^TB_1^TPEx_\tau \\ & - 2u^Tb^TPEx_\tau + \begin{bmatrix} q \\ p \end{bmatrix}^T \begin{bmatrix} S & T^T \\ T & -S \end{bmatrix} \begin{bmatrix} q \\ p \end{bmatrix}. \end{aligned}$$

By Lemma 1, the following inequalities hold.

$$\begin{aligned} 2x^TPbu \le \ & \tfrac{1}{\beta}x^TPbb^TPx + \beta\mu^2x^Tcc^Tx, \\ -2ub^TPEx_\tau \le \ & \tfrac{1}{\gamma}x_\tau^TE^TPb^TbPEx_\tau + \gamma\mu^2x^Tcc^Tx. \end{aligned}$$

Hence we have

$$\dot{\bar V}(x) \le X_2^T\Omega_2X_2 - \alpha x^TPx + \alpha\omega^T\omega,$$

where

$$X_2 = \begin{bmatrix} x^T & x_\tau^T & \omega^T & p^T \end{bmatrix}^T$$

$$\Omega_2 = \begin{bmatrix} \Pi & PA_d - A^TPE & PB & PB_1 + C_1^TSD_{11} + C_1^TT^T \\ A_d^TP - E^TPA & \Theta & -E^TPB & -E^TPB_1 \\ B^TP & -B^TPE & -\alpha I & 0 \\ B_1^TP + D_{11}^TSC_1 + TC_1 & -B_1^TPE & 0 & D_{11}^TSD_{11} + TD_{11} + D_{11}^TT^T - S \end{bmatrix}$$

with

$$\begin{aligned} \Pi = \ & A^TP + PA + \alpha P + C_1^TSC_1 + \tfrac{1}{\beta}Pbb^TP \\ & + \beta\mu^2cc^T + \gamma\mu^2cc^T, \\ \Theta = \ & -A_d^TPE - E^TPA_d^T + \tfrac{1}{\gamma}E^TPb^TbPE. \end{aligned}$$

Because $x^TPx > 1$ for $x \notin \Omega_P$, we obtain $\dot V(x) < 0$, if $\Omega_2 < 0$. By Schur complement, it is equivalent to (7). It's obvious that the ellipsoid $\Omega_P = \{x : x^TPx \le 1\}$ is a robust attractor of system (5).

As the proof of Theorem 1, system (5) has robust absolute $\rho$-performance and is robustly absolutely stable. ■

For the delay-dependent case, similar analysis is given.

**Theorem 4.** Given a positive scalar $\Gamma > 0$, if there exist symmetric positive de£nite matrices $P,\ Q_1,\ Q_2 \in \mathbf{R}^{n\times n}$, a symmetric matrix $S \in S_\Delta$, an antisymmetric matrix $T \in T_\Delta$, positive scalars $\alpha,\ \beta$ satisfying (3) and the following matrix inequality:

$$\begin{bmatrix} \Xi & \varpi & \pi & PB & 0 & PB_1 + C_1^TT^T & 0 & C_1^TS & Pb & \mu cb^TP & \Gamma\mu cb^TP \\ * & -\Gamma Q_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -E^TQ_2E & -E^TPB & 0 & -E^TPB_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\alpha I & \Gamma B^TP & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\Gamma Q_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \xi & \Gamma B_1^TP & D_{11}^TS & 0 & 0 & 0 \\ * & * & * & * & * & * & -\Gamma Q_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -S & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\beta & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -Q_2 & 0 \\ * & * & * & * & * & * & * & * & * & * & -\Gamma Q_1 \end{bmatrix} < 0, \quad (8)$$

where

$$\begin{aligned} \Xi = \ & (A + A_d)^TP + P(A + A_d) + 4\Gamma A_d^TQ_1A_d + 2E^TQ_2E \\ & + \alpha P + \beta\mu^2cc^T, \\ \xi = \ & -S + TD_{11} + D_{11}^TT^T,\ \varpi = \Gamma(A + A_d)^TP, \\ \pi = \ & -(A + A_d)^TPE, \end{aligned}$$

then for any $\tau : 0 \le \tau \le \Gamma$ the ellipsoid $\Omega_P = \{x : x^TPx \le 1\}$ is a robust attractor of system (5), $\Omega_P \subset \Omega(\rho)$ and system (5) has robust absolute $\rho$-performance. Moreover (7) guarantees that the system is robustly absolutely stable.

*Proof:* The proof of Theorem 4 can be easily obtained from Theorem 2 and Theorem 3 and is thus omitted. ■

## IV. AN ILLUSTRATIVE EXAMPLE

To illustrate the ef£ciency of our proposed approach, now we consider both of delay-independent and delay-dependent case for system (1). We choose the following parameters:

$$A = \begin{bmatrix} -5 & -1 \\ 1 & -5 \end{bmatrix},\ B = \begin{bmatrix} -0.1 & 1 \\ 1 & 0 \end{bmatrix},$$

$$A_d = \begin{bmatrix} -0.02 & 0.01 \\ 0.01 & -0.02 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0.2 \\ 1 & 0 \end{bmatrix},$$

$$D = \begin{bmatrix} 0.2 & -0.01 \\ -0.3 & 0.2 \end{bmatrix}, \; E = \begin{bmatrix} 0.01 & 0.1 \\ 0 & 0.01 \end{bmatrix},$$

$$b = \begin{bmatrix} 0.3 \\ 0.4 \end{bmatrix}, \; c = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}, \; \alpha = 0.9,$$

$\rho = 1.1$, and $\mu = 2$.

For delay-independent case we obtain $P$, $Q$, $\beta$, $\gamma$ that satisfy Theorem 1:

$$P = \begin{bmatrix} 3.4468 & 0.0948 \\ 0.0948 & 3.0140 \end{bmatrix}, \; Q = \begin{bmatrix} 9.2443 & 1.5439 \\ 1.5439 & 8.7514 \end{bmatrix},$$

$$\beta = 17.4333, \quad \gamma = 18.5293.$$

Hence we can obtain a robust attractor $\Omega_P$, system $(1)$ has absolute $\rho$-performance with $\rho = 1.1$ and is delay-independently stable for any $\tau > 0$.

For delay-dependent case, with the same parameters, We obtain $\Gamma \leq 14.0999$ and for $\Gamma = 14.0999$ we have the following solution to Theorem 2:

$$P = \begin{bmatrix} 1.3721 & 0.5132 \\ 0.5132 & 1.2202 \end{bmatrix}, \; Q_1 = \begin{bmatrix} 642.9098 & 634.0728 \\ 634.0728 & 795.8060 \end{bmatrix},$$

$$Q_2 = 10^5 \times \begin{bmatrix} 0.0310 & -0.2672 \\ -0.2672 & 2.3249 \end{bmatrix},$$

$\beta = 0.5732$. Hence if the delay size is smaller than $\Gamma$, system $(1)$ is delay-dependently stable for given $\tau : \; 0 \leq \tau \leq \Gamma$ and it has absolute $\rho$-performance with $\rho = 1.1$.

## V. CONCLUSIONS

For Lurie systems of the neutral type, using Lyapunov-Krasovskii functional method, we studied the problem of absolute stability and persistent bounded disturbance rejection performance for delay-dependent and delay-independent case. Suf£cient conditions on this problem were given in terms of LMIs. For Lurie system of the neutral type with uncertainty, similar analysis was given. Finally, a numerical example was given to illustrate the ef£ciency of the proposed approach.

## VI. ACKNOWLEDGMENTS

The authors gratefully acknowledge the contribution of reviewers' comments.

## REFERENCES


[1] J. Abedor, K. Nagpal and K. Poolla, "A linear matrix inequality approach to peak-to-peak gain minimization," *Int. J. Robust and Nonlinear Control,* vol. 6, 1996, pp. 899–927.

[2] M. A. Aizerman and F. R. Gantmacher, "Absolute stability of Regular Systems," Holden-Day, San Fransisco, 1964.

[3] F. Blanchini, "Persistent Disturbances Rejection via Static State Feedback," *IEEE. Trans. Automa. Contr.,* vol. 32(2), 1994, pp. 428–433.

[4] F. Blanchini, "Feedback control for linear time-invariant systems with state and control bounds in the persistence of disturbance," *IEEE. Trans. Automa. Contr.,* 1990, pp. 1131-1135.

[5] P. A. Bliman, "Absolute stability of nonautonomous delay systems: delay-dependent and delay-independent criteria," *Proceedings of the 38th IEEE Conference on Decision and Control,* 1999, pp. 2005–2010.

[6] S. Boyd, L. E., Ghaoui, E. Feron and V. Balakrishnan, "Linear matrix inequalities in system and control theory," Philadelpha: SIAM, 1994.

[7] M. A. Dahleh and J. B. Pearson, "$L^1$-optimal compensators for continuous-time systems," *IEEE. Trans. Automa. Contr.,* vol. 32(10), 1987, pp. 889–895.

[8] M. A. Dahleh and J. B. Pearson, "$l^1$-optimal feedback controllers for MIMO discrete-time systems," *IEEE. Trans. Automa. Contr.,* vol. 32(4), 1987, 314–322.

[9] Q. -L. Han, "Robust stability of uncertain delay-differential systems of neutral type," *Automatica,* 2002, pp. 719–723.

[10] F. Hao, T. Chu and L. Huang, "Disturbance rejection analysis for structured uncertainty Lur'e systems," in 15th Triennial World Congress, Barcelona, Spain, Paper No 706. 2002 IFAC.

[11] F. Hao, T. Chu, and L. Huang, "Persistent bounded disturbance rejection for Lurie systems," Appl. Math. Mech. vol. 24(3), 2003, pp. 318–325.

[12] C. -H. Lien, "New stability criterion for a class of uncertain nonlinear neutral time-delay systems," *Int. J. Systems Sci.,* vol. 32(2), 2001, pp. 215–219.

[13] H. Logemann and S. Townley, "The effect of small delays in the feedback loop on the stability of neutral systems," *Systems and Control Letters,* vol. 27, 1996, 267–274.

[14] W. -M. Lu, "Rejection of persistent $L_\infty$-bounded disturbances for nonlinear systems," *IEEE. Trans. Automa. Contr.,* vol. 43(12), 1998, pp. 1692–1702.

[15] M. S. Mahmoud, "Robust $H_\infty$ control of linear neutral systems," *Automatica,* 2000, pp. 757–764.

[16] J. H. Park and S. Won, "A note on stability of nertral delay-differential systems," *Journal of the Franklin Institute,* vol. 336, 1999, pp. 543–548.

[17] J. H. Park and S. Won, "A note on stability analysis of nertral systems with multiple time-delays," *International Journal of Systems Science,* vol. 32(4), 2001, pp. 409–412.

[18] I. R. Petersen, "A stabilization algorithm for a class of uncertain linear systems," *Systems and Control Letters,* vol. 8, 1987, pp. 351–357.

[19] S. Xu, J. Lam and C. Yang, "$H_\infty$ and positive real control for linear neutral delay systems," *IEEE. Trans. Automa. Contr.,* vol. 46, 2001, pp. 1321–1326.

[20] M. Vidyasagar, "Optimal rejection of persistent bounded disturbances,"*IEEE. Trans. Automa. Contr.,* vol. 31(6), 1986, pp. 527–534.