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Robust Stabilization for Singular Systems with Time-Delays and Saturating Controls

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***Abstract*—We studied the delay-dependent stabilization problem for a class of uncertain singular system with time-delays and saturating controls. Theorems derived give sufficient conditions for delay-dependent stabilization of the singular systems with a combination of saturating controls and multiple time-delays in both state and control; we assumed the delays to be constant bounded but unknown, moreover, the uncertainties are also described to be unknown but bounded and the nonlinear terms included in the systems are fallen into a set. Under these sufficient conditions, the solution of the uncertain singular system is regular, impulse free, and locally asymptotically stable for all admissible uncertainties. Furthermore, the results based on several Linear Matrix Inequalities (LMIs) are developed to guarantee stability and be computed effectively. Finally, we advance an example to demonstrate the superiority of this method.**

I. INTRODUCTION

CONYTOL of singular systems has been extensively studied in the past years due to the fact that singular systems better describe physical systems than regular ones. A great number of results based on the theory of regular systems (or state-space systems) have been extended to the area of singular systems [1]-[2]. Recently, robust stability and robust stabilization for uncertain singular systems with time-delays have been considered [3]-[5].

Moreover, the problem of stabilizing linear systems with saturating controls has been widely studied these last years because of its practical interest [6] and [7]. However, to the best of our knowledge, the problems of robust stabilization for uncertain singular system included both time-delays and saturating controls have not been fully investigated yet.

In this paper, we concerned with the delay-dependent robust stabilization of a continuous-time subject to multiple time-delays in both state and control, saturating controls and nonlinear terms. The synthesis problem addressed is to design a memoryless state feedback control law such that the resulting closed-loop system is regular, impulse free and stable for all admissible uncertainties, and a sufficient condition for the existence of such a control law is presented in terms of several linear matrix inequalities (LMIs).

II. SYSTEM DESCRIPTION AND DEFINITIONS

Consider the following uncertain singular systems with time-delays and saturating controls described by

$$
\begin{aligned}
(\Sigma):\ E\dot{x}(t) &= (A_0+\Delta A_0(x,t))x(t)+\sum_{i=1}^{k}(A_i+\Delta A_i(x,t)) \\
&\quad\times x(t-h_i(t))+E_{10}f(\sigma(t))+B_{10}w(t) \\
&\quad+(B_{20}+\Delta B_{20}(x,t))u'(t) \\
&\quad+\sum_{i=1}^{k}(B_{2i}+\Delta B_{2i}(x,t))u'(t-g_i(t)),
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
&u'(t)=sat(u(t)), x(t)=\phi(t),\ t\in[-\tau,0], \sigma(t)=Cx(t) \\
&sat(u(t))=[sat(u_1(t))\ \ sat(u_2(t))\ \ \cdots\ \ sat(u_m(t))],
\end{aligned}
$$

where $x(t)\in R^n$ is the state vector, $u(t)\in R^m$ is control input vector to the actuator (emitted from the designed controller), $u'(t)\in R^m$ is the control input vector to the plant, $w(t)\in R^p$ is the disturbance input vector from $L_2[0,\infty)$. The matrix $E\in R^{n\times n}$ may be singular, we shall assume that rank $E=r\le n$. The matrices $A_0, A_i, E_{10}, B_{10}, B_{20}$ and B_{2i} are known real constant matrices with appropriate dimensions. The matrices $\Delta A_0(\bullet), \Delta A_i(\bullet), \Delta B_{20}(\bullet)$ and $\Delta B_{2i}(\bullet)$ are time-invariant matrices representing norm-bounded parameter uncertainties, and are assumed to be of the following form:

$$
\begin{aligned}
&\begin{bmatrix}\Delta A_0(\bullet) & \Delta A_i(\bullet) & \Delta B_{20}(\bullet) & \Delta B_{2i}(\bullet)\end{bmatrix} \\
&=GF(x,t)\begin{bmatrix}H_1 & H_{2i} & H_3 & H_{4i}\end{bmatrix}
\end{aligned} \tag{2}
$$

where G, H_1, H_{2i}, H_3 and H_{4i} are known real constant matrices with appropriate dimensions. The uncertain matrix $F(x(t),t)$ with Lebesgue measurable elements satisfies

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$$F^T(x(t),t)F(x(t),t) \le I. \quad (3)$$

The input vector is assumed to satisfy actuator limitations, i.e. $u(t) \in U \subset R^m$ with

$$U = \{u(t) \in R^m; -u_{0(i)} \le u_{(i)}(t) \le u_{0(i)}, u_{0(i)} > 0, i = 1,\cdots,m\} \quad (4)$$

The actuator is described by the nonlinearity

$$sat(u_{(i)}(t)) = \begin{cases} u_{0(i)} & \text{if } u_{(i)}(t) > u_{0(i)} \\ u_{(i)}(t) & \text{if } -u_{0(i)} \le u_{(i)}(t) \le u_{0(i)} \\ -u_{0(i)} & \text{if } u_{(i)}(t) < -u_{0(i)} \end{cases} \quad (5)$$

$h_i(t)$ and $g_i(t)$ are unknown scalars denoting the delays in the state and control, respectively, and it is assumed that there exist positive numbers h, g and τ such that

$$0 \le h_i(t), g_i(t) \le h, g \le \tau \quad (6)$$

for all t, $i = 1,\cdots,k$. $\phi(t)$ is smooth vector-valued continuous initial function defined in the Banach space C_τ. In this paper, every nonlinear term is assumed to be of the form as follows

$$\begin{aligned} f_j(\bullet) \in K_j[0,k_j] = \{f_j(\sigma_j) \,|\, f_j(0) = 0, 0 < \sigma_j f_j(\sigma_j) \\ \le k_j\sigma_j^2 \ (\sigma_j \ne 0)\}, j = 1,2,\cdots,n \end{aligned} \quad (7)$$

where k_j are positive scalars.

The nominal unforced singular delay systems (1) can be written as

$$E\dot{x}(t) = A_0x(t) + \sum_{i=1}^{k} A_i x(t - h_i(t)) + E_{10}f(\sigma(t)). \quad (8)$$

Using the Leibniz-Newton formula [7], then the singular delay system (8) can be written as

$$\begin{aligned} E\dot{x}(t) &= (A_0 + \sum_{i=1}^{k} A_i)x(t) + E_{10}f(\sigma(t)) \\ &\quad - \sum_{i=1}^{k} A_i \int_{-h_i(t)}^{0} [A_0x(t+\theta) + \sum_{i=1}^{k} A_i \\ &\quad \times x(t - h_i(t) + \theta) + E_{10}f(\sigma(t+\theta))]d\theta \end{aligned} \quad (9)$$

$$x(t) = \phi(t),\ t \in [-2\tau, 0]$$

Throughout this paper, we shall use the following concepts and introduce the following useful lemmas.

Definition 1. The singular delay systems (8) is said to be regular and impulse free if the pair $(E, A + \sum_{i=1}^{k} A_i)$ is regular and impulse free [3].

Lemma 1 (Krasovskii theorem [1]). The singular delay systems (8) is said to be locally asymptotically stable if there exists a positive definite symmetric matrix P, positive scalars π_1, π_2, π_3, v and γ, for any initial condition $\phi(t) \in C_\tau^v$, the trajectories of the singular delay systems (8) remain confined in the set $\Omega(P,E,\gamma) = \{x(t) \in R^n \,|\, x^T(t)P^{-1}Ex(t) < \gamma^{-1}, \gamma > 0\}$, whereas, for a continuous function $V(x(t),t): R^n \times R^+ \to R$ such that $\pi_1 \|x(t)\|^2 \le V(x(t),t) \le \pi_2 \|x(t)\|^2$, and if $\dot{V}(x(t),t) \le -\pi_3 \|x(t)\|^2$, $V(x(t),t) \le V(\phi(0),0)$.

Definition 2. The uncertain singular systems (Σ) is said to be robustly stable if the systems (Σ) with $u(t) \equiv 0, u(t - g_i(t)) \equiv 0, w(t) \in L_2[0,\infty)$ is regular, impulse free and locally asymptotically stable for all admissible uncertainties.

Definition 3. The uncertain singular delay systems (Σ) is said to be robustly stabilizable if there exists a linear state feedback control law $u(t) = \Lambda x(t), \Lambda \in R^{m\times n}$ such that the resultant closed-loop system is robustly stable in the sense of Definition 3. In this case, $u(t) = \Lambda x(t)$ is said to be a robust state feedback control law for system (Σ).

Lemma 2 [8]. Given vector x, y, a positive definite symmetric matrix R with appropriate dimensions, then for any scalar $\varepsilon > 0$, we have

$$\pm 2x^T y \le \varepsilon x^T R x + \varepsilon^{-1} y^T R^{-1} y.$$

Lemma 3 [8]. Given matrices A, Θ, Ξ, Γ and $F(\sigma)$ of appropriate dimensions and with Θ symmetrical and $F(\sigma)$ satisfying $F^T(\sigma)F(\sigma) \le I$. Then we have:

a) If the following inequality holds,

$$\Theta + \Gamma F(\sigma)\Xi + (\Gamma F(\sigma)\Xi)^T < 0$$

if and only if there exists a scalar $\varepsilon > 0$ such that

$$\Theta + \varepsilon\Gamma\Gamma^T + \varepsilon^{-1}\Xi^T\Xi < 0$$

b) For any symmetric matrix $P > 0$ and scalar $\varepsilon > 0$ such that $\varepsilon I - \Xi P \Xi^T > 0$, then

$$\begin{aligned} &(A + \Gamma F(\sigma)\Xi)P(A + \Gamma F(\sigma)\Xi)^T \\ &\le APA^T + AP\Xi^T(\varepsilon I - \Xi P \Xi^T)^{-1}\Xi PA^T + \varepsilon\Gamma\Gamma^T \end{aligned}$$

III. ANALYSIS OF ROBUST STABILITY

A. Analysis of robust Stability of Systems (8)

The main result is derived as follows, it gives the sufficient condition of robust stability for the singular delay system (8).The proof of it is similar to [2] and [9] and is omitted.

Theorem 1. If there exists a series of positive definite symmetric $Q, Q_{1i}, Q_{2i}, Q_{3i}, i = 1\cdots k$, a matrix P, and the scalars ε, γ and τ such that

$$EP^T = PE^T \ge 0 \quad (10a)$$

$$M = \begin{bmatrix} W & \tau N_1 & \tau N_2 & \tau N_3 \\ \tau N_1^T & \tau\Omega_1 & 0 & 0 \\ \tau N_2^T & 0 & \tau\Omega_2 & 0 \\ \tau N_3^T & 0 & 0 & \tau\Omega_3 \end{bmatrix} < 0 \quad (10b)$$

$$\begin{bmatrix} Q & P^T \\ P & I \end{bmatrix} \ge 0 \quad (10c)$$

where

$$W = (A_0 + \sum_{i=1}^{k} A_i)P^T + P(A_0 + \sum_{i=1}^{k} A_i)^T$$
$$+\tau \sum_{i=1i}^{k} A_i(Q_{1i} + Q_{2i} + Q_{3i})A_i^T + \varepsilon E_{10}E_{10}^T + \varepsilon^{-1}PC^T K^T KCP^T$$
$$N_1 = [PA_0^T \ \cdots \ PA_0^T],\ N_2 = [PA_1^T \ \ PA_2^T \ \cdots \ PA_k^T],$$
$$N_3 = [PC^T K^T E_{10}^T \ \ PC^T K^T E_{10}^T \ \cdots \ PC^T K^T E_{10}^T],$$
$$\Omega_1 = -diag\{Q_{11}, Q_{12}, \cdots, Q_{1k}\}, K = diag\{k_1, \cdots k_n\},$$
$$\Omega_2 = -diag\{Q_{21}, Q_{22}, \cdots, Q_{2k}\},\ \Omega_3 = -diag\{Q_{31}, Q_{32}, \cdots, Q_{3k}\},$$

then the singular delay system (8) is regular, impulse free and locally asymptotically stable for any initial condition belonging to the set $\Phi_0 = \{\phi(t) \| \| \phi(t) \|^2 \le \delta\}$ with $\delta = \lambda_{\min}(Q)/\gamma\pi_2$

where

$$\pi_2 = \lambda_{\max}(EP^T) + \frac{k\tau^2}{2}\max_i \lambda_{\max}[PA_0^T Q_{1i}^{-1} A_0 P^T]$$
$$+\frac{3k\tau^2}{2}\max_i \lambda_{\max}[PA_i^T Q_{2i}^{-1} A_i P^T]$$
$$+\frac{k\tau^2}{2}\max_i \lambda_{\max}[PC^T K^T E_{10}^T Q_{3i}^{-1} E_{10} KCP^T])$$

B. Disturbance-Free Case (with $w(t) = 0$)

When $w(t) = 0$, for the uncertain singular system (Σ), introduce the control law $u(t) = 2\Lambda x(t)$, where the control law gain matrix $\Lambda \in R^{m\times n}$ is to be found, and the closed-loop system is

$$(\Sigma'):\ E\dot{x}(t) = (A_{0\Delta} + B_{20\Delta}\Lambda)x(t) + \sum_{i=1}^{k} A_{i\Delta}x(t - h_i(t))$$
$$+ E_{10}f(\sigma(t)) + \sum_{i=1}^{k} B_{2i\Delta}\Lambda x(t - g_i(t)) \qquad (11)$$
$$+ B_{20\Delta}\eta(t) + \sum_{i=1}^{k} B_{2i\Delta}\eta(t - g_i(t))$$
$$x(t) = \phi(t),\ t \in [-\tau, 0]$$

where $[\bullet]_\Delta = [\bullet] + \Delta[\bullet]$ ($[\bullet]$ denoting the matrix), and

$$\eta(t) = sat(2\Lambda x(t)) - \Lambda x(t),$$
$$\eta(t - g_i(t)) = sat(2\Lambda x(t - g_i(t)) - \Lambda x(t - g_i(t))$$

Obviously, the vector function $\eta(t)$ satisfies the following inequality

$$\eta^T(t)\eta(t) \le x^T(t)\Lambda^T\Lambda x(t),$$
$$\eta(t - g_i(t)) \le x^T(t - g_i(t))\Lambda^T\Lambda x(t - g_i(t)) \qquad (12)$$

By (4), one has $x(t) \in S(u_0, 1_m)$, where

$$S(u_0, 1_m) = \{x(t) \in R^n \mid \{u(t) \in R^m; -u_{0(i)}$$
$$\le \Lambda_{(i)}x(t) \le u_{0(i)}, i = 1, \cdots, m\} \qquad (13)$$

Using the same method as theorem 1 and taking into (12), (13) and [7], furthermore introducing the idea of generalized quadratic stability and generalized quadratic stabilization in [3], one can deduce the following corollary.

Corollary 1 (Disturbance-free case). If there exist a series of positive definite symmetric $Q, Q_{1i}, Q_{2i}, Q_{3i}$, $Q_{4i}, Q_{5i}, Q_{6i}, R_{1i}$, $R_{2i}, R_{3i}, R_{4i}, R_{5i}, R_{6i}, i = 1\cdots k$, a matrix P, and the scalars $\varepsilon_2, \varepsilon_{3i}, \gamma, i = 1, \cdots, k$ and τ such that $M_\Delta < 0, \Xi_1 \ge 0, \Xi_2 \le 0$ and the expression of $EP^T = PE^T \ge 0$ hold, then the uncertain singular delay system (1) is regular, impulse free and locally asymptotically stable for any initial condition belonging to the set $\Phi_0 = \{\phi(t) \| \| \phi(t) \|^2 \le \delta\}$ with $\delta = \dfrac{\lambda_{\min}(Q)}{\gamma\pi_2}$,

where

$$\pi_2 = \lambda_{\max}(EP^T) + k\tau\lambda_{\max}(P\Lambda^T\Lambda P^T) + \frac{k\tau^2}{2}\max_i \lambda_{\max}[P(A_{0\Delta}$$
$$+ B_{20\Delta}\Lambda)^T(Q_{1i}^{-1} + R_{1i}^{-1})(A_{0\Delta} + B_{20\Delta}\Lambda)P^T]$$
$$+\frac{3k\tau^2}{2}\max_i \lambda_{\max}[PA_{i\Delta}^T(Q_{2i}^{-1} + R_{2i}^{-1})A_{i\Delta}P^T]$$
$$+\frac{k\tau^2}{2}\max_i \lambda_{\max}[PC^T K^T E_{10}^T(Q_{3i}^{-1} + R_{3i}^{-1})E_{10}KCP^T]$$
$$+\frac{3k\tau^2}{2}\max_i \lambda_{\max}[P(B_{2i\Delta}\Lambda)^T(Q_{4i}^{-1} + R_{4i}^{-1})(B_{2i\Delta}\Lambda)P^T]$$
$$+\frac{k\tau^2}{2}\max_i \lambda_{\max}[P(B_{20\Delta}\Lambda)^T(Q_{5i}^{-1} + R_{5i}^{-1})$$
$$\times(B_{20\Delta}\Lambda)P^T] + \frac{3k\tau^2}{2}\max_i \lambda_{\max}[P(B_{2i\Delta}\Lambda)^T$$
$$\times(Q_{6i}^{-1} + R_{6i}^{-1})(B_{2i\Delta}\Lambda)P^T]$$

and

$$\Xi_1 = \begin{bmatrix} P^{-1}E & \Lambda_i^T \\ \Lambda_i & \gamma u_{0i}^2 \end{bmatrix} \ge 0 \qquad (14a)$$

$$\Xi_2 = \begin{bmatrix} Q & P^T \\ P & I \end{bmatrix} \le 0 \qquad (14b)$$

and M_Δ is shown in the next page, where N_3, Ω_1, Ω_2 and Ω_3 are the same as theorem 1, and

$$W_\Delta = (A_{0\Delta} + \sum_{i=1}^{k} A_{i\Delta} + B_{20\Delta}\Lambda + \sum_{i=1}^{k} B_{2i\Delta}\Lambda)P^T + P(A_{0\Delta}$$
$$+ \sum_{i=1}^{k} A_{i\Delta} + B_{20\Delta}\Lambda + \sum_{i=1}^{k} B_{2i\Delta}\Lambda)^T + \tau\sum_{i=1}^{k} A_{i\Delta}(Q_{1i}$$
$$+ Q_{2i} + Q_{3i} + Q_{4i} + Q_{5i} + Q_{6i})A_{i\Delta}^T + \tau\sum_{i=1}^{k} B_{2i\Delta}\Lambda(Q_{1i}$$
$$+ Q_{2i} + Q_{3i} + Q_{4i} + Q_{5i} + Q_{6i})(B_{2i\Delta}\Lambda)^T$$
$$+ \varepsilon_1 E_{10}E_{10}^T + \varepsilon_1^{-1}PC^T K^T KCP^T + \varepsilon_2 B_{20\Delta}B_{20\Delta}^T$$
$$+ \varepsilon_2^{-1}P\Lambda^T\Lambda P^T + \sum_{i=1}^{k} \varepsilon_{3i}B_{2i\Delta}B_{2i\Delta}^T + \sum_{i=1}^{k} \varepsilon_{3i}^{-1}P\Lambda^T\Lambda P^T$$

$$N_{1\Delta} = [P(A_{1\Delta} + B_{20\Delta}\Lambda)^T \ \cdots \ P(A_{1\Delta} + B_{20\Delta}\Lambda)^T],$$
$$N_{20} = [PA_{1\Delta}^T \ PA_{1\Delta 2}^T \ \cdots \ PA_{k\Delta}^T]$$

$$N_{4\Delta}=[P(B_{21\Delta}\Lambda)^T \quad \cdots \; P(B_{2k\Delta}\Lambda)^T]=N_{6\Delta},$$
$$N_{5\Delta}=[P(B_{20\Delta}\Lambda)^T \quad \cdots \; P(B_{20\Delta}\Lambda)^T],$$
$$\Omega_j=-diag\{Q_{j1},Q_{j2},\cdots,Q_{jk}\},\ j=4,5,6,$$
$$\Omega'_j=-diag\{R_{j1},R_{j2},\cdots,R_{jk}\},\ j=1,2,\cdots,6.$$

$$M_\Delta=\begin{bmatrix} W_\Delta & \tau N_{1\Delta} & \tau N_{2\Delta} & \tau N_3 & \tau N_{4\Delta} & \tau N_{5\Delta} & \tau N_{6\Delta} & \tau N_{1\Delta} & \tau N_{2\Delta} & \tau N_3 & \tau N_{4\Delta} & \tau N_{5\Delta} & \tau N_{6\Delta} \\ \tau N_{1\Delta}^T & \tau\Omega_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \tau N_{2\Delta}^T & 0 & \tau\Omega_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \tau N_3^T & 0 & 0 & \tau\Omega_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \tau N_{4\Delta}^T & 0 & 0 & 0 & \tau\Omega_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \tau N_{5\Delta}^T & 0 & 0 & 0 & 0 & \tau\Omega_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \tau N_{6\Delta}^T & 0 & 0 & 0 & 0 & 0 & \tau\Omega_6 & 0 & 0 & 0 & 0 & 0 & 0 \\ \tau N_{1\Delta}^T & 0 & 0 & 0 & 0 & 0 & 0 & \tau\Omega'_1 & 0 & 0 & 0 & 0 & 0 \\ \tau N_{2\Delta}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tau\Omega'_2 & 0 & 0 & 0 & 0 \\ \tau N_3^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tau\Omega'_3 & 0 & 0 & 0 \\ \tau N_{4\Delta}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tau\Omega'_4 & 0 & 0 \\ \tau N_{5\Delta}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tau\Omega'_5 & 0 \\ \tau N_{6\Delta}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tau\Omega'_6 \end{bmatrix}<0 \qquad (14c)$$

In the following we shall discuss how to solve the control law gain matrix Λ in the following analysis by using LMI technology. Assume that there exist scalars $\beta_0,\beta_i,\beta_{ji},\delta_{ji}>0$ and positive definite symmetric matrices T_0,T_i,T_{ji},P_{ji} such that the following inequalities are satisfied:

$$B_{20}B_{20}^T+B_{20}H_4^T(\beta_0 I-H_3H_3^T)^{-1}H_3B_{20}^T+\beta_0 GG^T\le T_0 \qquad (15)$$
$$B_{2i}B_{2i}^T+B_{2i}H_{4i}^T(\beta_i I-H_4H_{4i}^T)^{-1}H_{4i}B_{2i}^T+\beta_i GG^T\le T_i \qquad (16)$$
$$\begin{aligned}&A_iQ_{ji}A_i^T+A_iQ_{ji}H_{2i}^T(\beta_{ji}I-H_{2i}Q_{ji}H_{2i}^T)^{-1}H_{2i}Q_{ji}A_i^T\\&+\beta_{ik}GG^T\le T_{ji}\end{aligned} \qquad (17)$$
$$\begin{aligned}&B_{2i}Z_{ji}B_{2i}^T+B_{2i}Z_{ji}H_{4i}^T(\delta_{ji}I-H_{4i}Z_{ji}H_{4i}^T)^{-1}\\&\times H_{4i}Z_{ji}B_{2i}^T+\delta_{ji}GG^T\le P_{ji}\end{aligned} \qquad (18)$$

where $Z_{ji}\ge\Lambda Q_{ji}\Lambda^T,\ j=1,\cdots,6,\ i=1,\cdots,k$, and

$$\begin{aligned}&\beta_0 I-H_3H_3^T>0,\beta_i I-H_{4i}H_{4i}^T>0,\\&\beta_{ij}I-H_{2i}Q_{ji}H_{2i}^T>0,\delta_{ji}I-H_{4i}Z_{ji}H_{4i}^T>0\end{aligned} \qquad (19)$$

Using Lemma 3b, we have

$$\begin{aligned}&B_{20\Delta}B_{20\Delta}^T\le T_0,B_{2i\Delta}B_{2i\Delta}^T\le T_i,\\&A_{i\Delta}Q_{ji}A_{i\Delta}^T\le T_{ji},B_{2i\Delta}\Lambda Q_{ji}\Lambda^T B_{2i\Delta}^T\le P_{ji}\end{aligned} \qquad (20)$$

By Corollary 1, (2) and (20), it follows that

$$M_\Delta=\tilde{M}+\Theta_1F(x(t),t)\Theta_2+(\Theta_1F(x(t),t)\Theta_2)^T<0 \qquad (21)$$

where $\Theta_1=diag\{G,G,\cdots,G\}$, $\Theta_2=\begin{bmatrix}\tilde{\Theta}_2 & 0 & \cdots & 0\end{bmatrix}$, and

$$\tilde{\Theta}_2^T=[P(H_1+\sum_i H_{2i}+H_3\Lambda+\sum_i H_{4i}\Lambda)^T,\ \tilde{\Theta}_{21}^T,\ \tilde{\Theta}_{21}^T],$$
$$\begin{aligned}\tilde{\Theta}_{21}^T=[&P(H_1+H_3\Lambda)^T,\ \cdots,\ P(H_1+H_3\Lambda)^T,PH_{21}^T,\ \cdots,\\&PH_{2k}^T,\ 0,\ \cdots,0,P\Lambda^T H_{41}^T,\cdots,P\Lambda^T H_{4k}^T,\ P\Lambda^T H_3^T,\\&\cdots,P\Lambda^T H_3^T,P\Lambda^T H_{41}^T,\ \cdots,P\Lambda^T H_{4k}^T],\end{aligned}$$
$$\begin{aligned}\tilde{M}'=\tilde{W}'&-\tau N_1(\Omega_1^{-1}+\Omega_1'^{-1})N_1^T-\tau N_2(\Omega_2^{-1}+\Omega_2'^{-1})N_2^T\\&-\tau N_3(\Omega_3^{-1}+\Omega_3'^{-1})N_3^T-\tau N_4(\Omega_4^{-1}+\Omega_4'^{-1})N_4^T\\&-\tau N_5(\Omega_5^{-1}+\Omega_5'^{-1})N_5^T-\tau N_6(\Omega_6^{-1}+\Omega_6'^{-1})N_6^T.\end{aligned}$$

where the matrix N_i is the matrix $N_{i\Delta}(i=1,\cdots,6)$ without uncertainty in corollary 1, and

$$\begin{aligned}\tilde{W}'=&(A_0+\sum_{i=1}^k A_i+B_{20}\Lambda+\sum_{i=1}^k B_{2i}\Lambda)P^T+P(A_0+\sum_{i=1}^k A_i\\&+B_{20}\Lambda+\sum_{i=1}^k B_{2i}\Lambda)^T+\varepsilon_1E_{10}E_{10}^T+\varepsilon_1^{-1}PC^TK^TKCP^T\\&+\tau\sum_{j=1}^6\sum_{i=1}^k P_{ji}+\tau\sum_{j=1}^6\sum_{i=1}^k T_{ji}+\varepsilon_2T_0+\varepsilon_2^{-1}P\Lambda^T\Lambda P^T\\&+\sum_{i=1}^k\varepsilon_{3i}T_i+\sum_{i=1}^k\varepsilon_{3i}^{-1}P\Lambda^T\Lambda P^T\end{aligned}$$

By Lemma 3a and (21), we can obtain that there exists a scalar $\alpha>0$ such that

$$\tilde{M}'+\alpha\Theta_1\Theta_1^T+\alpha^{-1}\Theta_2^T\Theta_2<0 \qquad (22)$$

For simplicity we introduce the matrix $\Upsilon\in R^{n\times(n-r)}$ satisfying $E\Upsilon=0$ and rank $\Upsilon=n-r$. It's easy to see that there exist invertible matrices L_1 and $L_2\in R^{n\times n}$ from the proof of Theorem 1 such that

$$\bar{P}=L_1PL_2^{-T}=\begin{bmatrix}\bar{P}_{11} & \bar{P}_{12}\\ 0 & \bar{P}_{22}\end{bmatrix}$$

where $\bar{P}_{11}=\bar{P}_{11}^T\ge 0,\bar{P}_{12}\in R^{r\times(n-r)},\bar{P}_{22}\in R^{(n-r)\times(n-r)}$. On the other hand, from $E\Upsilon=0$ and rank $\Upsilon=n-r$, it implies that there exists an invertible matrix $\Gamma\in R^{(n-r)\times(n-r)}$ such that

$$\Upsilon=L_2\begin{bmatrix}0\\ I_{n-r}\end{bmatrix}\Gamma$$

Hence

$$\begin{aligned}P=&L_1^{-1}\begin{bmatrix}\bar{P}_{11} & \bar{P}_{12}\\ 0 & \bar{P}_{22}\end{bmatrix}L_2^T=(L_1^{-1}\begin{bmatrix}I_r & 0\\ 0 & 0\end{bmatrix}L_2^{-1})\\&\times(L_2\begin{bmatrix}\bar{P}_{11} & 0\\ 0 & I_{n-r}\end{bmatrix}L_2^T)+(L_1^{-1}\begin{bmatrix}\bar{P}_{12}\\ \bar{P}_{22}\end{bmatrix}\Gamma^{-T})\\&\times(\Gamma^T\begin{bmatrix}0 & I_{n-r}\end{bmatrix}L_2^T)\triangleq EX+Y\Upsilon^T\end{aligned}$$

where

$$X=L_2\begin{bmatrix}\bar{P}_{11} & 0\\ 0 & I_{n-r}\end{bmatrix}L_2^T>0,\quad Y=L_1^{-1}\begin{bmatrix}\bar{P}_{12}\\ \bar{P}_{22}\end{bmatrix}\Gamma^{-T}$$

Furthermore

$$EP^T=E(EX+Y\Upsilon^T)^T=EXE^T=(EX+Y\Upsilon^T)E^T=PE^T\ge 0$$

Define $\Psi=\Lambda(EX+Y\Upsilon^T)^T\triangleq\Lambda Z^T(X,Y)$. Without loss of generality, we can assume that $Z(X,Y)=EX+Y\Phi^T$ is invertible. Define matrix $\tilde{M}''$, as shown at the top of the next page, where

$$\begin{aligned}\tilde{W}''=&(A_0+\sum_{i=1}^k A_i)Z^T(X,Y)+B_{20}\Psi+\sum_{i=1}^k B_{2i}\Psi+Z(X,Y)\\&\times(A_0+\sum_{i=1}^k A_i)^T+(B_{20}\Psi+\sum_{i=1}^k B_{2i}\Psi)^T+\varepsilon_1E_{10}E_{10}^T\\&+\varepsilon_1^{-1}Z(X,Y)C^TK^TKCZ^T(X,Y)^T+\tau\sum_{j=1}^6\sum_{i=1}^k P_{ji}\end{aligned}$$

$$
\tilde{M}''=\begin{bmatrix}
\tilde{W}'' & \tau N_1 & \tau N_2 & \tau N_3 & \tau N_4 & \tau N_5 & \tau N_6 & \tau N_1 & \tau N_2 & \tau N_3 & \tau N_4 & \tau N_5 & \tau N_6 \\
\tau N_1^T & \tau\Omega_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\tau N_2^T & 0 & \tau\Omega_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\tau N_3^T & 0 & 0 & \tau\Omega_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\tau N_4^T & 0 & 0 & 0 & \tau\Omega_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\tau N_5^T & 0 & 0 & 0 & 0 & \tau\Omega_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\tau N_6^T & 0 & 0 & 0 & 0 & 0 & \tau\Omega_6 & 0 & 0 & 0 & 0 & 0 & 0 \\
\tau N_1^T & 0 & 0 & 0 & 0 & 0 & 0 & \tau\Omega_1' & 0 & 0 & 0 & 0 & 0 \\
\tau N_2^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tau\Omega_2' & 0 & 0 & 0 & 0 \\
\tau N_3^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tau\Omega_3' & 0 & 0 & 0 \\
\tau N_4^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tau\Omega_4' & 0 & 0 \\
\tau N_5^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tau\Omega_5' & 0 \\
\tau N_6^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tau\Omega_6'
\end{bmatrix} \tag{23}
$$

$$
\begin{aligned}
\tilde{W}'' &= (A_0+\sum_{i=1}^{k}A_i)Z^T(X,Y)+B_{20}\Psi+\sum_{i=1}^{k}B_{2i}\Psi+Z(X,Y)\\
&\quad\times(A_0+\sum_{i=1}^{k}A_i)^T+(B_{20}\Psi+\sum_{i=1}^{k}B_{2i}\Psi)^T+\varepsilon_1E_{10}E_{10}^T\\
&\quad+\varepsilon_1^{-1}Z(X,Y)C^TK^TKCZ^T(X,Y)^T+\tau\sum_{j=1}^{6}\sum_{i=1}^{k}P_{ji}\\
&\quad+\tau\sum_{j=1}^{6}\sum_{i=1}^{k}T_{ji}+\varepsilon_2T_0+\varepsilon_2^{-1}\Psi^T\Psi+\sum_{i=1}^{k}\varepsilon_{3i}T_i+\sum_{i=1}^{k}\varepsilon_{3i}^{-1}\Psi^T\Psi
\end{aligned}
$$

$$
\begin{aligned}
N_1 &= [Z(X,Y)A_0^T+\Psi^{\mathrm{T}}B_{20}^T \quad \cdots Z(X,Y)A_0^T+\Psi^{\mathrm{T}}B_{20}^T\ ],\\
N_2 &= [Z(X,Y)A_1^T \ \ Z(X,Y)A_2^T \ \ \cdots\ Z(X,Y)A_k^T],\\
N_3 &= [Z(X,Y)C^TK^TE_{10}^T \ \ Z(X,Y)C^TK^TE_{10}^T\\
&\qquad \cdots\ Z(X,Y)C^TK^TE_{10}^T],\\
N_4 &= [\Psi^{\mathrm{T}}B_{21}^T \quad \cdots\Psi^{\mathrm{T}}B_{2k}^T\ ]=N_6,\\
N_5 &= [\Psi^{\mathrm{T}}B_{20}^T \quad \cdots\ \Psi^{\mathrm{T}}B_{20}^T],\\
\Omega_j &= -diag\left\{Q_{j1},Q_{j2},\cdots,Q_{jk}\right\},\\
\Omega_j' &= -diag\left\{R_{j1},R_{j2},\cdots,R_{jk}\right\},\ j=1,2,\cdots,6.
\end{aligned}
$$

Through the upper analysis, now we can give the main result in this section.

Theorem 2*:* If there exist a series of positive definite symmetric $X,Q_{ji},R_{ji},Z_{ji},\bar{Z}_{ji},T_0,T_i,T_{ji},P_{ji},V_{ji},Q$, a matrix Y, Ψ, and the scalars $\varepsilon_1,\varepsilon_2,\varepsilon_{3i},\beta_0,\beta_i,\beta_{ji},\delta_{ji},\alpha,\mu_{ji},\gamma$ and τ, $i=1,\cdots,k$, $j=1,\cdots,6$ satisfying

$$EX+Y\Upsilon^T\ge\bar{Z}_{ji} \tag{24a}$$

$$\bar{Z}_{ji}\ge Q_{ji} \tag{24b}$$

$$\begin{bmatrix} Z_{ji} & \Psi \\ \Psi^T & \bar{Z}_{ji}\end{bmatrix}\ge 0 \tag{24c}$$

$$\begin{bmatrix} B_{20}B_{20}^T+\beta_0GG^T-T_0 & B_{20}H_3^T \\ H_3B_{20}^T & H_3H_3^T-\beta_0I\end{bmatrix}\le 0 \tag{24d}$$

$$\begin{bmatrix} B_{2i}B_{2i}^T+\beta_iGG^T-T_i & B_{2i}H_{4i}^T \\ H_{4i}B_{2i}^T & H_{4i}H_{4i}^T-\beta_iI\end{bmatrix}\le 0 \tag{24e}$$

$$\begin{bmatrix} A_iQ_{ji}A_i^T+\beta_{ji}GG^T-T_{ji} & A_iQ_{ji}H_{2i}^T \\ H_{2i}Q_{ji}A_i^T & H_{2i}Q_{ji}H_{2i}^T-\beta_{ji}I\end{bmatrix}\le 0 \tag{24f}$$

$$\begin{bmatrix} B_{2i}Z_{ji}B_{2i}^T+\delta_{ji}GG^T-P_{ji} & B_{2i}Z_{ji}H_{4i}^T \\ H_{4i}Z_{ji}B_{2i}^T & H_{4i}Z_{ji}H_{4i}^T-\delta_{ji}I\end{bmatrix}\le 0 \tag{24g}$$

$$\begin{bmatrix} EZ^T(X,Y) & \Psi_{(i)}^T \\ \Psi_{(i)} & \gamma u_{0(i)}^2\end{bmatrix}\ge 0 \tag{24h}$$

$$\begin{bmatrix} Q & Z^T(X,Y) \\ Z(X,Y) & I\end{bmatrix}\le 0 \tag{24i}$$

$$\begin{bmatrix} \tilde{M}'' & \Theta_1 & \Theta_2'^{\mathrm{T}} \\ \Theta_1^T & -\alpha^{-1}I & \\ \Theta_2' & & -\alpha I\end{bmatrix}< 0 \tag{24j}$$

where $\Theta_2'=[\Theta_2\ 0\cdots 0]$ and Θ_2 is defined in (21) setting $P=Z(X,Y)$, $\Lambda P^T=\Psi$, then the uncertain delay singular system (1) with the feedback gain $\Lambda=\Psi Z^{-T}(X,Y)$ is robust stable for any initial condition belonging to the set $\Phi_0=\{\phi(t)\big|\|\phi(t)\|^2\le\delta\}$ with $\delta=\dfrac{\lambda_{\min}(Q)}{\gamma\pi_2}$, where

$$
\begin{aligned}
\pi_2 &= \lambda_{\max}(EZ^T(X,Y))+k\tau\lambda_{\max}(\Psi^T\Psi)\\
&\quad+\frac{k\tau^2}{2}\max_i\lambda_{\max}[V_{1i}+V_{3i}+V_{5i}]\\
&\quad+\frac{3k\tau^2}{2}\max_i\lambda_{\max}[V_{2i}+V_{4i}+V_{6i}]
\end{aligned}
$$

The positive definite symmetric matrix V_{ji} and the scalars μ_{ji} can be solved by the following matrices inequalities

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{12}^T & \Pi_{22}\end{bmatrix}\le 0 \tag{25a}$$

$$
\begin{aligned}
\Pi_{11} &= (Z(X,Y)A_0^T+\Psi^TB_{20}^T)(Q_{1i}^{-1}+R_{1i}^{-1})(A_0Z^T(X,Y)\\
&\qquad+B_{20}\Psi)+\mu_{1i}GG^T-V_{1i}\\
\Pi_{12} &= (Z(X,Y)A_0^T+\Psi^TB_{20}^T)(Q_{1i}^{-1}+R_{1i}^{-1})(H_1Z^T(X,Y)+H_3\Psi),\\
\Pi_{22} &= (H_1Z^T(X,Y)+H_3\Psi)^T(Q_{1i}^{-1}+R_{1i}^{-1})(H_1Z^T(X,Y)\\
&\qquad+H_3\Psi)-\mu_{1i}I
\end{aligned}
$$

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{12}^T & \Theta_{22}\end{bmatrix}\le 0 \tag{25b}$$

$$
\begin{aligned}
\Theta_{11} &= Z(X,Y)A_i^T(Q_{2i}^{-1}+R_{2i}^{-1})A_0Z^T(X,Y)+\mu_{2i}GG^T-V_{2i},\\
\Theta_{12} &= Z(X,Y)A_i^T(Q_{2i}^{-1}+R_{2i}^{-1})H_{2i}Z^T(X,Y),\\
\Theta_{22} &= Z(X,Y)H_{2i}^T(Q_{1i}^{-1}+R_{1i}^{-1})H_{2i}Z^T(X,Y)-\mu_{2i}I
\end{aligned}
$$

$$\begin{bmatrix} V_{3i} & C^TK^TE_{10}^T \\ E_{10}KC & -(Q_{3i}^{-1}+R_{3i}^{-1})^{-1}I\end{bmatrix}\ge 0 \tag{25c}$$

$$\begin{bmatrix} \Psi^TB_{2i}^T(Q_{4i}^{-1}+R_{4i}^{-1})B_{2i}\Psi+\mu_{4i}GG^T-V_{4i} & \Psi^TB_{2i}^T(Q_{4i}^{-1}+R_{4i}^{-1})H_{4i}\Psi \\ [\Psi^TB_{2i}^T(Q_{4i}^{-1}+R_{4i}^{-1})H_{4i}\Psi]^T & \Pi_{22}'\end{bmatrix}\le 0 \tag{25d}$$

$$\Pi_{22}'=(H_{4i}\Psi)^T(Q_{4i}^{-1}+R_{4i}^{-1})(H_{4i}\Psi)-\mu_{4i}I$$

$$\begin{bmatrix} \Psi^T B_{20}^T (Q_{5i}^{-1}+R_{5i}^{-1}) B_{20}\Psi + \mu_{5i} GG^T - V_{5i} & \Psi^T B_{20}^T (Q_{5i}^{-1}+R_{5i}^{-1}) H_3 \Psi \\ [\Psi^T B_{20}^T (Q_{5i}^{-1}+R_{5i}^{-1}) H_3 \Psi]^T & \Theta'_{22} \end{bmatrix} \le 0 \tag{25e}$$

$$\Theta'_{22} = (H_3\Psi)^T (Q_{5i}^{-1} + R_{5i}^{-1})(H_3\Psi) - \mu_{5i} I$$

$$\begin{bmatrix} \Psi^T B_{2i}^T (Q_{6i}^{-1}+R_{6i}^{-1}) B_{2i}\Psi + \mu_{6i} GG^T - V_{6i} & \Psi^T B_{2i}^T (Q_{6i}^{-1}+R_{6i}^{-1}) H_{4i} \Psi \\ [\Psi^T B_{2i}^T (Q_{6i}^{-1}+R_{6i}^{-1}) H_{4i} \Psi]^T & \Xi_{22} \end{bmatrix} \le 0 \tag{25f}$$

C. Disturbance Case (with $w(t) \neq 0$)

When $w(t) \neq 0$, we assume that $w^T(t)w(t) \le w_0^{-1}$. In the conditions described below, the matrices $\tilde{M}'', \Theta_1$ and Θ'_2 are the matrices defined in Theorem 2 with $j = 1, \cdots, 7$.

Theorem 3. For given $w_0 > 0$, if there exist a series of positive definite symmetric $X, Q_{ji}, R_{ji}, Z_{ji}, \bar{Z}_{ji}, T_0, T_i, T_{ji}, P_{ji}, V_{ji}, Q$, a matrix Y, Ψ, and the scalars $\varepsilon_1, \varepsilon_2, \varepsilon_{3i}, \beta_0, \beta_i, \beta_{ji}, \delta_{ji}, \alpha, \mu_{ji}, \gamma, \nu, \upsilon$ and τ, satisfying (35a-i) for $i = 1, \cdots, k$, $j = 1, \cdots, 7$ and

$$\begin{bmatrix} W_{11} & \Theta_1 & \Theta_2'^T & B_{10} Z^T(X,Y) & 0 & 0 \\ \Theta_1^T & -\alpha^{-1} I & 0 & 0 & 0 & 0 \\ \Theta'_2 & 0 & -\alpha I & 0 & 0 & 0 \\ Z(X,Y) B_{10}^T & 0 & 0 & -\upsilon I & \tau N_7 & \tau N_7 \\ 0 & 0 & 0 & \tau N_7^T & \tau\Omega_7 & 0 \\ 0 & 0 & 0 & \tau N_7^T & 0 & \tau\Omega'_7 \end{bmatrix} < 0 \tag{26a}$$

$$-\nu w_0 + \upsilon\gamma \le 0 \tag{26b}$$

$$-w_0 + \frac{\tau}{2}\upsilon\gamma \le 0 \tag{26c}$$

where 0 denotes the zero matrix with appropriate dimensions, and

$$W_{11} = \tilde{M}'' + \begin{bmatrix} \nu E Z^T(X,Y) & 0_{n\times 12kn} \\ 0_{12kn\times n} & 0_{12kn \times 12kn} \end{bmatrix},$$

$$N_7 = [Z(X,Y)B_{10}^T \quad \cdots \quad Z(X,Y)B_{10}^T],$$

$$\Omega_7 = -diag\{Q_{71}, Q_{72}, \cdots, Q_{7k}\},$$

$$\Omega'_7 = -diag\{R_{71}, R_{72}, \cdots, R_{7k}\}.$$

then the feedback gain $\Lambda = \Psi Z^{-T}(X,Y)$, the scalar δ defining in the set $\Phi_0 = \{\phi(t) \mid \|\phi(t)\|^2 \le \delta\}$ with $\delta = \dfrac{\lambda_{\min}(Q)(w_0 - \frac{\tau}{2}\upsilon\gamma)}{\gamma w_0 \pi_2}$, where π_2 is defined in Theorem 2, and the closed-loop trajectories remain confined in the set $\Omega(P,E,\gamma) = \left\{ x(t) \in R^n \,\middle|\, x^T(t) P^{-1} E x(t) < \gamma^{-1}, \gamma > 0 \right\}$.

IV. ILLUSTRATIVE EXAMPLE

Consider an uncertain time-delay singular system (1) with $k = 1$ with an actuator saturated at level ± 1 and a dynamic described as follows:

$$E = \begin{bmatrix} -1 & 1 \\ -2 & 2 \end{bmatrix}, A_0 = \begin{bmatrix} -1 & 0 \\ -1 & 2 \end{bmatrix}, A_1 = \begin{bmatrix} -0.1 & -0.1 \\ 0.3 & 0.2 \end{bmatrix}, w_0 = 1.2,$$

$$B_{10} = \begin{bmatrix} 0.7 \\ 0.8 \end{bmatrix}, B_{20} = \begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix}, B_{21} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, E_{10} = \begin{bmatrix} 0.4 & 0.3 \\ -0.3 & 0.2 \end{bmatrix},$$

$$K = \begin{bmatrix} 0.4 & \\ & 0.6 \end{bmatrix}, C = \begin{bmatrix} -1 & 1 \\ 0.6 & -2 \end{bmatrix}, G = \begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix}, u_0 = 10,$$

$$H_1 = \begin{bmatrix} 0.2 & 0.5 \end{bmatrix}, H_{21} = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}, H_3 = 0.3, H_{41} = 0.2.$$

It's easy to see $\Phi = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$. Applying Theorem 3 to this uncertain time-delay singular system, it is found, using the software package LMI lab, that this system is regular, impulse free and locally asymptotically stable for any time-delay $\tau \le 0.6684$, When $\tau = 0.5$, the corresponding calculation results are as follows:

$$X = \begin{bmatrix} 6.0657 & 4.7504 \\ 4.7504 & 6.0657 \end{bmatrix}, \quad Y = \begin{bmatrix} -1.1697 \\ -2.3394 \end{bmatrix},$$

$$\Psi = \begin{bmatrix} -0.6008 & -1.2016 \end{bmatrix},$$

$$\Lambda = \begin{bmatrix} 0.2284 & -0.2284 \end{bmatrix}, \quad \gamma = 1.0953$$

Hence, the corresponding optimal value of δ is 0.7849. Owing to be out of tuning of parameters, it is obvious to see that the process of calculation is simple as the method presented in [7].

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