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Modelling and Distributed Control of Mobile Offshore Bases

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Abstract

Distributed modelling and control of Mobile Offshore Base systems is considered. Specifically, a spatial array system paradigm is used with a particular structure in spatial and temporal dimensions. For the purpose of linearization, a suitable coordinate transformation is applied to the model. Comparisons between centralized and distributed control strategies for the robust control of the system are made. The distributed modelling approach results in lower order models than the centralized approach, thus making the control design both easier and faster, with no apparent degradation in performance.

I. Introduction

A wide variety of systems consists of similar units directly interacting with each other or with their nearest neighbors. These systems are often referred to as **spatial array systems**. Examples of spatial array systems include vehicular platoons (autonomous vehicle strings) on a highway [12] and Unpiloted Air Vehicle (UAV's) in formation flight [11], as well as MEMS and smart structure systems. MEMS consist of a wide array of micro devices which are capable of sensing, actuating, computing and telecommunicating [8] . All of these systems, though clearly diverse in nature, share the feature that both the measurements and the controls are spatially distributed; that is, each unit is equipped with sensing and actuating capabilities. The concept of spatial array systems can also be applied to flow control and heat transfer problems [9] .

In this paper, we consider the modelling and control of a specific vehicle string system, namely a Mobile Offshore Base (MOB) system, which is, fundamentally, a string of marine vessels (see Fig. 1). Mobile Offshore Base presents a new application for the use of distributed $\mathcal{H}_\infty$ control algorithms for spatial array systems. A typical MOB consists of a series of semi-submersible modules which may or may not be physically connected. Each module is equipped with on-board sensors, actuators and controllers - these may or may not interact with each other. The primary task of the controllers is to maintain the alignment of the modules so as to form a runway in the sea.

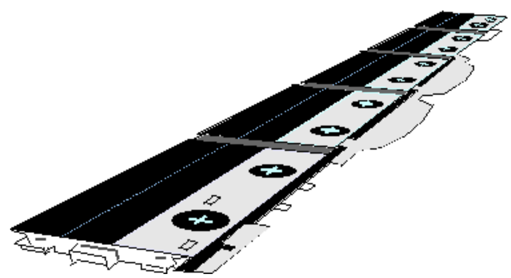

Fig. 1. Mobile Offshore Base - five modules aligned

A. Centralized versus De-centralized Control

The strategy employed to control spatial array systems may be centralized, fully de-centralized or distributed [5] . In a centralized framework, all computations are performed by a single controller and the control signals are transmitted to each individual unit in the array; that is, sensor and actuator information is shared globally.

Alternatively, de-centralized control schemes call for some distribution of the computation across the network of systems. In a completely de-centralized scenario, sensors and actuators on the individual modules are connected only to local controllers, which operate independently; that is information is not shared globally. However, there may be dynamic interactions between neighboring modules, making the overall system interactions more complex.

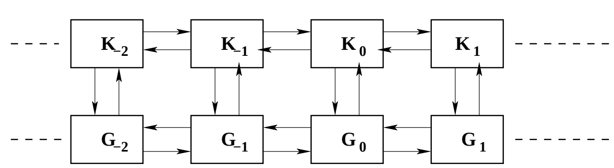

Fig. 2. Spatial Array systems - Interaction between the controllers

One factor which prohibits the widespread adoption of de-centralized control is the lack of global performance and stability guarantees[10]. Recently, techniques have been developed for application of *distributed* control in spatial array systems[3],[5]. This type of control implementation is neither centralized nor completely de-centralized (Fig.2).

Each individual unit in the array is equipped with a controller which has a particular spatial structure. The structure determines the extent to which local information is shared between the neighboring units. We consider the application of such techniques to MOB systems.

II. MODELLING MOBILE OFFSHORE BASE SYSTEMS

MOBs are modular semi-submersible floating bases which can be easily deployed in the sea. Apart from forming a floating interconnected runway, an MOB may also provide flight maintenance, supply and other logistics support at sea.

MOBs may consist of any number of semi-submersible units. In our study, we consider 5 modules, each around 200-400 m long, with no explicit physical connectors. Various configurations have been considered; see for example, [1], [2]. In all cases, the air base is formed by aligning the MOB platforms in an end-to-end fashion. MOB control strategies previously considered and evaluated are discussed in [14].

A. Single Vessel Model

The non-linear equations of motion for a single module in a MOB can be written in *body-fixed coordinates*[6] as

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu = \tau + w \tag{1}$$

where

- $M = M_{RB} + M_A$ is the inertia matrix which consists of the rigid body mode mass and the added mass matrices;
- $C(\nu) = C_{RB} + C_A$ is the Coriolis matrix which consists of the rigid body and added mass matrices of the Coriolis and centripetal terms;
- $D(\nu) = D_p(\nu) + D_v(\nu)$ is the damping matrix which consists of the radiation induced potential damping terms and the viscous damping (skin-friction, wave drift and vortex shedding);
- τ is the force vector which contains the body-fixed components of the external forces and moments (thruster forces, viscous drag etc),
- w is the disturbance due to wind and wave forces.
- $\nu = [u, v, r]$ is the velocity vector containing the surge, sway rate and rotation components.

This model assumes that the MOB modules travel with a negligible forward speed (u=0), and only horizontal motion is being accounted for (i.e. we neglect heave, pitch and roll dynamics in this discussion). Additional assumptions include homogeneous mass distribution, and placement of the origin of the body-fixed coordinate system along the center line of the ship.

1) Transformation of coordinates: The equations of motion derived above lead to a non-linear model. For the purpose of distributed control design, we prefer a linear system model for the MOB. As it turns out, this linearization can be achieved by suitable transformation of coordinates under the low-speed assumptions which we have already stated. One obvious choice for the coordinate transformation is the *Earth-fixed coordinate* system[6]. This is an inertial frame of reference with its origin at a fixed pre-determined point.

To obtain the Earth-fixed representation, we apply the transformation matrix $J(\eta)$, denoting

$$\dot{\eta} \;=\; J(\eta)\nu\;;\;\; where\;\; \begin{bmatrix} \dot{\eta} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} \tag{2}$$

and

$$J(\eta) = \begin{bmatrix} cos(\psi) & sin(\psi) & 0 \\ sin(\psi) & -cos(\psi) & 0 \\ 0 & 0 & -1 \end{bmatrix}. \tag{3}$$

The transformation matrix $J(\eta)$ transforms the equations of motion from the right-handed body-fixed coordinate system to the Earth-fixed coordinates, that is

$$\dot{\eta} = J(\eta)\nu \iff \nu = J^{-1}(\eta)\dot{\eta}$$
$$\ddot{\eta} = J(\eta)\dot{\nu} + \dot{J}(\eta)\nu \iff \nu = J^{-1}(\eta)[\ddot{\eta} - \dot{J}(\eta)J^{-1}(\eta)\dot{\eta}].$$

Using this transformation matrix, the Earth-fixed representation obtained is as follows

$$M_\eta(\eta)\ddot{\eta} + C_\eta(\nu,\eta)\dot{\eta} + D_\eta(\nu,\eta)\dot{\eta} = \tau_n + w \tag{4}$$

where

- $M_\eta(\eta) = JMJ^T$,
- $C_\eta(\nu,\eta) = J(CJ^T - MJ^T\dot{J}J^T)$,
- $D_\eta(\nu,\eta) = JDJ^T$,
- $\tau_n = J\tau$.

The transformed system model is also non-linear, as was the case with model resulting in the body fixed frame.

A more suitable choice for coordinate transformation for our purposes is the *vessel parallel coordinate system* [7]. This is a coordinate frame which is fixed to the vessel with its axes parallel to an Earth-fixed reference frame. The motion of the MOB is completely characterized by 3 degrees of freedom. Using the low-speed assumptions, we infer that:

$$\dot{\eta} = J(\eta)\nu \approx P(\psi)\nu \tag{5}$$

where

$$P(\psi) = J(\eta). \tag{6}$$

The vessel parallel system is defined by:

$$\eta_p = P^T(\psi)\eta \tag{7}$$

where η_p is the position in the body coordinates and $P(\psi)$ is the transformation matrix; clearly $P^T(\psi)P(\psi) = I_{3X3}$. For low-speed applications, we have

$$\begin{aligned} \dot{\eta_p} &= \dot{P^T}(\psi)\eta + P^T(\psi)\dot{\eta} \\ &= \dot{P^T}(\psi)P(\psi)\eta_p + P^T(\psi)P(\psi)\nu \\ &= rS\eta_p + \nu \end{aligned} \tag{8}$$

where $r = \dot{\psi}$. Using low speed assumptions implies $r \approx 0$. So finally we have

$$\begin{aligned} \dot{\eta_p} &\approx \nu \\ D(\nu)\nu &= D\nu \\ C(\nu)\nu &= 0. \end{aligned} \tag{9}$$

Thus, the model is transformed into the following system of equations:

$$\begin{aligned} \dot{\eta_p} &= \nu \\ M\dot{\nu} + D\nu &= \tau. \end{aligned} \tag{10}$$

One of the main advantages of transforming the system of equations into the vessel parallel coordinates is that the transformed system is now linear in ν. We can then directly apply distributed $\mathcal{H}_\infty$control tools for the purpose of controller design.

At any time, the Earth-fixed positions can be computed from the values of η_p by using the transformation:

$$\eta = P(\psi)\eta_p. \tag{11}$$

So essentially the control system is based on the information from the states η_p, ν and τ. At the same time, η contains the information that describes the degree of misalignment.

B. MOB Modelling

For the purpose of controller design, we convert the vessel parallel coordinate system model to state-space form. In order to account for the thruster dynamics, we use a simple first-order model for the three thrusters which are mounted on each unit of the MOB [7]. Thus, we incorporate the limitations of the propellers. Three time constants in surge, sway and yaw directions ($T_{surge}, T_{sway}, T_{yaw}$) are used for this purpose. The thruster model which we use is:

$$\dot{\tau} = A_{thr}(\tau - \tau_{com}) \tag{12}$$

where τ_{com} is the commanded thrust by the controller and A_{thr} is the thruster matrix determined by the 3 time constants, i.e.,

$$A_{thr} = \begin{bmatrix} -\frac{1}{T_{surge}} & 0 & 0 \\ 0 & -\frac{1}{T_{sway}} & 0 \\ 0 & 0 & -\frac{1}{T_{yaw}} \end{bmatrix}. \tag{13}$$

To determine a state space realization for the equations of motion, we consider the states $\eta_p = [x_p \ y_p \ \psi_p]^T$, $\nu = [u \ v \ r]^T$ and $\tau = [\tau_x \ \tau_y \ \tau_\psi]$. Using these states (positions, velocities and forces/torques), the state space representation of the system can be written in the following form:

$$\dot{x} = Ax + B\tau_{com} \tag{14}$$

where

$$A = \begin{bmatrix} 0 & I & 0 \\ 0 & -M^{-1}D & M^{-1} \\ 0 & 0 & A_{thr} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ -A_{thr} \end{bmatrix} \tag{15}$$

and

$$x = [\eta_p{}^T, \nu^T, \tau^T]^T. \tag{16}$$

Here τ is the actual control force/moment which the thrusters apply on the MOB units. The thrusters will provide the required force and moment to steer the MOB to the required position. Note that τ can be written as

$$u = \tau = \begin{bmatrix} F_x & F_y & F_\psi \end{bmatrix}^T \tag{17}$$

where F_x, F_y and F_ψ are the thruster (actuator) forces in the X and Y directions, and the thruster torque in the ψ direction, respectively. The task of the controller will essentially be to calculate these thruster forces and moments, and transmit this information to the actuators.

C. Physical Details

We have assumed the following in our modelling and control efforts (for each module in the MOB)

$$Length = 200\ m \tag{18}$$

$$Mass = 11000\ kg \tag{19}$$

$$M = \begin{bmatrix} 11937.3 & 0 & 0 \\ 0 & 27582.5 & -899140 \\ 0 & -899140 & 94732000 \end{bmatrix} \tag{20}$$

$$D = \begin{bmatrix} 210.73 & 0 & 0 \\ 0 & 185.638 & 7357.311 \\ 0 & 7357.311 & 30208.82 \end{bmatrix} \tag{21}$$

$$A_{thr} = \begin{bmatrix} -0.33 & 0 & 0 \\ 0 & -0.25 & 0 \\ 0 & 0 & -0.167 \end{bmatrix}. \tag{22}$$

Wind and wave disturbances of varying frequencies and amplitudes are also considered as acting on the MOB; these disturbances correspond to conditions up to Sea-state 7 (moderate gale conditions)[7].

III. $\mathcal{H}_\infty$ Control Design for Single Vessel

The first stage in our control design process for a string of MOB modules is the study of a single floating module. Equation (14) describes the single vessel model in state-space form. For the purpose of control design for a single vessel, we assume full state feedback to the controller. The plant-controller interaction is modelled as an LFT. The origin (of the Earth-fixed coordinate system) is chosen as the vessel's desired final point. Thus, the error, denoted by z, is measured as the distance/angle of the vessel's center point from the origin. The task of the controller is to steer the MOB to the origin in the presence of the unknown wind and wave disturbances. Additionally, to account for the low speed assumptions that we have made in the modelling process, we need to monitor the velocity of the MOB module. This is achieved by considering the velocity ν as an additional error variable, separate from η. Clearly, our error criterion will be to minimize the error z. Such a criterion will ensure that the module is steered to the origin at low-speed even in the presence of wind and wave forces.

Naturally, disturbances play a major role in the design of any robust controller. For the purpose of simulation, we model wind and wave disturbances as sinusoids of varying frequencies and amplitudes, which enter the model as forces/torques in the surge, sway and yaw directions.

IV. Centralized $\mathcal{H}_\infty$ Controller Design

The single vessel controller design forms the basis for the centralized controller synthesis. We now consider a string of 5 vessels, initially in proximity to one another, but not aligned. Each of these vessels is equipped with thrusters which can exert forces/torques in the 3 directions. A central controller collects sensor information from all the modules, based on which it provides each vessel with a control force/torque command. The equations of motion for all 5 MOB modules, together with their actuator models, form the centralized model. Since a single vessel model has 9 states, the string of 5 vessels requires a total of 45 states for modelling the system, i.e.,

$$x = [{\eta_{1p}}^T {\eta_{2p}}^T ... {\eta_{5p}}^T {\nu_1}^T {\nu_2}^T ... {\nu_5}^T {\tau_1}^T {\tau_2}^T ... {\tau_5}^T]^T. \quad (23)$$

The error signals considered for the $\mathcal{H}_\infty$ control design are:

$$\begin{aligned} z_1 &= \eta_1 \\ z_2 &= \eta_1 - \eta_2 \\ z_3 &= \eta_2 - \eta_3 \\ z_4 &= \eta_3 - \eta_4 \\ z_5 &= \eta_4 - \eta_5. \end{aligned} \quad (24)$$

The criteria is thus to minimize the relative misalignment between adjacent vessels. For the purpose of simulation, we again consider full state feedback. The response of the MOB system to the wind and wave disturbances has been evaluated via simulations. Simulation results are shown in Figure 3. The difference in the X, Y and ψ positions of the adjacent vessels is evaluated. As can be seen from Fig. 3, the controller drives the relative misalignment to zero, roughly within 0.67 minutes. The position of the first MOB module is compared with the origin (its final position). Note that the performance index γ (which is a measure of disturbance rejection) obtained for the centralized $\mathcal{H}_\infty$ controller design is 3.04. The system responded similarly for all disturbances up to sea-state 7. For larger disturbances, the MOB's final position oscillated around the origin.

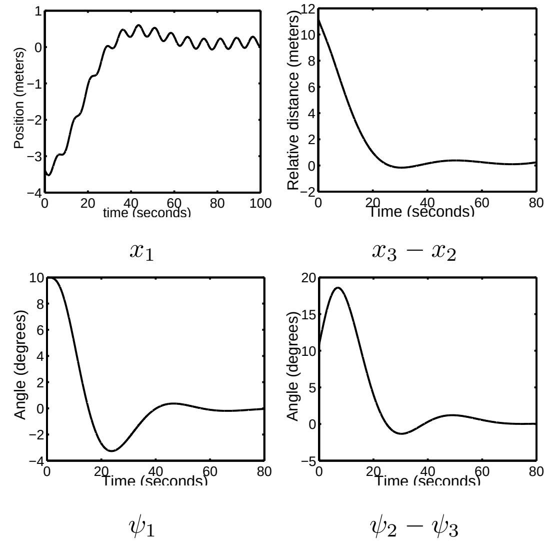

Fig. 3. Centralized Controller Performance

V. Distributed $\mathcal{H}_\infty$ Control Design

The aim of the centralized model approach is to incorporate different MOB modules together in a single system. Alternatively, we now model the MOB system as a distributed system with identical sensing and actuating capabilities at each module [3] [5]. The model thus obtained is far less complex than the large centralized model of the previous section.

An additional point to note is that the distributed synthesis which we consider uses local information sharing in both directions within the array. Recent work has indicated that information shared in only one direction (e.g. a "lookahead" scheme) is not sufficient to guarantee stability [13].

We have employed the spatial shift-operator multi-dimensional modelling technique (discussed in [4], [5]) for modelling the string of vessels which form the MOB. Using this approach, an infinite extent system realization is captured by finite dimensional state matrices, which can thus be used for controller design. Clearly, an infinite string of MOB modules can be considered a spatially invariant distributed system with one spatial variable (the positions of the various modules). As has been pointed out in [3], the infinite string approximation may be sufficient when we have a large number of units in the spatial array. Additionally, if the infinite system is well-posed, stable and contractive, then all periodic interconnections of that system inherit the same properties [5]. To describe the modelling of a string of modules, first we define the following:

The forward shift operator $\mathbf{S_i}$ given by

$$(\mathbf{S_i} u(t))(s) := u(t, s_1, .., s_i + 1, .., s_L), \quad i = 1, ...L$$

Similarly, the backward shift operator ${\mathbf{S_i}}^{-1}$ is given by

$$({\mathbf{S_i}}^{-1} u(t))(s) := u(t, s_1, .., s_i - 1, .., s_L), \quad i = 1, ...L$$

The differential operator λ is defined as $\lambda x = \frac{dx}{dt}$, with the inverse mapping denoted by λ^{-1}.

Equation (10) describes the motion of a single vessel. For the purpose of distributed control design, we consider an infinite string of MOB platforms such that each module at the spatial coordinate s has two distinct neighbors $s-1$ and $s+1$. The equations of motion for the platform at the spatial coordinate s now becomes,

$$\begin{aligned} \eta_p(s) &= \nu(s) \\ M\dot{\nu}(s) + D\nu(s) &= \tau(s). \end{aligned} \quad (25)$$

To generate a multi-dimensional state-space representation for these equations, we take the first three states as the position in the vessel parallel coordinate system $[x_p \ \ y_p \ \ \psi_p],^T$ the next three states as velocities $[u \ \ v \ \ r]^T$ and the final three as forces/torques $[\tau_x \ \ \tau_y \ \ \tau_\psi].^T$ Thus, the resulting state matrices A and B for each unit are same as before. The

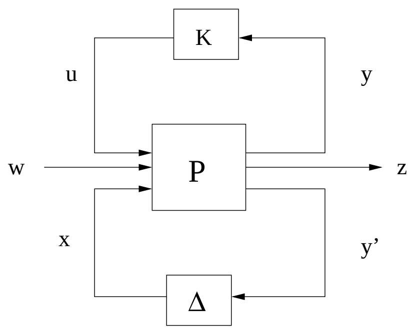

Fig. 4. Plant(with the spatial and temporal structure)- Controller Interconnection

C and D matrices depend on the number of measurements the sensors perform. To capture the above system in a realization of the form $\mathcal{M} = \{A, B, C, D, \mathbf{m}\}$ (using the notation of [5]), the first nine states are defined as above. The remaining six states are defined by the spatial relationships amongst the units, namely

$$\begin{aligned} x_{10} &= \mathbf{S_1} x_p = \mathbf{S_1} x_1 \\ x_{11} &= \mathbf{S_1} y_p = \mathbf{S_1} x_2 \\ x_{12} &= \mathbf{S_1} \psi_p = \mathbf{S_1} x_3 \\ x_{13} &= \mathbf{S_1}^{-1} x_p = \mathbf{S_1}^{-1} x_1 \\ x_{14} &= \mathbf{S_1}^{-1} y_p = \mathbf{S_1}^{-1} x_2 \\ x_{15} &= \mathbf{S_1}^{-1} \psi_p = \mathbf{S_1}^{-1} x_3. \end{aligned} \tag{26}$$

The resulting 15 state model thus defines the infinite-extent system. The spatial operator modelling provides us with a tool for evaluating a physical quantity at different spatial locations. Thus, we are able to capture the behavior of the entire spatial array in a single set of finite-dimensional state matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{D}$.

The next step in the robust control design is to determine a performance criterion for the system. This is defined in terms of the error z which we would like to minimize. An obvious choice for the error is the degree of misalignment between adjacent modules of the MOB string. Minimizing this error will lead to alignment of the string of MOBs in an end-to-end fashion. The error is defined as:

$$\begin{aligned} z_1 &= [x_p(s+1) - x_p(s)] - [x_p(s) - x_p(s-1)] \\ &= \mathbf{S_1} x_p(s) + \mathbf{S_1}^{-1} x_p(s) - 2x_p(s) \\ &= x_{10} + x_{13} - 2x_1. \end{aligned} \tag{27}$$

Similarly, we define $z_2 = x_{11} + x_{14} - 2x_2$ and $z_3 = x_{12} + x_{14} - 2x_3$. To account for the physical limitations of the thrusters, we need to keep the thruster forces/torques under a certain pre-specified limit. In order to model this constraint, we choose the other three output signals as the forces/torque applied by the thrusters in the following way:

$$\begin{aligned} z_4 &= u = x_4 \\ z_5 &= u = x_5 \\ z_6 &= u = x_6. \end{aligned} \tag{28}$$

The feedback signal y is chosen to be the position and the velocity information provided by the sensors. Noise terms have been added to each of these feedbacks to account for the sensor and measurement noise, i.e.,

$$\begin{aligned} y_1 &= x_1 + d_4 \\ y_2 &= x_2 + d_5 \\ y_3 &= x_3 + d_6 \\ y_4 &= x_4 + d_7 \\ y_5 &= x_5 + d_8 \\ y_6 &= x_6 + d_9. \end{aligned} \tag{29}$$

The differential operator λ operates on the first nine states. Our structured operator matrix is thus defined as follows:

$$\Delta = \begin{bmatrix} \lambda^{-1} I_9 & 0 & 0 \\ 0 & S_1 I_3 & 0 \\ 0 & 0 & {S_1}^{-1} I_3 \end{bmatrix}. \tag{30}$$

The finite dimensional realization for the infinite dimensional system can now be written in the form: $\mathcal{M} = (A, B, C, D, \mathbf{m})$, with $\mathbf{m} = [9, 3, 3]$. The system matrices A, B and C can now be split into temporal and spatial parts giving $A_{TT}, A_{TS}, A_{ST}, A_{SS}, B_T, B_S, C_T$ and C_S. The $\mathcal{H}_\infty$ synthesis methods proposed in [5] are directly applicable to this model.

The controller design for the distributed system involves solving a system of LMIs in both spatial and temporal variables. During the synthesis, it is assumed that the controller has the same structure as that of the plant. For the MOB model, the resulting controller has the structure

$$\Delta_K = \begin{bmatrix} \lambda^{-1} I_3 & 0 & 0 \\ 0 & S_1 I_3 & 0 \\ 0 & 0 & {S_1}^{-1} I_3 \end{bmatrix}. \tag{31}$$

Simulation results are given in Figures 5 and 6. Figure 5 shows the misalignment (distance between adjacent units) in the X-direction. Figure 6 shows the instantaneous yaw angles (ψ) of the modules. A non-zero initial condition is assumed. As can be seen from the figures, the controller drives the relative misalignment to zero, roughly within 1.2 minutes. Thus the controlled units become closely aligned in an end-to-end fashion and form a runway, even in the presence of moderate gale forces. Simulations were carried out for disturbances up to sea-state 7. For higher disturbances, the system did not perform well and the final position of the MOB again oscillated around the origin, as was seen in the centralized case. The performance index γ, of the distributed $\mathcal{H}_\infty$ controller obtained in this case was 1.12. An important point to note here is that we have included the control effort u in our error criterion with a

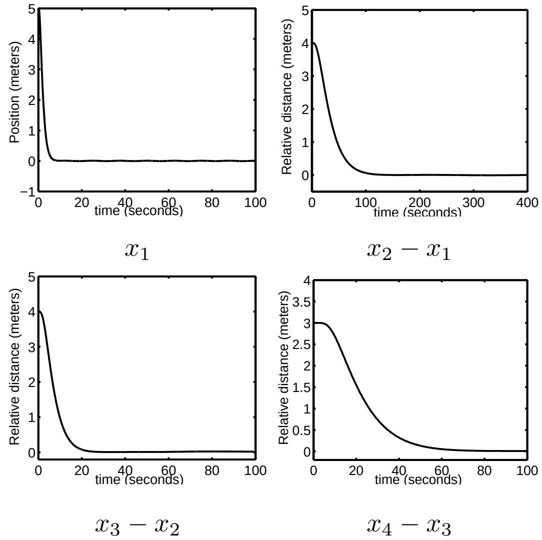

Fig. 5. Performance of the Distributed Controller - Relative positions between adjacent modules.

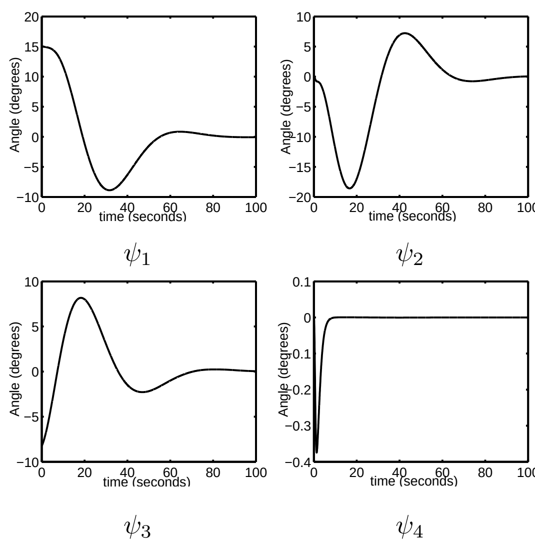

Fig. 6. Performance of the Distributed Controller - Yaw (ψ) angles.

unit weighting. In order to directly compare the distributed and the centralized approaches (with respect to the control effort required), the weighting on u should be modified. The weighting factor should be chosen such that the control effort required in the distributed case is essentially the same as that in the centralized case.

VI. Comparison of Centralized and Distributed Techniques

We have completed both centralized and distributed control designs for our system. Table I shows a comparison of the control orders and computational efforts, which we have obtained for these cases. The distributed model is far less complex than the centralized model as it requires only 15 states, as opposed to the 45 states required in case of the latter. Additionally, the controller synthesis for the distributed case took approximately 6.47 seconds (on a Pentium-IV 1.3 Ghz computer) to synthesize as compared to the centralized controller which took 16.2 seconds to synthesize. Note that virtually no loss of performance results with respect to desired position, which is demonstrated in Figures 3, 5 and 6. However the distributed design does require more control effort than the centralized model, but it is well within limits of the maximum permissible control. One of the additional advantages of such a distributed control design approach is scalability. We can clearly add additional units to the same system without having to modify the control algorithm, or alternatively an MOB configuration consisting of a large number of smaller units could easily be handled possibly allowing for greater overall flexibility in the design of such MOBs.

Type of Model	**Plant Order**	**Controller Order**	**Time for synthesis**
Single Vessel	9	8	2.4 s
Centralized	45	45	16.2 s
Distributed	15	9	6.47 s

TABLE I

Comparison of Controllers

Acknowledgements

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References

[1] *Operational Availability Assessment Model for Mobile Offshore Bases*. Bechtel National, Inc., December 1999.

[2] *Design Technologies for Mobile Offshore Base, Generalized Sea Cargo Transfer Model, Final Project Report, User Manual and Analysis Results*. McDermott Technology, Inc., May 2000.

[3] B. Bamieh, F. Paganini, and M. Dahleh. Distributed control of spatially invariant systems. *Submitted to IEEE Transaction on Automatic Control*, 1998.

[4] R. D'Andrea. A linear matrix inequality approach to decentralized control of distributed parameter systems. *Proc. of American Control Conference*, pages 1350–1354, 1998.

[5] R. D'Andrea and G. E. Dullerud. Distributed control design for spatially interconnected systems. *IEEE Transactions on Automatic Control*, 48(9):1478–1495, 2003.

[6] T. I. Fossen. *Guidance and Control of Ocean Vehicles*. Wiley, 1994.

[7] T. I. Fossen. *Marine Control Systems - Guidance, Navigation and Control of ships, rigs and underwater vehicles*. Marine Cybernetics, Trondheim, Norway, 2002.

[8] R. S. Smith H. T. Banks and Y. Wang. *Smart Material Structures: Modeling , Estimation and Control*. John Wiley and Sons, 1996.

[9] C. Ho and Y. Tai. Review: MEMS and its applications for flow control. *ASME Journal of Fluid Engineering*, 1996.

[10] B. A. Francis J. C. Doyle and A. R. Tannenbaum. *Feedback Control Theory*. Mcmillan Publishing Company, 1992.

[11] D. F. Chichka J. D. Wolfe and J. L. Speyer. Decentralized controllers for unmanned aerial vehicle formation flight. *American Institute of Aeronautics and Astronautics*, 1996.

[12] H. Raza and P. Ioannou. Vehicle following control design for automated highway systems. *IEEE Control Systems*, 16(6):43–60, 1996.

[13] P. J. Seiler. *Coordinated Control of Unmanned Air Vehicles*. PhD thesis, University of California, Berkeley, 2001.

[14] S. Spry and K. Hedrick. A centralized control strategy for a mobile offshore base. *Proc. of American Control Conference*, pages 1482–1486, 1999.