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Task-space Adaptive Setpoint Control for Robots with Uncertain Kinematics and Actuator Model

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***Abstract*— In this paper, we proposed a new task-space setpoint control scheme for robots with uncertainties in kinematics, actuators and dynamics. The stability problem of the robot in the presence of these uncertainties is formulated and solved. Sufficient conditions for choosing the feedback gains and approximate models are given to guarantee the convergence of the task-space position error. Simulation results based on a 3-link robot are presented to illustrate the performance of the proposed scheme.**

I. Introduction

A great many control schemes for robotic manipulators have been developed in the literature during the past few decades. In most of the control methods [1-11], the controllers are designed at the torque input level and the actuator part is neglected. However, as shown by Good $et.al.$ [12], the actuator model constitutes an important part of the complete robot system and may cause detrimental effects when neglected in the design procedure. Some research work that deal with this problem can be found in [13-20]. The control schemes proposed thereby can deal with the dynamic and actuator model uncertainties existing in the robot systems.

In most applications of robots, the desired path of the robot manipulator is specified in task space. Therefore, one principal limitation associated with the joint-space controllers including the results mentioned above [1-20] is that the desired joint position must be obtained by solving the inverse kinematics problem. To avoid the problem of solving the inverse kinematics, Takegaki and Arimoto [1] proposed a task-space controller for setpoint control in Cartesian space using a transposed Jacobian matrix. Many other task-space control schemes are proposed later [21-24]. Recently Cheah [25] proposed a task-space control scheme that can deal with actuator model uncertainty. In these methods, inverse kinematics problem is avoided and the feedback errors of the control law are defined and computed directly in the task space such as Cartesian space and visual space. However, to apply these task-space control schemes, an exact knowledge of the Jacobian matrix from joint space to task space is required. If uncertainties exist in the kinematics, these controllers [1-25] may give degraded performance and may incur instability. To overcome the problem of uncertain kinematics, Cheah $et.al.$[26-29] proposed several task-space feedback laws with uncertain kinematics from joint space to task space. However, it is again assumed in these papers [26-29] that the actuator model is known exactly.

The objective of this paper is to develop task-space control scheme that can deal uncertainties in kinematics and actuator model at the same time. To our knowledge, this problem has not been studied before. Hence, it is unknown whether the stability of the robot's motion can still be guaranteed in the presence of these uncertainties. We propose an adaptive SP-D control law for the task of setpoint control with uncertainties existing in kinematics and actuator model. Sufficient conditions for choosing the feedback gains, estimated Jacobian matrix and estimated actuator model are given to guarantee the stability. Simulation results are presented to illustrate the performance of the proposed control scheme.

II. Robot Kinematics and Dynamics

In order to describe a task for the robot manipulator, the desired path for the end effector is usually specified in task space. Let $X \in R^m$ represents the position vector of the manipulator in task space defined by [22], [26]:

$$X = h(q), \tag{1}$$

where $q \in R^n$ is a vector of generalized joint coordinates, $h(\cdot) \in R^n \rightarrow R^m$ $(m \leq n)$ is generally a nonlinear transformation describing the relation between the joint and task space. The velocity vector $\dot{X}$ is therefore related to $\dot{q}$ as:

$$\dot{X} = J(q)\dot{q}, \tag{2}$$

where $J(q) \in R^{m \times n}$ is the Jacobian matrix of mapping from joint space to task space. Note that if the robot's kinematics is uncertain, the Jacobian matrix becomes uncertain too.

The equations of motion of the robotic manipulator with n degrees of freedom in joint-space is given as [6], [22]

$$M(q)\ddot{q} + (\frac{1}{2}\dot{M}(q) + S(q,\dot{q}))\dot{q} + g(q) = \tau, \tag{3}$$

where $M(q) \in R^{n \times n}$ denotes a positive definite inertia matrix, $g(q) \in R^n$ denotes a gravitational force vector, $\tau \in R^n$ denotes the control inputs, $S(q,\dot{q})$ is a skew-symmetric matrix,

$$S(q,\dot{q})\dot{q} = \frac{1}{2}\dot{M}(q)\dot{q} - \frac{1}{2}\{\frac{\partial}{\partial q}\dot{q}^T M(q)\dot{q}\}^T, \tag{4}$$

$$g(q) = (\partial P/\partial q_1, \cdots, \partial P/\partial q_n)^T, \tag{5}$$

and $P(q)$ is the potential energy due to gravitational force. The gravitational force can be completely characterized by a set of parameters $\phi = (\phi_1, \cdots, \phi_p)^T$ [2, 3, 6] as

$$g(q) = Z(q)\phi, \tag{6}$$

where $Z(q) \in R^{n \times p}$ is the gravity regressor

If a permanent-magnet DC motor driven by an amplifier operating in $current\ mode$ is used as an actuator at the i^{th} joint, then the differential equation of motion describing the rotational behavior of the motor is given by [6], [22]:

$$J_{mi}\ddot{\theta}_i + B_{mi}\dot{\theta}_i = K_{\tau i} I_{ai} - r_i \tau_i, \tag{7}$$

where θ_i denotes the angle of the motor rotor shaft, J_{mi} the inertia moment, B_{mi} the rotor damping coefficient, I_{ai} the motor armature current, $K_{\tau i}$ the motor torque constant. r_i is the transmission gear ratio defined as:

$$q_i = r_i \theta_i. \tag{8}$$

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From equations (3), (7) and (8), the dynamics of the robot with actuators can be given as:

$$(M_0+M(q))\ddot{q}+(B_0+\frac{1}{2}\dot{M}(q)+S(q,\dot{q}))\dot{q}+g(q)=K_\tau I_a, \quad (9)$$

where $M_0 = diag(J_{m1}/{r_1}^2,\cdots,J_{mn}/{r_n}^2)$, $B_0 = diag(B_{m1}/{r_1}^2,\cdots,B_{mn}/{r_n}^2)$, $K_\tau = diag(K_{\tau 1}/r_1,\cdots,K_{\tau n}/r_n)$, $I_a=(I_{a1},\cdots,I_{an})^T$.

If a permanent-magnet DC motor driven by a $voltage$ amplifier is used as the joint actuator, the differential equation of motion of the motor is described by [6], [22]:

$$J_{mi}\ddot{\theta}_i + B_{oi}\dot{\theta}_i = K_{oi}v_i - r_i\tau_i, \quad (10)$$

where v_i is the armature voltage, $B_{oi} = B_{mi} + K_{\tau i}K_{bi}/R_{ai}, K_{oi} = K_{\tau i}/R_{ai}, R_{ai}$ is the armature resistance and K_{bi} the constant of motor back electromotive force. In this case, we assume that armature inductances are negligible, because the electrical time constant is much smaller than the mechanical time constant [6], [22]. Then, from equations (3), (8) and (10), we can get the dynamics of the robot with actuators as follows:

$$(M_0+M(q))\ddot{q}+(B_1+\frac{1}{2}\dot{M}(q)+S(q,\dot{q}))\dot{q}+g(q)=K_v v_a, \quad (11)$$

where $B_1 = diag(B_{o1}/{r_1}^2,\cdots,B_{on}/{r_n}^2)$, $K_V = diag(K_{\tau 1}/R_{a1}r_1,\cdots,K_{\tau n}/R_{an}r_n)$, $v_a=(v_1,\cdots,v_n)^T$.

Since the dynamic equation (9) is in a similar form as equation (11), we can write the dynamics of robot with actuators in a general form as:

$$(M_0+M(q))\ddot{q}+(B+\frac{1}{2}\dot{M}(q)+S(q,\dot{q}))\dot{q}+g(q)=Ku, \quad (12)$$

where $u=I_a, K=K_\tau, B=B_0$ if current amplifiers are used, and $u=v_a, K=K_v, B=B_1$ if voltage amplifiers are used.

In actual implementations of the robot controllers, calibration is necessary to identify the exact parameters of matrix K in equation (12). However, since both $K_{\tau i}$ and R_{ai} are temperature sensitive, the actuator model K may change as temperature varies due to overheating of motor or changes in ambient temperature. In addition, the robot kinematics may be uncertain in the presence of modeling error and when the robot picks up a tool of unknown length or gripping point. Hence, in the presence of modeling uncertainties or calibration errors of both actuator and robot kinematics, position error may be resulted. It is also unknown whether the stability of the system can still be guaranteed in the presence of these uncertainties. In this paper, we solve this problem with an adaptive SP-D control scheme with approximate models of kinematics and actuators. We shall show that the proposed controller can guarantee the asymptotic convergence of the robot motion.

III. Adaptive Saturated-Proportional, Differential (SP-D) Control with Approximate Jacobian Matrix and Actuator Model

In this section, we propose the adaptive SP-D control scheme for setpoint control of robotic manipulators in the presence of uncertainties in both kinematics and actuator model. The gravitational force vector in the dynamic equation (12) can be completely characterized by a set of parameters $\phi=(\phi_1,\cdots,\phi_p)^T$ as [6]

$$g(q)=Z(q)\phi=[z_1(q)\phi,\cdots,z_n(q)\phi]^T, \quad (13)$$

where $z_i(q)\in R^{1\times p}$ for $i=1,\cdots,n$, ϕ is the $p\times 1$ unknown parameter vector of $Z(q)$. In the presence of uncertainty in the parameters of the gravitational force, we have

$$\hat{g}(q)=Z(q)\hat{\phi}=[z_1(q)\hat{\phi},\cdots,z_n(q)\hat{\phi}]^T, \quad (14)$$

where $\hat{\phi}\in R^p$ is an estimated parameter of ϕ which will be updated by an updating law.

Let $\hat{K}$ and $\hat{J}(q)$ be the approximate actuator model and the approximate Jacobian matrix respectively chosen so that

$$\|I-K\hat{K}^{-1}\|\le\bar{\beta}, \quad (15)$$

$$\|J^T(q)-\hat{J}^T(q)\|\le\bar{\gamma}, \quad (16)$$

where $\bar{\beta}$ and $\bar{\gamma}$ are positive constants to be defined later. Using the approximate Jacobian matrix and actuator model and the exact gravity regressor the control input is proposed as:

$$u=-\hat{K}^{-1}[\hat{J}^T(q)K_p s(e)+\hat{J}^T(q)K_v\dot{\hat{X}} -Z(q)\hat{\phi}-Y(q,\hat{\phi})\hat{\varphi}], \quad (17)$$

$$\dot{\hat{\phi}}=-L_1Z^T(q)(\dot{q}+\alpha\hat{J}^+(q)s(e)), \quad (18)$$

$$\dot{\hat{\varphi}}=-L_2Y(q,\hat{\phi})(\dot{q}+\alpha\hat{J}^+(q)s(e)), \quad (19)$$

where $\dot{\hat{X}}=\hat{J}(q)\dot{q}$ is the estimated task velocity vector, $e=X-X_d=(e_1,\cdots,e_m)^T$ is a positional deviation from a desired position $X_d\in R^m$ and $s_i(\cdot), i=1,\cdots,n$ are saturated functions of e, X is measured by sensor [29], K_p, and K_v are positive definite diagonal feedback gains for the position and velocity respectively, $L_1,\ L_2$ are positive definite diagonal matrices, α is a positive constant, $\hat{J}^+(q)=\hat{J}^T(q)(\hat{J}(q)\hat{J}^T(q))^{-1}$ is the pseudo-inverse of $\hat{J}(q)$ such that $\hat{J}(q)\hat{J}^T(q)\in R^{m\times m}$ is non-singular, and $\hat{J}(q)\hat{J}^+(q)=I$. The regressor $Y(q,\hat{\phi})$ is a diagonal matrix whose i^{th} diagonal element is the i^{th} entry of estimated gravity force vector $\hat{g}(q)=[z_1(q)\hat{\phi},\cdots,z_n(q)\hat{\phi}]^T$ (see equation (14)):

$$Y(q,\hat{\phi})=\begin{bmatrix} z_1(q)\hat{\phi} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & z_n(q)\hat{\phi}\end{bmatrix}, \quad (20)$$

and $\hat{\varphi}\in R^n$ is the adaptive parameter vector whose updating law is given by equation (19).

Remark 1: In the controller, the gravity regressor $Z(q)$ and $\hat{\phi}$ are used to cope with the uncertainty in gravity force and the regressor $Y(q,\hat{\phi})$ and $\hat{\varphi}$ are used to compensate the uncertainty in actuator model. The role of $\hat{\varphi}$ would be clearer in the later development. It is interesting to note that the novel regressor $Y(q,\hat{\phi})$ makes use of the updated information from $\hat{\phi}$ instead of fixed information.

Let us define a scalar function $S_i(e)$ and its derivative $s_i(e)$ as shown in Figure 1 and with the following properties [6]:

1) $S_i(e)>0$ for $e\neq 0$ and $S_i(0)=0$.
2) $S_i(e)$ is twice continuously differentiable, and the derivative $s_i(e)=\frac{dS_i(e)}{de}$ is strictly increasing in e for $|e|<\gamma_i$ with some γ_i and saturated for $|e|\ge\gamma_i$, i.e. $s_i(e)=\pm s_i$ for $e\ge\pm\gamma_i$, and $e\le-\gamma_i$ respectively where s_i is a positive constant.
3) There are constants $\hat{c}_i>0$ such that for $e\neq 0$,

$$S_i(e)\ge\hat{c}_i{s_i}^2(e). \quad (21)$$

Some examples of the saturated function can be found in [6], [9].

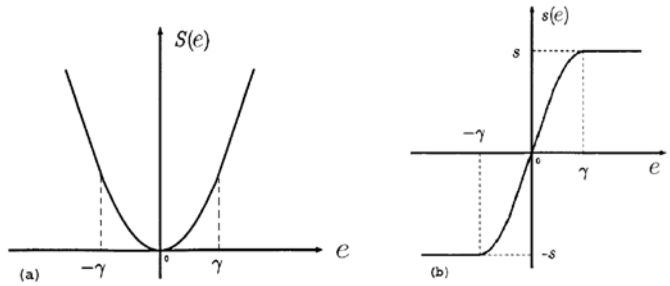

Fig. 1. (a)Quasi-natural potential: $S(e)$ (b)derivative of $S(e)$: $s(e)$

Substituting equation (17) into equation (12), we have the closed-loop dynamic equation

$$
\begin{aligned}
&(M_0+M(q))\ddot{q}+(B_0+\tfrac{1}{2}\dot{M}(q)+S(q,\dot{q}))\dot{q} \\
&+K\hat{K}^{-1}\hat{J}^T(q)K_v\dot{\hat{X}}+K\hat{K}^{-1}\hat{J}^T(q)K_p s(e)+g(q) \\
&-K\hat{K}^{-1}Z(q)\hat{\phi}-K\hat{K}^{-1}Y(q,\hat{\phi})\hat{\varphi}=0. \qquad (22)
\end{aligned}
$$

Since $(I-K\hat{K}^{-1})$ is a diagonal matrix, according to the definition of $Y(q,\hat{\phi})$ in equation (20), we have

$$
\begin{aligned}
& (I-K\hat{K}^{-1})Z(q)\hat{\phi} \\
= \; & (I-K\hat{K}^{-1})(z_1(q)\hat{\phi},\cdots,z_n(q)\hat{\phi})^T \\
= \; & Y(q,\hat{\phi})\phi_k, \qquad (23)
\end{aligned}
$$

where $\phi_k=(1-\frac{k_1}{\hat{k}_1},\cdots,1-\frac{k_n}{\hat{k}_n})^T$ is unknown since the exact actuator model is unknown, and k_i and $\hat{k}_i$ are the i^{th} diagonal elements of K and $\hat{K}$ respectively.

Substituting equation (23) into (22) and using equation (13), the dynamic equation can be written as:

$$
\begin{aligned}
&(M_0+M(q))\ddot{q}+(B_0+\tfrac{1}{2}\dot{M}(q)+S(q,\dot{q}))\dot{q} \\
&+K\hat{K}^{-1}\hat{J}^T(q)K_v\dot{\hat{X}}+K\hat{K}^{-1}\hat{J}^T(q)K_p s(e)+Z(q)\phi \\
&-Z(q)\hat{\phi}+Y(q,\hat{\phi})\phi_k-Y(q,\hat{\phi})K\hat{K}^{-1}\hat{\varphi}=0, \qquad (24)
\end{aligned}
$$

where we note that $Y(q,\hat{\phi})$, $K\hat{K}^{-1}$ are diagonal matrices.

Next, we define a Lyapunov function candidate V as:

$$
\begin{aligned}
V=&\tfrac{1}{2}\dot{q}^T(M_0+M(q))\dot{q}+\alpha\dot{q}^T(M_0+M(q))\hat{J}^+(q)s(e) \\
&+\textstyle\sum_{i=1}^m(\alpha k_{vi}+k_{pi})S_i(e_i)+\tfrac{1}{2}(\phi-\hat{\phi})^T L_1^{-1}(\phi-\hat{\phi}) \\
&+\tfrac{1}{2}(\hat{K}K^{-1}\phi_k-\hat{\varphi})^T L_2^{-1}K\hat{K}^{-1}(\hat{K}K^{-1}\phi_k-\hat{\varphi}), \qquad (25)
\end{aligned}
$$

where k_{pi}, k_{vi} denote the i^{th} diagonal elements of K_p and K_v respectively, $L_2^{-1}K\hat{K}^{-1}$ is a positive diagonal matrix. Since

$$
\begin{aligned}
&\tfrac{1}{4}\dot{q}^T(M_0+M(q))\dot{q}+\alpha\dot{q}^T(M_0+M(q))\hat{J}^+(q)s(e) \\
&\quad+\textstyle\sum_{i=1}^m(\alpha k_{vi}+k_{pi})S_i(e_i) \\
=&\tfrac{1}{4}(\dot{q}+2\alpha\hat{J}^+(q)s(e))^T(M_0+M(q))(\dot{q}+2\alpha\hat{J}^+(q)s(e)) \\
&-\alpha^2 s(e)^T(\hat{J}^+(q))^T(M_0+M(q))\hat{J}^+(q)s(e) \\
&+\textstyle\sum_{i=1}^m(\alpha k_{vi}+k_{pi})S_i(e_i) \\
\geq&\textstyle\sum_{i=1}^m(\alpha k_{vi}\hat{c}_i+k_{pi}\hat{c}_i-\alpha^2\lambda_m)s_i^2(e_i), \qquad (26)
\end{aligned}
$$

where α can be chosen small enough or k_{pi} and k_{vi} can be chosen large enough to satisfy the inequality,

$$
\alpha k_{vi}\hat{c}_i+k_{pi}\hat{c}_i-\alpha^2\lambda_m>0, \qquad (27)
$$

and $\lambda_m=\lambda_{max}[(\hat{J}^+(q))^T(M_0+M(q))\hat{J}^+(q)]$, $\lambda_{max}[A]$ denotes the maximum eigenvalue of a matrix A and $\lambda_{min}[A]$ denotes the minimum eigenvalue.

If we substitute the inequalities (26) and (27) into equation (25), we have

$$
\begin{aligned}
V\geq&\tfrac{1}{4}\dot{q}^T(M_0+M(q))\dot{q}+\tfrac{1}{2}(\phi-\hat{\phi})^T L_1^{-1}(\phi-\hat{\phi}) \\
&+\textstyle\sum_{i=1}^m(\alpha k_{vi}\bar{c}_{i1}+k_{pi}\bar{c}_{i2}-\alpha^2\lambda_m)s_i^2(e_i) \\
&+\tfrac{1}{2}(\hat{K}K^{-1}\phi_k-\hat{\varphi})^T L_2^{-1}K\hat{K}^{-1}(\hat{K}K^{-1}\phi_k-\hat{\varphi})>0. \qquad (28)
\end{aligned}
$$

Hence, V is positive definite. Note that α must be chosen sufficiently small or K_v, K_p must be chosen sufficiently large to guarantee the positive definiteness of V.

Differentiating V with respect to time and substituting equations (18) and (19) into it, we can get

$$
\begin{aligned}
\tfrac{d}{dt}V=&\dot{q}^T(M_0+M(q))\ddot{q}+\tfrac{1}{2}\dot{q}^T\dot{M}(q)\dot{q} \\
&+\alpha s^T(e)(\hat{J}^+(q))^T\dot{M}(q)\dot{q}+\alpha s^T(e)(\hat{J}^+(q))^T(M_0+M(q))\ddot{q} \\
&+\alpha\dot{q}^T(M_0+M(q))\dot{\hat{J}}^+(q)s(e)+\dot{X}^T K_p s(e) \\
&+\alpha\dot{q}^T(M_0+M(q))\hat{J}^+(q)\dot{s}(e)+\alpha\dot{X}^T K_v s(e) \\
&+(\dot{q}+\alpha\hat{J}^+(q)s(e))^T Z(q)(\phi-\hat{\phi}) \\
&+(\dot{q}+\alpha\hat{J}^+(q)s(e))^T Y(q,\hat{\phi})(\phi_k-K\hat{K}^{-1}\hat{\varphi}). \qquad (29)
\end{aligned}
$$

Substituting $(M_0+M(q))\ddot{q}$ from equation (24) into equation (29), we have:

$$
\frac{d}{dt}V=-W, \qquad (30)
$$

where

$$
\begin{aligned}
W=&\dot{q}^T\{B_0+K\hat{K}^{-1}\hat{J}^T(q)K_v\hat{J}(q)\}\dot{q}-\dot{q}^T\{J^T(q)K_p \\
&-K\hat{K}^{-1}\hat{J}^T(q)K_p-\alpha\hat{J}^T(q)K_v\hat{J}(q)\hat{K}^{-1}K\hat{J}^+(q) \\
&+\alpha J^T(q)K_v\}s(e)+\alpha s^T(e)(\hat{J}^+(q))^T K\hat{K}^{-1}\hat{J}^T(q)K_p s(e) \\
&+\alpha\{s^T(e)(\hat{J}^+(q))^T(B_0-\tfrac{1}{2}\dot{M}(q)+S(q,\dot{q}))\dot{q} \\
&-\dot{s}^T(e)(\hat{J}^+(q))^T(M_0+M(q))\dot{q} \\
&-s^T(e)(\dot{\hat{J}}^+(q))^T(M_0+M(q))\dot{q}\}. \qquad (31)
\end{aligned}
$$

From the last term of equation (31), since $s(e)$ is bounded, there exist constants $c_0>0$ and $c_1>0$ so that [6]:

$$
\begin{aligned}
&\alpha|s(e)^T(\hat{J}^+(q))^T[B_0-\tfrac{1}{2}\dot{M}(q)+S(q,\dot{q})]\dot{q} \\
&-s(e)^T(\dot{\hat{J}}^+(q))^T(M_0+M(q))\dot{q} \\
&-\dot{s}(e)^T(\hat{J}^+(q))^T(M_0+M(q))\dot{q}|\geq-\alpha c_0\|\dot{q}\|^2-\alpha c_1\|s(e)\|^2. \qquad (32)
\end{aligned}
$$

Substituting inequality (32) into equation (31) and defining $\Delta_K=I-K\hat{K}^{-1}$, $\Delta_J=J^T(q)-\hat{J}^T(q)$, we have

$$
\begin{aligned}
W\geq&\dot{q}^T[B_0+\hat{J}^T(q)K_v\hat{J}(q)-\Delta_K\hat{J}^T(q)K_v\hat{J}(q)-\alpha c_0 I]\dot{q} \\
&+\alpha s(e)^T[K_p-(\hat{J}^+(q))^T\Delta_K\hat{J}^T(q)K_p-\alpha c_1 I]s(e) \\
&-\dot{q}^T[\Delta_K\hat{J}^T(q)K_p+\Delta_J K_p+\alpha\hat{J}^T(q)K_v\hat{J}(q)\Delta_K\hat{J}^+(q) \\
&+\alpha\Delta_J K_v]s(e)\geq\{\lambda_{min}[B_0+\hat{J}^T(q)K_v\hat{J}(q)]-c_2\bar{\beta}\lambda_{max}[K_v] \\
&-\alpha c_0\}\|\dot{q}\|^2+\{\alpha\lambda_{min}[K_p]-\alpha c_5\bar{\beta}\lambda_{max}[K_p]-\alpha c_1\}\|s(e)\|^2 \\
&-\{c_3\bar{\beta}\lambda_{max}[K_p]+\bar{\gamma}\lambda_{max}[K_p]+\alpha c_4\bar{\beta}\lambda_{max}[K_v] \\
&+\alpha\bar{\gamma}\lambda_{max}[K_v]\}\|\dot{q}\|\|s(e)\|, \qquad (33)
\end{aligned}
$$

Since,

$$
-\|s(e)\|\cdot\|\dot{q}\|\geq-\frac{1}{2}(\|s(e)\|^2+\|\dot{q}\|^2), \qquad (34)
$$

we have

$$\begin{aligned}W \geq \{&\lambda_{min}[B_0 + \hat{J}^T(q)K_v\hat{J}(q)] - c_2\bar{\beta}\lambda_{max}[K_v]\\ &-\tfrac{1}{2}c_3\bar{\beta}\lambda_{max}[K_p] - \tfrac{1}{2}\bar{\gamma}\lambda_{max}[K_p] - \tfrac{1}{2}\alpha c_4\bar{\beta}\lambda_{max}[K_v]\\ &-\tfrac{1}{2}\alpha\bar{\gamma}\lambda_{max}[K_v] - \alpha c_0\}\|\dot{q}\|^2 + \{\alpha\lambda_{min}[K_p]\\ &-\alpha c_5\bar{\beta}\lambda_{max}[K_p] - \tfrac{1}{2}c_3\bar{\beta}\lambda_{max}[K_p] - \tfrac{1}{2}\bar{\gamma}\lambda_{max}[K_p]\\ &-\tfrac{1}{2}\alpha c_4\bar{\beta}\lambda_{max}[K_v] - \tfrac{1}{2}\alpha\bar{\gamma}\lambda_{max}[K_v] - \alpha c_1\}\|s(e)\|^2, \quad (35)\end{aligned}$$

where $c_2 = b_{\hat{J}^T}b_{\hat{J}}$, $c_3 = b_{\hat{J}^T}$, $c_4 = b_{\hat{J}^{+T}}b_{\hat{J}^T}b_{\hat{J}}$, $c_5 = b_{\hat{J}^{+T}}b_{\hat{J}^T}$, and $b_{\hat{J}^T}$, $b_{\hat{J}}$, $b_{\hat{J}^+}$, $b_{\hat{J}^{+T}}$ are the bounds for $\hat{J}^T(q)$, $\hat{J}(q)$, $\hat{J}^+(q)$, $(\hat{J}^+(q))^T$ respectively. Then we have

$$W \geq (\lambda_{max}[K_v]l_a - \alpha c_0)\|\dot{q}\|^2 + (\lambda_{max}[K_v]l_b - \alpha c_1)\|s(e)\|^2, \quad (36)$$

where

$$\begin{aligned}l_a &= \hat{\lambda}_1 - \bar{\beta}(c_2 + \tfrac{1}{2}c_3a_1 + \tfrac{1}{2}\alpha c_4) - \tfrac{1}{2}\bar{\gamma}(a_1 + \alpha),\\ l_b &= \alpha\hat{\lambda}_2a_1 - \bar{\beta}(\alpha c_5a_1 + \tfrac{1}{2}c_3a_1 + \tfrac{1}{2}\alpha c_4) - \tfrac{1}{2}\bar{\gamma}(a_1 + \alpha),\end{aligned}$$

$$\hat{\lambda}_1 = \frac{\lambda_{min}[B_0+\hat{J}^T(q)K_v\hat{J}(q)]}{\lambda_{max}[K_v]}; \quad \hat{\lambda}_2 = \frac{\lambda_{min}[K_p]}{\lambda_{max}[K_p]}; \quad a_1 = \frac{\lambda_{max}[K_p]}{\lambda_{max}[K_v]}.$$

Hence, if the following conditions are satisfied:

$$\hat{\lambda}_1 - \bar{\beta}(c_2 + \frac{1}{2}c_3a_1 + \frac{1}{2}\alpha c_4) - \frac{1}{2}\bar{\gamma}(a_1 + \alpha) > 0, \quad (37)$$

$$\alpha\hat{\lambda}_2a_1 - \bar{\beta}(\alpha c_5a_1 + \frac{1}{2}c_3a_1 + \frac{1}{2}\alpha c_4) - \frac{1}{2}\bar{\gamma}(a_1 + \alpha) > 0, \quad (38)$$

That is

$$min\{\frac{2\hat{\lambda}_1 - \bar{\gamma}(a_1 + \alpha)}{c_3a_1 + 2c_2 + \alpha c_4}, \frac{2\alpha\hat{\lambda}_2a_1 - \bar{\gamma}(a_1 + \alpha)}{2\alpha c_5a_1 + c_3a_1 + \alpha c_4}\} > \bar{\beta}, \quad (39)$$

$$min\{\frac{2\hat{\lambda}_1 - \bar{\beta}(2c_2 + c_3a_1 + \alpha c_4)}{\alpha + a_1}; \frac{2\alpha\hat{\lambda}_2a_1 - \bar{\beta}(2\alpha c_5a_1 + c_3a_1 + \alpha c_4)}{\alpha + a_1}\} > \bar{\gamma}, \quad (40)$$

then $l_a > 0$ and $l_b > 0$ and hence K_v can be chosen large enough so that

$$l_a - \frac{\alpha c_0}{\lambda_{max}[K_v]} > 0, \qquad l_b - \frac{\alpha c_1}{\lambda_{max}[K_v]} > 0, \quad (41)$$

and hence W is positive definite in $\dot{q}$ and $s(e)$.

Graphical illustrations of conditions (39) and (40) are shown in Figure 2 and 3. Figure 2(a) shows the relation between $\bar{\beta}$ and a_1 with $\bar{\gamma} = 0$, i.e., with the actuator model uncertainty only, and figure 2(b) shows the relation between $\bar{\gamma}$ and a_1 with $\bar{\beta} = 0$, i.e., with Jacobian matrix uncertainty only. The shaded area is the region where stability can be guaranteed. The figures show that with one kind of uncertainty existing, a smaller a_1 allows more uncertainty in the system. Figure 3 shows a 3-D illustration of the conditions when both uncertainties exist. Surface S_1 is described by condition (37) and surface S_2 is described by condition (38). The region between the two surfaces as indicated by the arrow is the region where stability can be guaranteed. From the figure, we can see that the allowable bound $\bar{\beta}$ of actuator model uncertainty and bound $\bar{\gamma}$ of Jacobian uncertainty are inversely proportional to the ratio a_1 of maximum eigenvalue of positional feedback gain to that of velocity feedback gain. If the actuator model uncertainty $\bar{\beta}$ and/or the Jacobian uncertainty $\bar{\gamma}$ increase, a smaller a_1 is required. Hence, to allow more uncertainty in Jacobian matrix and/or actuator model, a_1 should be kept smaller.

We are now in a position to state the following Theorem:

Theorem *The closed-loop system described by equation (18), (19) and (22) gives rise to the convergence of $X \to X_d$ and $\dot{q} \to 0$ as $t \to \infty$ if the feedback gains K_p and K_v are chosen to satisfy conditions (27), (39), (40), (41), $\hat{K}$ and $\hat{J}(q)$ are chosen to satisfy the condition (15) and (16) respectively.*

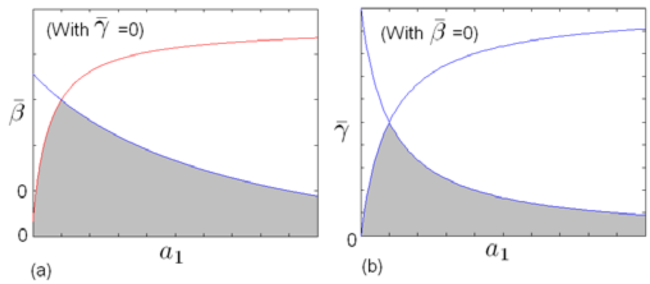

Fig. 2. (a)Variation of $\bar{\beta}$ with a_1 (with actuator uncertainty only) (b)Variation of $\bar{\gamma}$ with a_1 (with Jacobian uncertainty only)

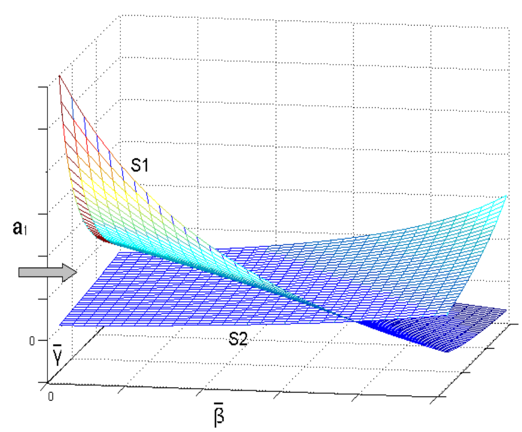

Fig. 3. Variation of $\bar{\beta}$ and $\bar{\gamma}$ with a_1(with both actuator uncertainty and Jacobian uncertainty)

Proof Since V and W are positive definite in $s(e)$ and $\dot{q}$, from equation (30), we have

$$\frac{d}{dt}V = -W \leq 0. \quad (42)$$

Hence, V is a Lyapunov function whose time derivative is negative definite in $(s(e), \dot{q})$. Since $W = 0$ implies that $\dot{q} = 0$ and $e = X - X_d = 0$, by LaSalle's invariance Theorem, the proof is complete.

Remark 2: The stability conditions (27), (39), (40), (41), presented in the Theorem are sufficient conditions to guarantee the stability of robot system in presence of uncertain actuator model and uncertain kinematics and dynamics. The conditions are simple conditions to achieve in practice. Conditions (27), (41) simply mean that the feedback gains should be chosen sufficiently large. Conditions (15), (16), (39) and (40) imply that the feedback gain K_p should be small as compared to K_v (see Figure 2 and Figure 3 also). Hence tuning can be established easily in practice.

IV. Simulation Results

In this section, we present some simulation results to illustrate the performances of the proposed controllers. Let us consider a 3-link planar robotic manipulator holding an object as shown in Figure 4.

$$J(q) = \frac{f_1}{z - f_1} \begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix} \begin{bmatrix} cos\delta & sin\delta \\ -sin\delta & cos\delta \end{bmatrix} \begin{bmatrix} -l_1 s_1 - l_2 s_{12} - (l_3 + l_o) s_{123} & -l_2 s_{12} - (l_3 + l_o) s_{123} & -(l_3 + l_o) s_{123} \\ l_1 c_1 + l_2 c_{12} + (l_3 + l_o) c_{123} & l_2 c_{12} + (l_3 + l_o) c_{123} & (l_3 + l_o) c_{123} \end{bmatrix} \quad (41)$$

$$\hat{J}(q) = \frac{\hat{f}_1}{\hat{z} - \hat{f}_1} \begin{bmatrix} \hat{\beta}_1 & 0 \\ 0 & \hat{\beta}_2 \end{bmatrix} \begin{bmatrix} cos\hat{\delta} & sin\hat{\delta} \\ -sin\hat{\delta} & cos\hat{\delta} \end{bmatrix} \begin{bmatrix} -\hat{l}_1 s_1 - \hat{l}_2 s_{12} - (\hat{l}_3 + \hat{l}_o) s_{123} & -\hat{l}_2 s_{12} - (\hat{l}_3 + \hat{l}_o) s_{123} & -(\hat{l}_3 + \hat{l}_o) s_{123} \\ \hat{l}_1 c_1 + \hat{l}_2 c_{12} + (\hat{l}_3 + \hat{l}_o) c_{123} & \hat{l}_2 c_{12} + (\hat{l}_3 + \hat{l}_o) c_{123} & (\hat{l}_3 + \hat{l}_o) c_{123} \end{bmatrix} \quad (42)$$

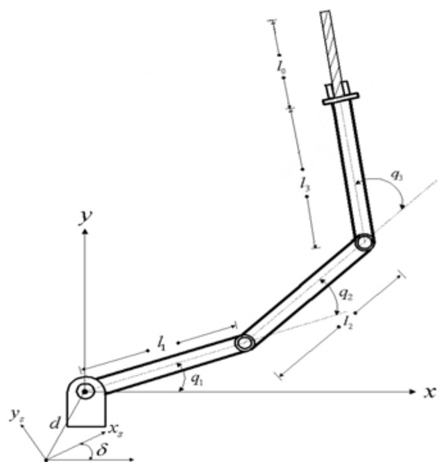

Fig. 4. A three-link planar robot

Using a fixed camera placed some distance away from the robot as the external sensor, the task space is defined in the vision coordinates described by $X = [x_s, y_s]^T$. The Jacobian matrix $J(q)$ of mapping from joint space to visual space is given by equation (41) [11]. Where $s_1 = sin(q_1), s_{12} = sin(q_1 + q_2)$, $s_{123} = sin(q_1 + q_2 + q_3)$ and $c_1 = cos(q_1), c_{12} = cos(q_1 + q_2)$, $c_{123} = cos(q_1+q_2+q_3)$. $\beta_1,\ \beta_2$ denote scaling factors in pixels/m, δ represents the angle of rotation of the vision coordinates relative to Cartesian coordinates, the offset of the origins of the coordinates $d = (d_x, d_y)^T$ (see Figure 4) were set to 0 m, z is the perpendicular distance between the robot and the camera, f_1 is the focal length of the camera, and l_o is the length of the object held, l_i is the length of link i.

The masses of the links $m_1,\ m_2,\ m_3$ were chosen as $1kg$, $l_1,\ l_2$ and l_3 were set as $0.5m$; and the mass of the object m_o was chosen as $0.5kg$ with a length of $0.3m$. f_1 was chosen as $16mm$, z was chosen as $1.5m$ and δ chosen as $45°$, $\beta_1 = \beta_2 = 78333$. The amplifiers were operating in $current$ mode. The exact parameters of the actuator as mentioned in section II were set as $K_{\tau 1} = 18, K_{\tau 2} = 14, K_{\tau 3} = 16, r_1 = r_2 = r_3 = 1$. Hence, $K = diag(K_{\tau 1}/r_1, K_{\tau 2}/r_2, K_{\tau 3}/r_3) = diag(18, 14, 16)$. The robot was required to move from an initial position of $[x(0), y(0)] = [480, 120]$ pixels to a desired position of $[x_d, y_d] = [975, 720]$ pixels in image space.

In the simulation, the camera parameters are estimated as $\hat{f}_1 = 12mm$, $\hat{z} = 1.2m$, the scaling factors are estimated as $\hat{\beta}_1 = \hat{\beta}_2 = 80000$, the lengths of the links and the object were estimated as $\hat{l}_1 = 0.55m, \hat{l}_2 = 0.45m, \hat{l}_3 = 0.7m, \hat{l}_o = 0.4m$ respectively.

First, the rotation angle δ was estimated as $\hat{\delta} = 60°$ and the actuator model as $\hat{K} = diag(10, 10, 10)$, the approximate Jacobian matrix $\hat{J}(q)$ can be obtained according to equation (42). To show the effects of actuator model uncertainty on the system performance, simulation result with only kinematic updating is shown in figure 5 with $K_p = 0.0001I$, $K_v = 0.0001I$ and $\alpha = 1$. Figure 6 shows the improved performance after adding in the actuator model updating algorithm. Note that the overall feedback gains are not small because the entries of $\hat{J}(q)K_p$ and $\hat{J}(q)K_v$ are multiplied by the large scaling factors $\beta_1,\ \beta_2$.

Next, $\hat{K}$ and estimated rotation angle $\hat{\delta}$ were varied to examine the effects of different uncertainties in kinematics and actuator model on the robot's motion. Simulation result with $\hat{\delta} = 80°$, $\hat{K} = diag(60, 35, 35), K_p = 0.0001I, K_v = 0.0001I$ and $\alpha = 1$ is shown in figures 7.

The results of the simulation study show that the control scheme we proposed in this paper is effective in dealing with uncertainties in the kinematics, dynamics and actuator model of the robot system and convergence of the position errors is guaranteed.

V. Conclusion

In this paper, we proposed a task-space controller for setpoint control of robotic manipulator with approximate models. The main advantage of the proposed control scheme is that exact knowledge of both Jacobian matrix and actuator model is not required. Sufficient conditions for choosing the feedback gains, approximate models were presented to guarantee the stability. The performance of the proposed controller was illustrated by simulation results.

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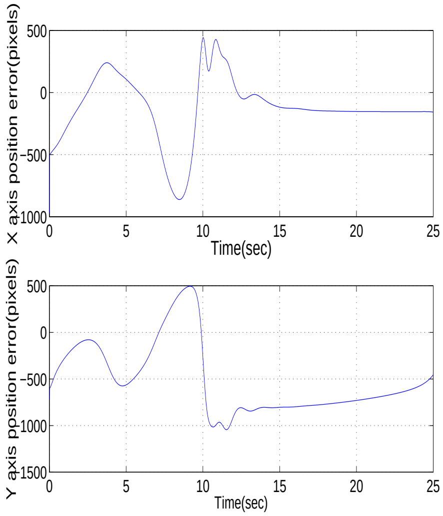

Fig. 5. Response with $\delta = 60°$, $\hat{K} = diag(10, 10, 10)$, $L_1 = 100I$ and $L_2 = 0$

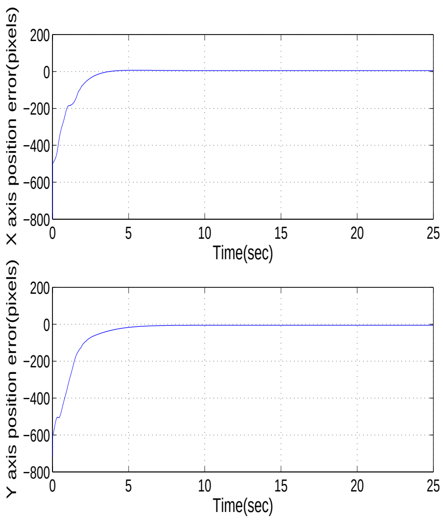

Fig. 6. Stable response with $\delta = 60°$, $\hat{K} = diag(10, 10, 10)$, $L_1 = L_2 = 100I$

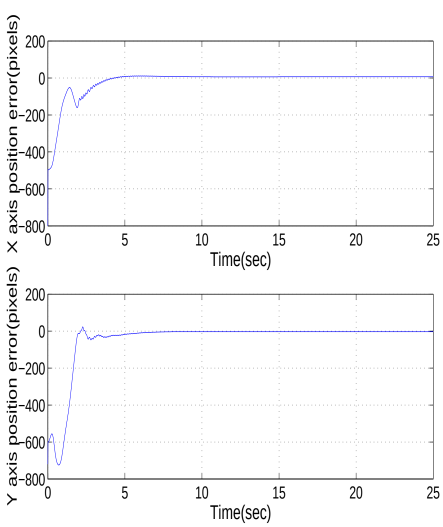

Fig. 7. Stable response with $\delta = 80°$, $\hat{K} = diag(60, 35, 35)$, $L_1 = L_2 = 100I$

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