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Monotonic Regression Filters for Trending Deterioration Faults

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Abstract— **This paper describes optimal nonlinear filtering algorithms for recovering trends of system performance variables (fault intensities) from noisy sensor data.**

A key underlying assumption for the algorithms is that the performance can only deteriorate with time, never improve. This assumption describes accumulating damage to the system components. Mathematically, the trend is obtained as a maximum likelihood estimate of an orbit in a hidden Markov model from the noisy output data. The empirical signal model and the overall problem setup are very close to optimal Kalman filtration. The main difference is that instead of a gaussian noise driving the random model of the fault a one sided exponentially distributed noise is assumed. Such a statistical model leads to a nonlinear batch filter. The trend is estimated by solving a quadratic programming problem. Unlike Kalman filters that can be implemented through recursive computations, the developed algorithms run in a batch mode. Though being more complex computationally, the developed trending algorithms demonstrate performance superior to Kalman filters in the fault trending applications.

I. INTRODUCTION

The focus of this work is on trending fault parameter estimates for system health management applications, in particular for predictive maintenance. The specific problem statement considered herein follows from this application but is quite fundamental. The health state estimates are computed from the data collected in a serviced equipment unit. So far, the applications have been in the aerospace area but the methodology should be applicable to health management of ground vehicles, process plants, and other complex and maintenance-critical systems.

Ii is assumed that the collected data is stored in a computer memory and processed by a trending computer between the usage cycles. Since the processing is done off-line, computational complexity of the processing algorithm is not a major issue.

This paper describes algorithms for estimating the trends of system performance variables (fault intensities) from noisy sensor data. A key assumption in the basis of the algorithms is that the performance can only deteriorate with time, never improve. This assumption is reasonable for the performance losses associated with accumulating mechanical damage to the system components. The paper shows that trending algorithms based on this monotonicity assumptions allow reliable detection and estimation of weak trends in very noisy data.

The simplest approach to trending is to perform the estimation of the fault parameters independently at each cycle and then perform a low-pass filtering of the data, e.g., see [2]. In this approach a single parameter (filter factor) provides a tradeoff between noise rejection and a delay in detecting a trend. Low-pass filtering is an *ad hoc* approach and it does not allow for incorporating a prior understanding of how the faults develop with time. When the filtering is heavy enough to reject the noise, the trend estimation would have a significant phase lag meaning a fault-caused deterioration might not be detected early enough.

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A more sophisticated approach to the trending is to use a Kalman Filter approach. Kalman filtering formulation defines easy-to-understand statistical model handles (covariances of various gaussian noises) and provides an optimized way for incorporating the prior information about the fault evolution into the filtering framework. The Kalman Filter approach was demonstrated to be sufficiently practical in many industrial applications. For instance, most modern navigation systems routinely use Kalman Filters with first or second-order dynamical models for trending motion of mobile platforms.

The prior information about fault variable being monotonic non-increasing can be utilized in the framework of *monotonic regression*. Monotonic regression is an advanced statistical regression method that has been extensively studied and applied for some time, e.g., see [6], [7]. The existing monotonic regression methods, provide ad hoc solutions to the problem. These solutions are guaranteed to be monotonic but are not guaranteed to be optimal in some sense. There is no regular way of incorporating additional information about the data model and fault evolution with these monotonic regression methods into the framework. For an optimal solution developed herein, additional assumptions can be always consistently incorporated by changing the models, constraints, or optimality index.

In this paper, the trend is determined as a maximum likelihood estimate of the orbit in a hidden Markov model from the noisy output data. The empirical signal model of the trend and the overall problem setup are very close to optimal Kalman filtration. The main difference is that instead of a gaussian noise driving the random model of the fault (performance variable) a one-sided exponentially distributed noise is assumed. Such a statistical model leads to a nonlinear filter, different from the Kalman least square regression solution. The trend is estimated by the filter through a solution of a QP (quadratic programming) problem. Unlike the Kalman filter that can be implemented through recursive computations, the developed algorithm processes the data in a batch mode. The formulation of a maximum likelihood estimate in a hidden Markov model presented herein is a special case of one found in [1].

To the best of the author's knowledge and literature search ability, the monotonic regression ideas - though

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simple - have not been considered in system health management, diagnostics, prognostics, and performance trending areas. The main contributions of this work are in (i) formulating specific nonlinear monotonic regression filtering problems that are simple and especially suitable for performance (fault) trending applications, (ii) demonstrating practically acceptable solutions of these problems through use of QP technology. The results of this work were practically implemented and are used in aerospace systems health management applications.

II. Data model

Consider a univariate case of trending an estimate of a single fault. The trending algorithms described below allow a straightforward extension towards a case of multiple faults and multiple measurements. The single fault case herein affords for better clarity of presentation.

Let $x(t)$ be a scalar performance deterioration (gradual) fault parameter at the usage cycle number t. As one example the performance parameter $x(t)$ can describe aerodynamic efficiency of a turbomachine stage. Let $y(t)$ be an estimate of the parameter $x(t)$ calculated from the data collected at this usage cycle. The estimate could be based on data about ambient conditions as well as data from internal sensors in the equipment unit.

Because of the modeling errors, sensor noise, and ambient condition variation, the estimate $y(t)$ contains an error and differs from $x(t)$

$$y(t) = x(t) + \gamma(t), \tag{1}$$

where $\gamma(t)$ is a scalar 'noise' variable. The data model (1) is used as a basis for the estimation and trending algorithms in this paper.

Consider the data sequences $x(t)$, $y(t)$ in (1), on the interval $t = 1, \ldots, N$ and denote them as

$$Y_N = \{y(1), \ldots, y(n)\} \tag{2}$$

$$X_N = \{x(1), \ldots, x(n)\} \tag{3}$$

The fault trending problem is to build an estimate of the underlying fault parameter sequence X_N (3) based on the observed data sequence Y_N (2). This is the main problem studied in the paper.

In most practical applications of trending the contribution of the noise $\gamma(t)$ in the model residual (1) is significant compared to the faults that need to be estimated. Thus, carefully designed statistical estimation of $x(t)$ is required.

III. First-order trending filter

In what follows, it is assumed that $\gamma(t)$ is an uncorrelated (white) noise sequence, where variable γ is zero mean gaussian distributed with the covariance Γ

$$\gamma \sim N(0, \Gamma) \tag{4}$$

To formulate a filtering problem, the statistical model of the observation noise (4) should be complemented by a statistical model of the underlying trend sequence $x(t)$. As a baseline, next subsection considers a classical random walk model. An optimal estimation of the trend in this case is given by a Kalman Filter. The following subsection considers a non-standard model of the random walk driven by a random sequence with an exponential distribution for each term. This second model leads to the nonlinear optimal estimator implementing the monotonic regression.

A. Gaussian noise and Kalman Filter

One of established approaches to probabilistic modeling of an unknown data sequence $x(t)$ in (1) is given by a Random Walk model

$$x(t+1) = x(t) + \xi(t), \tag{5}$$

where $\xi(t)$ in an uncorrelated gaussian noise sequence with covariance Ξ.

Since the random variables $\xi(t)$ are independent, the probabilistic model (1), (5) describes a Markov chain. The distribution of the chain state $x(t)$ at time t fully defines its future statistics evolution. As usual, to complete the model there is a need to describe the probabilistic properties of the initial conditions. The initial state is assumed to be normally distributed with the mean x_0 and covariance Q_0

$$x(t=1) \sim N(x_0, Q_0) \tag{6}$$

Given the model (1)–(6), the problem is to estimate the underlying trend $x(t)$ from the noisy data $y(t)$. This problem is known as an estimation of the *orbit* $x(t)$ of the Markov chain. Since variables are gaussian, a Maximum A posteriori Probability (MAP) estimate of $x(t)$ can be found by solving the batch least square problem: $p_{X_N|Y_N} \to \max$. Denote $J = -\log p_{X_N|Y_N}$. Then the problem is (see [1] for derivation)

$$J = \frac{[x(1)-x_0]^2}{2Q_0} + \sum_{t=1}^{N} \frac{[x(t)-y(t)]^2}{2\Gamma} + \sum_{t=2}^{N} \frac{[x(t)-x(t-1)]^2}{2\Xi} \to \min \tag{7}$$

In trending, the decisions are usually made based on most recent estimate $x(N)$. Instead of solving the orbit estimation problem, the last estimate can be found as a solution of a filtering problem. In this case a Kalman Filter provides the recursion for the optimal estimate. A derivation of the Kalman Filter is well-known and can be found in [4]. A Riccati equation describing the filter gain evolution converges to a steady state solution relatively quickly after an initial transient process and a stationary Kalman Filter can be used with little loss of performance. For the system (1)–(6), the stationary Kalman Filter equation can be presented in the form

$$\hat{x}(t+1) = \hat{x}(t) + K_* \left[y(t) - \hat{x}(t)\right], \tag{8}$$

where K_* is the filter gain. Since the noises are scalars, the stationary Ricatti equation can be solved analytically to yield the steady state gain

$$K_* = \sqrt{\alpha^2 + 2\alpha} - \alpha; \quad \alpha = \Xi/(2\Gamma), \tag{9}$$

where α has the meaning of the signal to noise ratio: Ξ is the covariance of the noise ξ driving the signal $x(t)$ and Γ is the covariance of the measurement noise γ. For small α, the filter gain K approaches zero. For $\alpha \to \infty$, the filter gain K approaches unity.

The steady-state Kalman Filter (8) is a simple exponential filter. An example of using such a filter for trending engine data can be found, for instance, in [2].

B. Monotonic regression

Consider now the random walk model of the form (5) where $\xi(t)$ in an uncorrelated noise sequence with each $\xi(t)$ distributed *exponentially*. In statistics, the exponential distribution is used to model the behavior of units that have a constant failure rate. This is the only memoryless random distribution. In that regard, one can think about the performance fault evolution as a process of accumulating independent microscopic failures.

Note that accumulation of the fault related damage described by (5) follows the spirit of Palmgren–Miner's cumulative damage theory, which is well known in the analysis of fatigue damage for mechanical elements [3], [5].

The exponential distribution depends on single parameter that has meaning of the average failure rate.

$$\xi \sim E(\lambda): \quad p(x) = \frac{1}{\lambda} e^{-x/\lambda} \tag{10}$$

The same probability distribution of the initial conditions (6) as in the previous subsection is assumed.

Consider now a problem of estimating the orbit $x(t)$ of the Markov chain (1), (4), (5), (6), (10). The orbit X_N in (3) should be estimated from the observed data Y_N (2).

Let us find a Maximal Likelihood (ML) estimate of the orbit $x(t)$. The Markov chain model is stationary – the update equations and the probability distributions do not depend on t. Hence, the Markov process in question is homogeneous and its statistical properties are completely defined by the *transition density* function

$$\phi(r; s) = p_{x(t)|x(t-1)}(r, s), \tag{11}$$

where $p_{x(t)|x(t-1)}(r, s)$ is the conditional probability density and the function $\phi(\cdot, \cdot)$ (11) is the same for any t. From (5), it follows that the conditional probability density is defined by the probability density (10) of the update noise and can be presented in the form

$$\phi(r; s) = \begin{cases} \frac{1}{\lambda} e^{-(r-s)/\lambda}, & r \geq s \\ 0, & r < s \end{cases} \tag{12}$$

The conditional expectation $p_{X_N|Y_N}$ can be calculated through the conditional probability density for the sequences (2), see [1] for more detail. The MAP estimate of the orbit $x(t)$ is obtained by solving the problem

$$-\log p_{X_N|Y_N} \to \min, \quad p_{X_N|Y_N} \neq 0 \tag{13}$$

Applying the Bayes' rule for conditional probabilities yields (see [1])

$$p_{X_N|Y_N} = p_0(x(1)) \cdot \prod_{t=1}^{N} p_\gamma(y(t) - x(t)) \phi(x(t); x(t-1)), \tag{14}$$

where $p_0(x(1))$ is the probability density function of the initial condition and $p_\gamma(\cdot)$ is the gaussian probability density of the noise (1).

The second inequality in (12) leads to the constraint

$$x(1) \leq x(2) \leq \ \dots \ \leq x(N) \tag{15}$$

If the monotonicity condition (15) is violated, then at least one of the multipliers $\phi(x(t); x(t-1))$ (12) contributing to the expression for $p_{X_N|Y_N}$ in (14) is zero. By substituting (6), (10), and (12) into (14) we get the problem of minimizing the loss index $J = -\log p_{X_N|Y_N}$ that needs to be solved for finding the sequence $x(t)$:

$$J = \frac{[x(1) - x_0]^2}{2Q_0} + \sum_{t=1}^{N} \frac{[x(t) - y(t)]^2}{2\Gamma} + \sum_{t=2}^{N} \frac{x(t) - x(t-1)}{\lambda} \to \min, \tag{16}$$

where (15) should be taken into account as a hard constraint. Note that all terms $x(t)$ in the last sum (16) cancel out, except for $x(1)$ and $x(N)$.

The problem (16) subject to (15) is a QP (Quadratic Programming) problem and very efficient computational methods, such as interior point methods, are available for such problems. A few QP-related codes are a part of Matlab Optimization Toolbox.

In case when no apriori information about x_0 is available, one can assume the initial condition covariance $Q_0 \to \infty$ and drop the first term in the r.h.s of (16). In that case, the MAP estimate of the orbit $x(t)$ becomes a ML estimate and depends on the single tuning knob parameter, $\beta = \lambda/\Gamma$).

The main difficulty with the problem (15), (16) is in the presence of the of the monotonicity constraints (15). Note, that the third term in the performance index (16) provides a penalty $\beta^{-1}[x(N) - x(1)]$ for the overall increase of the fault estimate x through the observation time. The weight at this penalty is essentially a ratio of the observation noise covariance Ξ to the fault driving noise covariance λ. The parameter β has the same essential meaning as the parameter α in the Kalman filter gain (9) and could be tuned empirically to achieve the desired performance of the filter, similar to how an exponential filter gain is tuned in practice.

C. Filter performance simulation and comparison

The developed monotonic regression trending algorithm was validated in extensive simulations. A random noise was added to systematic trends and the algorithm attempted recovering the underlying trend.

In addition to the source data, the trending results depend on the single tuning parameter β of the algorithm. This parameter $\beta = \lambda/\Gamma$ depends (i) on the covariance Γ of the gaussian observation noise and (ii) on the width (covariance) λ of the exponentially distributed innovation noise in the Markov chain model for the underlying trend. In the simulations, the observation noise was uniformly distributed, not gaussian; the underlying trend was a deterministic function, not a Markov chain realization. Thus, β was considered just as a tuning parameter of the algorithm without assigning to it any other special meaning.

An intuitive explanation of how β influences the results can be obtained by considering a case where the last data value is much larger than the second last trend value. For large β, the algorithm will draw a monotonic regression that jumps up in the end to accommodate this last data point. For small β, the algorithm assumes that the observed increase in the data is a random outlier and follows an average monotonic regression trend observed through many previous data points. Thus, β is essentially a smoothing parameter similar to the (inverse) time constant of an exponential filter.

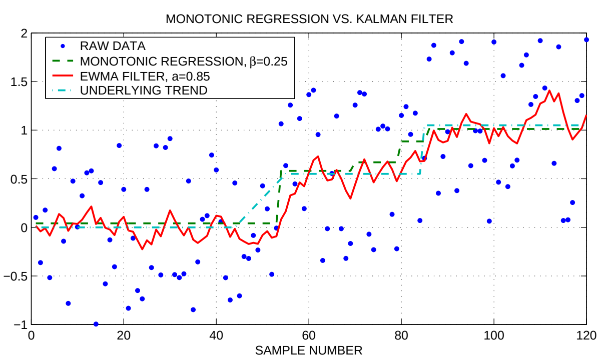

Fig. 1. Monotonic regression vs. exponential filtering. Dotted line - underlying trend. Solid line - monotonic regression with $\beta = 1/2$. Dashed line - exponential filtering with the filter factor 0.85. Dots - raw data.

For $\beta \to \infty$, the regularization penalty given by the last term in (16) vanishes. In that case the trending becomes very sensitive to outliers, especially those coming as a first or the last point in the data batch. Consider a data sequence where $y(1) \leq y(t)$ or $y(N) \geq y(t)$. Then, for $\lambda \to \infty$ (which is the same as $\beta \to \infty$) the minimum in (16) is achieved at $x(1) = y(1)$ or $x(N) = y(N)$ respectively. The filter does not have any smoothing action.

For $\beta \to 0$ we have $\lambda \to 0$ and the regularization penalty given by the last term in (16) dominates the optimization problem. In that case one can prove that an optimal estimate of the trend is given by a constant value $x(t) = \text{mean}(Y_N) = \text{const}$. The value of $\beta = 1/2$ was selected for the problem in hand. In Figure 1, this monotonic regression trend is compared against the exponential filtering results. By trial and error, the exponential filter factor of 0.85 was selected. This corresponds to the gain $K_* = 0.15$ in the stationary Kalman filter (8). By using (9), the signal-to-noise ratio parameter $\alpha = \Xi/(2\Gamma)$ can be found as $\alpha = 9/68 \approx 0.132$. In addition to the two estimated trends, Figure 1 shows the raw data and the underlying trend. The monotonic regression estimate is much closer to the underlying trend than the result of the exponential filtering.

IV. Second-order filter, prognostics

A. Second-order Kalman Filter, linear trend

The Random Walk model (5) of the unknown signal sequence $x(t)$ in (1) does not adequately describe systematic trends in the data. In fault diagnostics and prognostics, such trends can be an important indication of impending failure and might be used for evaluating the need for preventive maintenance.

A systematic way of modeling regular trends in the data for filtering, is through a second-order model

$$x_1(t+1) = x_1(t) + \xi_1(t), \qquad (17)$$

$$x_2(t+1) = a x_1(t) + x_2(t) + \xi_2(t) \qquad (18)$$

In (18), a is scalar parameter. The model (17)–(18) is a generalization of (5). The first equation, (17) describes the evolution of the fault growth rate. In the absense of the random excitation $\xi_1(t)$ this rate is assumed to be constant. The second equation, (18) describes the evolution of the fault itself.

In the absense of the random excitation $\xi_2(t)$ the fault parameter $x_2(t)$ grows at the rate $x_1(t)$.

The uncorrelated gaussian white noise sequences $\xi_1(t)$ and $\xi_2(t)$ in (17)–(18) are assumed to be independent and have covariances Ξ_1 and Ξ_2 respectively.

The signal model (17)–(18) should be complemented by a measurement model explaining the observed data sequence $y(t)$. This model is similar to (1) and has the form

$$y(t) = x_2(t) + \gamma(t), \qquad (19)$$

where $\gamma(t)$ is a Gaussian white noise described by (4).

Update equations (17)–(18) and observation equation (19) make a data model in the form suitable for Kalman filtering. A model of that form (constant velocity model) is commonly used in navigation, motion estimation, and tracking applications.

In order to formulate Kalman Filter equations similar to (8), consider the matrix form of (17)–(19). With an overload of notation, denote $x(t) = [x_1(t) \;\; x_2(t)]^T$, then

$$x(t+1) = Ax(t) + \xi(t) \qquad (20)$$

$$y(t) = Cx(t) + \gamma(t) \qquad (21)$$

where $\xi(t) = [\xi_1(t) \;\; \xi_2(t)]^T$ is a noise vector. With further overload of notation, the initial conditions and state noise are distributed as

$$x(t=1) \sim N(x_0, Q_0): \quad Q_0 = \text{diag}\{Q_{01}, Q_{02}\}, \qquad (22)$$

$$\xi \sim N(0, \Xi): \quad \Xi = \text{diag}\{\Xi_1, \Xi_2\} \qquad (23)$$

For the system (4), (20)–(23), the stationary Kalman Filter is a second-order filter that gives an estimate of the underlying trend $\hat{y}(t) = C\hat{x}(t)$ for the data $y(t)$ as

$$\hat{y} = C[Iz - A - K_* C]^{-1} K_* y \qquad (24)$$

The stationary Kalman filter gain K_* can be found from an algebraic Riccati equation. The filter gain K_* and, hence, the transfer function (24) depend on the following four design parameters: noise covariances Ξ_1, Ξ_2, Γ and the dynamic model parameter a. Note that only three parameters here are independent because of the possible variable change $x_1 \to ax_1$, $\xi_1 \to a\xi_1$. This variable change leads to the parameters scaled as $a \to 1$, $\Xi_1 \to a^2\Xi_1$. The second-order linear filter (24) can be applied to fault data trending in a straightforward way.

B. Second-order monotonic regression, secondary damage

The stochastic model (4), (20)–(23), allows modeling a regular trend in the data. This is expecially useful for prognostics applications of predictive trending. Yet, this model does not take into account inherent monotonicity (irreversibility) of the fault damage accumulation.

This section considers the model (17)–(18) with the initial conditions (22). The observation noise γ in (19) is again assumed to be gaussian distributed white noise (4). The state noises $\xi_1(t)$ and $\xi_2(t)$ are now assumed to have one sided distributions. At each t, the random variables $\xi_1(t)$ and $\xi_2(t)$ are independent and exponentially distributed in accordance with (10) as

$$\xi_1(t) \sim E(\lambda_1), \quad \xi_2(t) \sim E(\lambda_2) \tag{25}$$

The described model can be best explained as a primary and secondary damage model. It is based on a practically reasonable assumption that the system operates normally till an onset of fault condition. This condition shows up as a systematic deterioration trend of the performance variable $y(t)$. The systematic deterioration rate is $x_1(t)$ and it can only increase with time. The rate $x_1(t)$ must be watched and defines prognostics of the trend. In addition to the systematic and accelerating trend, the random variable $\xi_2(t)$ describes monotonic accumulation of the damage that follows the Palmgren–Miners rule.

The deterioration rate $x_1(t)$ could include a systematic average performance loss for the cycle. The model can be conveniently used for describing the accumulation of secondary damage in the system caused by a primary fault condition. In that case, $x_1(t)$ corresponds to the intensity of the primary fault condition and $x_2(t)$ describes the secondary damage accumulating because of this primary fault condition.

Having described and explained the second order monotonic regression model, let us consider the problem of estimating the orbit $x(t) = [x_1(t)\ x_2(t)]^T$ (3) of the Markov chain (4), (17)–(18), (22), (25) from the observed data sequence $y(t)$ (2). The derivation of the nonlinear filter largely repeats (and extends) the derivation for the first-order monotonic regression filter in the previous section.

To find a MAP estimate of the orbit $x(t)$, consider the transition density function (12) $\phi(r; s) = p_{x(t)|x(t-1)}(r, s)$, where $p_{x(t)|x(t-1)}(r, s)$ is the conditional probablity density. From (17)–(18), (25), it follows that the transition density can be presented in the form

$$\phi(r; s) = \frac{1}{\lambda_1} e^{-(r_1 - s_1)/\lambda_1} \frac{1}{\lambda_2} e^{-(r_2 - as_1 - s_2)/\lambda_2} \tag{26}$$

for $r_1 \geq s_1$, $r_2 \geq s_2$; and $\phi(r; s) = 0$ if $r_1 < s_1$ or $r_2 < s_2$.

The conditional expectation for MAP estimation yields the loss index $J = -\log p_{X_N|Y_N}$,

$$J = \frac{1}{2}(x(1) - x_0)^T Q_0^{-1}(x(1) - x_0) + \sum_{t=1}^{N} \frac{[y(t) - x_2(t)]^2}{2\Gamma} + \frac{x_1(N) - x_1(1)}{\lambda_1} + \frac{x_2(N) - x_2(1)}{\lambda_2} - \sum_{t=1}^{N-1} \frac{a x_1(t)}{\lambda_2} \tag{27}$$

where it is assumed that for $t = 1, \ldots, N-1$

$$x_1(t+1) - x_1(t) \geq 0, \quad x_2(t+1) - x_2(t) - ax_1(t) \geq 0 \tag{28}$$

The MAP estimate of the orbit $x(t)$ is obtained by solving the optimization problem $J \to \min$ as defined by (27) with the constraints (28). This is a QP (Quadratic Programming) problem. Note that unlike the first-order monotonic regression problem (16), (28), the second-order monotonic regression problem (27), (28), is ill-defined (underspecified). That is, the Hessian of the quadratic form (27) has only N nonzero singular values out of $2N$ total. Not every QP solver can deal with such problems.

Similar to the first-order monotonic regression problem, in most cases there is no information about the initial value x_0 of the trend. Thus, the initial condition covariance can be assumed infinite, $Q_0 = \infty$, and the second term in the loss index (27) disappears. Consider now the variable change $x_1 \to ax_1$, $\xi_1 \to a\xi_1$. It leads to the parameter change $a \to 1$, $\lambda_1 \to a\lambda_1$. With that in mind, the Maximum Likelihood estimate of orbit (assuming $Q_0^{-1} = 0$) can be multiplied through by Γ and shown to depend on two tuning knob parameters only:

$$\beta_1 = a\lambda_1/\Gamma, \quad \beta_2 = \lambda_2/\Gamma \tag{29}$$

The parameter β_2 provides a penalty of the fault estimate x through the observation time and is essentially similar to the parameter β in the first-order monotonic regression problem of previous section. The parameter β_1 provides a penalty for the linear trend in the data and characterizes the amplitude of the driving noise ξ_1 in (17). If $\beta_1 \to 0$, the second-order monotonic regression estimate coincides with the first-order monotonic regression. If $\beta_1 \to \infty$, an average linear trend only is estimated.

C. Filter performance simulation

The described second-order monotonic regression trending algorithm was validated in simulation. The data set was similar to one used for testing the first-order monotonic regression trending and included 80 points. The underlying trend held a constant value for 15 samples, then stepped up by 0.4 then was constant for 35 more samples, then started ramping up with the slope of 0.04 per sample. In the data set this underlying trend (the orbit) was distorted by adding an uncorrelated random noise uniformly distributed on the $[-1,\ 1]$ interval. The noise was produced by Matlab random number generator function `rand`.

The trending results depend on the tuning parameters of the algorithm: β_1 and β_2. These parameters $\beta_j = \lambda_j/\Gamma,\ j = 1,2$ are defined by the covariance Γ of the gaussian observation noise and the parameters λ_j of the exponential distribution for innovation noises in the second-order Markov model of the trend. In the test data set, the observation noise is uniformly distributed, not gaussian. In reality a trend is a deterministic function. Thus, we considered β_1 and β_2 as tuning parameter of the algorithm without assigning to them any other meaning. As explained above, β_1 and β_2 are smoothing parameters similar to the (inverse) time constant of the exponential filter.

For small β_2, the solver fits a concave piece-wise linear trend into the data. For large β_2, the second-order monotonic regression yields a piece-wise constant trend, which is similar to a first-order monotonic regression solution of the previous section. This is because large β_2 corresponds to large covariance λ_2 in the state noise model (10), (25). In turn, large state equation noise means slower filtering - this is well recognized in Kalman filtering. A slow filter for the coordinate x_2 means a piece-wise constant solution that is not very responsive to changes in the data. Similarly, small β_2 corresponds to small λ_2 and this leads to the part of the filter that follows the model for the coordinate x_1 providing the dominant (slow) dynamics yielding a piece-wise linear concave function.

Based on experimentation, the tuning knob values were selected as $\beta_1 = 1$, $\beta_2 = 1/2$. Figure 2 compares the designed second-order monotonic regression filter against the result for the stationary Kalman Filter described earlied in this section. By trial and error, the noise covariances $\Gamma = 10^4$, $\Xi_1 = 1$, $\Xi_2 = 50$ were found to provide the best trending quality for the Kalman Filter. In addition to the two estimated trends, Figure 2 shows the raw data and the underlying trend. As one can see, the second-order monotonic regression estimate recovers the underlying trend with by far superior quality of estimation compared to the second-order Kalman Filter.

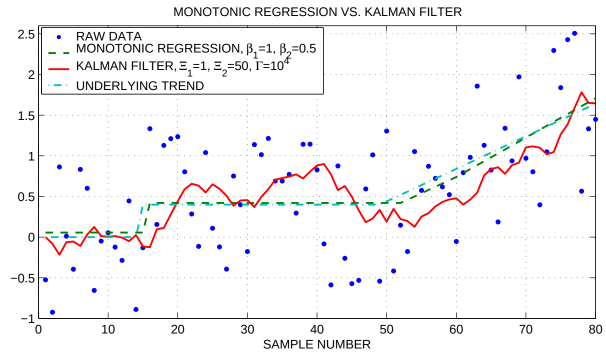

Fig. 2. Comparison of second order Kalman Filter and the second order monotonic regression filter. Solid line - Kalman filtering results. Dashed line - monotonic regression. Dash-dotted line - underlying trend.

V. Conclusions

Nonlinear filtering algorithms have been developed for trending fault estimate sequence. The fundamental statistical model used for the nonlinear filtering in obtaining trends is based on the assumption of monotonic increase of the fault parameters. The faults can only accumulate, and the fault condition would never improve unless a maintenance action is taken. The two fault models were discussed including a first-order model describing fault accumulation and a second order-model describing secondary damage caused by accumulating primary fault. The deterioration rate caused by the secondary damage can be assumed sustained and is used for the prognostics of the fault condition. The developed algorithms have similarity with basic Kalman filtering methods. Unlike linear Kalman filters that are based on gaussian noise models, the developed filters are nonlinear and are based on exponential one-sided noise statistics. Comparison with Kalman filters shows superiority of the developed trending approaches. They are suitable for a broad use in the trend monitoring applications.

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