

A Dead-zone Based Filter for Systems with Unknown Parameters

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Abstract

In this paper, we focus on parameter estimation in systems with output noise. By adding a dead-zone to the Linear Adaptive Estimator, it is shown that statistically the bounded output noise can be filtered out and that the true unknown parameters are estimated exactly. This time-domain noise filter which applies to systems with unknown parameters is denoted as filtered deadzone estimator and it is later extended to situation where the output noise is white noise. The difference between model disturbance and output noise is discussed and the extension to situation where both of them exist is proposed.

1 Introduction

Adaptive estimation algorithms have been developed for dynamic systems where the unknown parameters occur both linearly and nonlinearly over the past several errors. While stability properties of these estimators have been studied in [1]-[4], parameter convergence properties have been studied in [1]-[8]. In the presence of external disturbances and noise, it is well known that for linearly parameterized systems, either modifications in the adaptive law or persistently exciting reference inputs have to be introduced to establish robustness. The same however has not been established for nonlinearly parameterized systems thus far, and is addressed in this paper. In particular, we establish that when output noise is present, a modified algorithm that include a deadzone, similar to that in [1], can be used to establish boundedness. We also show that the deadzone algorithm filters the output noise statistically and guarantees the asymptotic convergence of the estimates to true unknown parameters, and is denoted as the filtered deadzone estimator (FDE). The paper is organized as follows. In section 2, problem formulation is proposed and the inability of the adaptive estimator to deal with output noise, without any modifications, is discussed. In section 3, the FDE is proposed. Proof of asymptotic convergence is also given. In section 4, comparison between output noise and model disturbance is discussed and the extension to situation where both of them exist is made. Section 5 shows simulation re-

sults.

2 Problem Formulation

We consider a nonlinearly parameterized dynamic system with bounded output noise such as

$$\begin{aligned}\dot{y} &= -\alpha y + \sum_{i=0}^N c_i(\omega^*)^i \\ y_n &= y + n(t)\end{aligned}\quad (1)$$

where c_i are measurable signals, $\omega^* \in \mathbb{R}$ is unknown parameter, $y \in \mathbb{R}$ is inaccessible state variable, output noise $n(t)$ is a stationary stochastic process and y_n is measured output signal.

We make the following assumptions regarding the stationary stochastic process $n(t)$.

Assumption 1: $|n(t)| \leq n_{max}$, $\forall t \geq 0$ where n_{max} is a known positive constant.

Assumption 2: $n(t_1)$ is independent of $y(t_2)$, $\forall t_1, t_2 \geq 0$.

Assumption 3: $n(t_1)$ and $n(t_2)$ are i.i.d. if $|t_1 - t_2| > \Delta$ for any $\Delta > 0$.

Assumption 4: $n(t)$ is piece-wise differentiable.

Assumption 4 implies that $\dot{n}(t)$ exists almost everywhere except on a set D of measure zero.

In what follows, we refer to $\dot{n}(t)$ only at points in the real line not including D .

Assumption 5: $|\dot{n}(t)| \leq m_{max}$.

Assumption 6: $\Delta^2 m_{max} \leq \eta$ for any $\Delta > 0, \eta > 0$.

It can be shown that a typical measurement noise due to effects of quantization satisfies assumptions 1-6.

2.1 Polynomial Adaptive Estimator (PAE)

In this section, we examine the properties of a N th order PAE with N auxiliary estimates $\hat{\omega}_1, \dots, \hat{\omega}_N$ that was proposed in [8] for (1) in the absence of noise.

Suppose the Lyapunov function candidate is chosen as

$$V = y_n^2/2 + \sum_{i=1}^N p_i(\tilde{\omega}_i), \quad \tilde{\omega}_i = \hat{\omega}_i - \omega^* \quad (2)$$

where

$$\begin{aligned} p_i(\tilde{\omega}_i) &= \frac{1}{i+1} \tilde{\omega}_i^{i+1} & \text{if } i \text{ is odd;} \\ p_i(\tilde{\omega}_i) &= \frac{1}{i} \tilde{\omega}_i^i + \frac{k_i}{i+1} \tilde{\omega}_i^{i+1} & \text{if } i \text{ is even} \end{aligned} \quad (3)$$

for $i = 1, \dots, N$, and k_i is to be chosen appropriately as in [8]. The corresponding g_i is the derivative of p_i w.r.t. $\tilde{\omega}_i$ as of

$$\begin{aligned} g_i(\tilde{\omega}_i) &= \tilde{\omega}_i^i & \text{if } i \text{ is odd;} \\ g_i(\tilde{\omega}_i) &= \tilde{\omega}_i^{i-1} + k_i \tilde{\omega}_i^i & \text{if } i \text{ is even.} \end{aligned} \quad (4)$$

We note that $\tilde{\omega}_i = \hat{\omega}_i - \omega^*$ and g_i is a i th order polynomial function of ω^* and it can be expressed as

$$g_i = \sum_{j=0}^i d_{ij}(\hat{\omega}_i)(\omega^*)^j. \quad (5)$$

The PAE is of the form

$$\begin{aligned} \dot{\hat{y}} &= -\alpha \hat{y} + \phi_0^* \\ \dot{\hat{\omega}}_i &= -\tilde{y}_n \phi_i^*, \quad i = 1, \dots, N \\ \tilde{y}_n &= \hat{y} - y_n \\ \phi^* &= A^{-1}C \end{aligned} \quad (6)$$

where $\phi^* = [\phi_0^*, \phi_1^*, \dots, \phi_N^*]^T$, $sat(\cdot)$ denote the saturation function, A is a non-singular $(N+1) \times (N+1)$ matrix as of

$$A = \begin{bmatrix} d_{00} & * & * & \dots & * \\ 0 & d_{11} & * & \dots & * \\ 0 & 0 & d_{22} & \dots & * \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & d_{NN} \end{bmatrix} \quad (7)$$

and

$$C = [c_0 \ c_1 \ \dots \ c_N]^T. \quad (8)$$

The element of i th row and j th column of matrix A in (7) is

$$A_{ij} = \begin{cases} 0 & i > j; \\ d_{(j-1)(i-1)} & i \leq j \end{cases}$$

where d_{ji} is defined as in (5).

Combining (1), (6) and (2), using the same derivation as in [8], the derivative of V follows as

$$\dot{V} = \tilde{y}_n(-\alpha \tilde{y} - \dot{\hat{y}})$$

and hence

$$\dot{V} = -\alpha \tilde{y} \tilde{y}_n - \tilde{y}_n \dot{\hat{y}}.$$

Since V cannot be guaranteed to be nonpositive in a compact set, it follows that V need not be bounded. Therefore modifications in the adaptive law are needed.

3 Filtered Deadzone Estimator

The result of no convergence of PAE with output noise in section 2.1 raises a problem for its application. To overcome this difficulty, we introduce a filtered deadzone estimator (FDE) as

$$\begin{aligned} \dot{\hat{y}} &= -\alpha \hat{y} + \phi_0^* \\ \dot{\hat{\omega}}_i &= -\tilde{y}_{n\epsilon} \phi_i^*, \quad i = 1, \dots, N \\ \tilde{y}_n &= \hat{y} - y_n \\ \tilde{y}_{n\epsilon} &= \tilde{y}_n - n_{max} \text{sat}\left(\frac{\tilde{y}_n}{n_{max}}\right) \\ \phi^* &= A^{-1}C \\ \hat{n} &= y_n - \hat{y} \end{aligned} \quad (9)$$

where \hat{n} is the filtered out output noise, $\phi^* = [\phi_0^*, \phi_1^*, \dots, \phi_N^*]^T$, A and C are defined as in (7) and (8), and $sat(\cdot)$ denotes the saturation function and is given by $sat(x) = sign(x)$ if $|x| \geq 1$ and $sat(x) = x$ if $|x| < 1$. In fact, the relationship between $\tilde{y}_{n\epsilon}$ and \tilde{y}_n is of

$$\tilde{y}_{n\epsilon} = \begin{cases} \tilde{y}_n - n_{max} & \text{if } \tilde{y}_n > n_{max}; \\ 0 & \text{if } -n_{max} \leq \tilde{y}_n \leq n_{max}; \\ \tilde{y}_n + n_{max} & \text{if } \tilde{y}_n < -n_{max} \end{cases} \quad (10)$$

and we use the Lyapunov function candidate

$$V = \tilde{y}_{n\epsilon}^2/2 + \sum_{i=1}^N p_i(\tilde{\omega}_i), \quad \tilde{\omega}_i = \hat{\omega}_i - \omega^* \quad (11)$$

where $p_i(\cdot)$ are the same as in (3).

The following properties can be derived for the FDE:

Property 1

- (i) $\tilde{y}_{n\epsilon} > 0 \Rightarrow \tilde{y} > \tilde{y}_{n\epsilon}$
- (ii) $\tilde{y}_{n\epsilon} < 0 \Rightarrow \tilde{y} < \tilde{y}_{n\epsilon}$

Proof of Property 1: First, let us consider case (i) in Property 1. $\tilde{y}_{n\epsilon} > 0$ implies from (10) that

$$\tilde{y}_n > n_{max} \text{ and } \tilde{y}_{n\epsilon} = \tilde{y}_n - n_{max}. \quad (12)$$

Because $\tilde{y}_n = \tilde{y} + n$ and $|n| \leq n_{max}$, it follows from (12) that

$$\tilde{y} > \tilde{y}_{n\epsilon}$$

which proves Case (i). Case (ii) of Property 1 can be proved in a similar manner. •

Property 2

$$\hat{y}(t) \text{ is independent of } \dot{n}(t). \quad (13)$$

Proof of Property 2: We note that n comes into the estimator by affecting $\hat{\omega}$ first and \hat{y} later. Consider the effects of n on $\hat{y}(t_1)$, it follows from Assumption 2 that $y(t_1)$ is independent of $n(t_2)$, $t_2 < t - \Delta$. It follows from [8] that ϕ_i^* is bounded by B_ϕ and thus the maximum effects of $n(t)$, $t \in [t_1 - \Delta, t_1]$ on $\hat{\omega}_i$ is bounded by

$$B_\phi n_{max} \Delta. \quad (14)$$

$\phi_0^*(t)$ is related with $\hat{\omega}_i(t)$ through a non-singular matrix, therefore, the maximum effects of $n(t)$, $t \in [t_1 - \Delta, t_1]$ on $\hat{y}(t_1)$ is bounded by

$$B n_{max} \Delta^2 \quad (15)$$

where B is some bounded constant which is independent of noise. It follows from Assumption 3 that (15) can be arbitrarily small which proves Property 2. •

Property 3 $E[\dot{n}(t)] = 0$.

Proof of Property 3: Since $|n(t)| \leq n_{max}$, it follows that

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_{\tau=0}^T \dot{n}(\tau) d\tau = \lim_{T \rightarrow \infty} \frac{n(T) - n(0)}{T} = 0. \quad (16)$$

Now that $n(t)$ is a stationary process, therefore

$$E[\dot{n}(t)] = 0. \quad \bullet$$

Property 4 $Prob[\int_0^\infty \tilde{y}_{n\epsilon} \dot{n}(t) dt < \infty] = 1$.

Proof of Property 4 can be found in [9].

In the following Theorem, we will show that the output error $\tilde{y}_{n\epsilon}$ will converge to zero with probability 1.

Theorem 1 For problem formulation in (1) and FDE as in (9), it follows that

$$Prob[\lim_{t \rightarrow \infty} \tilde{y}_{n\epsilon} = 0] = 1. \quad (17)$$

Proof of Theorem 1: For the FDE algorithm as in (9), and Lyapunov function candidate V as in (11), using the same derivation as in [8] the derivative of V follows as

$$\dot{V} = -\alpha \tilde{y}_{n\epsilon} \tilde{y} - \tilde{y}_{n\epsilon} \dot{n}. \quad (18)$$

It follows from (18) that

$$V(0) - V(\infty) = \int_0^\infty \alpha \tilde{y}_{n\epsilon} \tilde{y} d\tau + \int_0^\infty \tilde{y}_{n\epsilon} \dot{n} d\tau. \quad (19)$$

From Property 1, we have

$$\alpha \tilde{y}_{n\epsilon} \tilde{y} \geq \alpha \tilde{y}_{n\epsilon}^2 \geq 0. \quad (20)$$

Property 4 implies that

$$Prob[\int_0^\infty \tilde{y}_{n\epsilon} \dot{n} d\tau < \infty] = 1. \quad (21)$$

Now that $V(0)$ is bounded and $V(\infty) > 0$, it follows from (19), (20), and (21) that

$$\int_{t_0}^\infty \alpha \tilde{y}_{n\epsilon}^2 d\tau < \infty \quad (22)$$

with probability 1. Since derivative of $\tilde{y}_{n\epsilon}$ is bounded, it follows from (22) and Barbalat's lemma that

$$\lim_{t \rightarrow \infty} \tilde{y}_{n\epsilon} = 0 \quad (23)$$

Hence,

$$Prob[\lim_{t \rightarrow \infty} \tilde{y}_{n\epsilon} = 0] = 1,$$

which proves the Theorem. •

We note here that probability 1 implies that

$$\lim_{t \rightarrow \infty} \tilde{y}_{n\epsilon} = 0.$$

3.1 Parameter Convergence of FDE

Theorem 1 established that output error $\tilde{y}_{n\epsilon}$ will converge to zero and parameter estimate is steady. What remains is whether $\hat{\omega}$ will converge asymptotically to ω^* . First, we note that once Lyapunov function V , which is defined as in (11), reaches zero, it will rest there and never left. This is different from PAE where the noise will drive V away from zero. In this section, we will discuss if (17) implies $V = 0$ and under what conditions.

In the system in (1), we have no assumption about the statistical properties of the output noise $n(t)$. Now we assume $n(t)$ is of

$$n(t) = U[n_L, n_H] \quad \forall t \quad (24)$$

where $U[n_L, n_H]$ is the uniform distribution in region $[n_L, n_H]$. Of course it will satisfy

$$|n_L| \leq n_{max} \quad |n_H| \leq n_{max} \quad n_H > n_L. \quad (25)$$

We define a signal x which is a function of $\tilde{y} = \hat{y} - y$ when

$$x = \begin{cases} \tilde{y} - n_{max} + n_H & \text{if } \tilde{y} > n_{max} - n_H; \\ 0 & \text{if } -n_{max} - n_L \leq \tilde{y} \leq n_{max} - n_H; \\ -\tilde{y} - n_{max} - n_L & \text{if } \tilde{y} < -n_{max} - n_L. \end{cases} \quad (26)$$

About x , we have the following lemma.

Lemma 1 For problem formulation in (1), FDE in (9) and output noise as in (24), it follows that

$$Prob[\lim_{t \rightarrow \infty} x = 0] = 1. \quad (27)$$

Proof of Lemma 1: If (27) does not hold, it implies that

$$Prob[x(t) \neq 0] > 0 \quad (28)$$

as $t \rightarrow \infty$. $x(t) \neq 0$ implies that

$$\tilde{y} > n_{max} - n_H \quad \text{or} \quad \tilde{y} < -n_{max} - n_L. \quad (29)$$

Combining (24), (28) and (29), it follows that

$$Prob[\tilde{y}_{n\epsilon} \neq 0] > 0 \quad (30)$$

as $t \rightarrow \infty$ which contradicts Theorem 1. Therefore, lemma 1 must hold. •

Lemma 1 implies that

$$-n_{max} - n_L \leq \tilde{y} \leq n_{max} - n_H \quad (31)$$

if (24) and (25) hold.

In what follows, we will discuss the convergence of estimates for several cases.

Case 1: $n_L = -n_{max}$, $n_H = n_{max}$

It follows from (26) that $x = |\tilde{y}|$ and Lemma 1 states that

$$\lim_{t \rightarrow \infty} \tilde{y} = 0.$$

Thus, just the input signals satisfy the Nonlinear Persistent Condition established in [8], $\hat{\omega}$, which is derived from $\sum_{i=0}^N c_i \hat{\omega}^i = \phi_0^*$, will converge to ω^* asymptotically. In case 1 of the simulation results in section 5, the asymptotic convergence is illustrated.

Case 2: $n_L > -n_{max}$, $n_H = n_{max}$

It follows from Theorem 1 that $\tilde{y}_{n\epsilon}$ will converge to zero as $t \rightarrow \infty$ and hence $\hat{\omega}$ come to some steady value $\hat{\omega}^c$. It

follows from Lemma 1 that when $t \rightarrow \infty$, \tilde{y} will always be nonpositive. Instead of the NLPE condition, if for any t , there exists time constant T, ϵ_0 such that

$$\int_{\tau=t}^{t+T} \left(\sum_{i=0}^N c_i (\omega^i - (\omega^*)^i) \right) d\tau \geq \epsilon_0 \|\omega - \omega^*\|, \quad (32)$$

it will guarantee the asymptotic convergence of $\hat{\omega}$ to ω^* . The reason is that for any $\hat{\omega}^c \neq \omega^*$, if the input satisfies (32), \tilde{y} will always become significantly positive. Hence, $\hat{\omega}$ must converge to ω^* . In case 2 of the simulation results in section 5, the asymptotic convergence with biased output noise is illustrated.

Case 3: White Noise

When n is white noise which is not bounded, for a given n_{max} we can decompose n into 2 components

$$n = n_1 + n_2$$

where

$$\begin{aligned} n_1 &= n & n_2 &= 0 & |n| &\leq n_{max} \\ n_1 &= n_{max} & n_2 &= n - n_{max} & n &> n_{max} \\ n_1 &= -n_{max} & n_2 &= n - n_{max} & n &< -n_{max}. \end{aligned}$$

For white noise, n can be very large however just at very small measure in time. Choosing appropriate threshold value n_{max} to make

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_{t=0}^T |n_2| dt$$

small, we can treat the effects of additional n_2 as a disturbance which will perturb the convergence of $\hat{\omega}$. We note that FDE does not depend on the initial value of $\hat{\omega}$ and will correct the disturbance. Thus, $\hat{\omega}$ is not steady but perturbed at some amplitude. Choosing n_{max} which makes $\hat{\omega}$ perturbed in the desired precision and we are done. The trade-off here is just if we want higher precision of $\hat{\omega}$, we need to set the value of n_{max} bigger and therefore the time needed for convergence is longer.

3.2 Output Noise Filter

After $\hat{\omega}$ converges to ω^* , \tilde{y} converges to zero as well and the output noise can be evaluated exactly. We introduce the concept of the Filtered Deadzone Estimator as what follows.

Definition 1 In dynamic system with unknown parameters, the Filtered Deadzone Estimator (FDE) is the method which applies the deadzone adaptive estimator to estimate the unknown parameters and then filter out the output noise at the same time when parameter estimation converges.

In FDE, the estimation of output noise n is simply

$$\hat{n} = y_n - \hat{y}$$

and it follows that

$$\hat{n} - n = y - \hat{y}$$

which means that the errors of \hat{y} and \hat{n} to y and n are of the same amplitude and different sign. The convergence of them happens at the same time. Now that both y and n are not accessible, the indication of the convergence of \hat{y} and \hat{n} is that $\tilde{y}_{n\epsilon}$ converges to 0 and $\hat{\omega}$ keeps steady.

Another information which can be derived from the FDE is the derivative of y . Now that $\hat{\omega} \rightarrow \omega^*$ and $\hat{y} \rightarrow y$, it follows naturally that the estimation of \dot{y} is of

$$\hat{\dot{y}} = -\alpha\hat{y} + \phi_0^*.$$

It is noted that this estimation will converge to the true derivative \dot{y} and it is stable and free of noise. If we want to calculate the derivative directly from measured y_n , the uncertainty always exists and the derivative could be very noisy.

4 Model Disturbance

In [1], same structure of FDE is used to deal with model disturbance, which is

$$\dot{y} = -\alpha y + \sum_{i=0}^N c_i (\omega^*)^i + o \quad (33)$$

where $|o| \leq O$.

For systems where both output noise and model disturbance exist, i.e.

$$\begin{aligned} \dot{y} &= -\alpha y + \sum_{i=0}^N c_i (\omega^*)^i + o \\ y_n &= y + n \\ o &\leq O \\ n &\leq n_{max}, \end{aligned} \quad (34)$$

the modified FDE is of

$$\begin{aligned} \dot{\hat{y}} &= -\alpha\hat{y} + \phi_0^* - a_{max}^* \text{sat}\left(\frac{\tilde{y}_n}{n_{max}}\right) \\ \hat{\omega}_i &= -\tilde{y}_{n\epsilon} \phi_i^* \quad i = 1, \dots, N \\ \tilde{y}_{n\epsilon} &= \tilde{y}_n - n_{max} \text{sat}\left(\frac{\tilde{y}_n}{n_{max}}\right) \\ \tilde{y}_n &= \hat{y} - y_n \\ \phi^* &= A^{-1}C \\ a_{max} &= O, \end{aligned} \quad (35)$$

where $\phi^* = [\phi_0^*, \phi_1^*, \dots, \phi_N^*]^T$, A and C are defined as in (7) and (8), and $\text{sat}(\cdot)$ denotes the saturation function and

is given by $\text{sat}(x) = \text{sign}(x)$ if $|x| \geq 1$ and $\text{sat}(x) = x$ if $|x| < 1$.

Choosing Lyapunov candidate V as

$$V = \tilde{y}_{n\epsilon}^2/2 + \sum_{i=1}^N p_i(\tilde{\omega}_i) \quad (36)$$

where $p_i(\cdot)$ is defined as in (3), it follows that

$$\dot{V} = -\alpha\tilde{y}_{n\epsilon}\tilde{y} - \tilde{y}_{n\epsilon}\dot{n} + \tilde{y}_{n\epsilon} \left(-o - a_{max}^* \text{sat}\left(\frac{\tilde{y}_n}{n_{max}}\right) \right). \quad (37)$$

It can be checked easily that

$$\tilde{y}_{n\epsilon} \left(-o - a_{max}^* \text{sat}\left(\frac{\tilde{y}_n}{n_{max}}\right) \right) \leq 0 \quad (38)$$

since $|o| \leq a_{max}^*$. It follows from (37) and (38) that

$$\dot{V} \leq -\alpha\tilde{y}_{n\epsilon}\tilde{y} - \tilde{y}_{n\epsilon}\dot{n}. \quad (39)$$

Substituting (18) with (39) and using the same derivation as in Theorem 1, we have

$$\text{Prob}\left[\lim_{t \rightarrow \infty} \tilde{y}_{n\epsilon} = 0\right] = 1. \quad (40)$$

Therefore,

$$\lim_{t \rightarrow \infty} \tilde{y}_{n\epsilon} = 0$$

for problem formulation in (34) and modified FDE in (35).

We note that the difference of (35) from the FDE in (9) is the additional item $-a_{max}^* \text{sat}\left(\frac{\tilde{y}_n}{n_{max}}\right)$ which is used to balance the model disturbance.

5 Simulation Results

We consider a simple example

$$\dot{y} = -4y + u\omega^* + (u^2 - u)(\omega^*)^2$$

where $\omega^* = 1$ and input $u = \sin(0.2 * t)$. For the following two cases

$$\text{Case 1} \quad n(t) = U[0.5, 0.5]$$

$$\text{Case 2} \quad n(t) = 0.1 + 0.01U[-1, 1]$$

where $U[a, b]$ is uniformly distributed random variable in $[a, b]$, we run simulations for both PAE and FDE and compare the results. We note here that in Case 1, the mean value of noise is zero however in Case 2, it is a biased noise with mean at 0.1. In the simulation, we choose initial values of $[\hat{\omega}_1, \hat{\omega}_2]$ as $[\hat{\omega}_1, \hat{\omega}_2] = [0.9, 0.9]$. Figures 1 and 2 show simulation results for case 1 and 2 respectively. In each figure, for both PAE in (6) and FDE in (9), it plots the

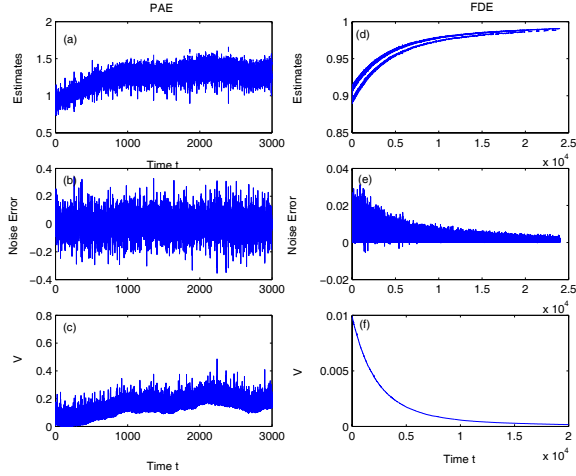


Figure 1: Comparison of PAE and FDE in *Case 1* - Unbiased Noise. Figures (a)-(c) show the trajectories of estimates $\hat{\omega}_1$ and $\hat{\omega}_2$, Noise filter error as of $\hat{n} - n$, and Lyapunov function V in PAE. Figures (d)-(f) show the trajectories of estimates $\hat{\omega}_1$ and $\hat{\omega}_2$, Noise filter error as of $\hat{n} - n$, and Lyapunov function V in FDE

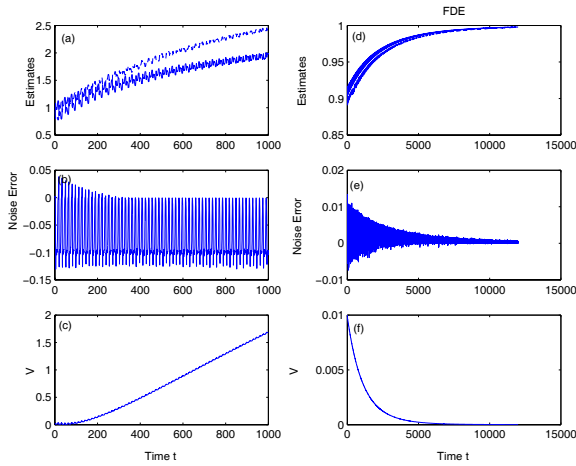


Figure 2: Comparison of PAE and FDE in *Case 2* - Biased Noise. Figures (a)-(c) show the trajectories of estimates $\hat{\omega}_1$ and $\hat{\omega}_2$, Noise filter error as of $\hat{n} - n$, and Lyapunov function V in PAE. Figures (d)-(f) show the trajectories of estimates $\hat{\omega}_1$ and $\hat{\omega}_2$, Noise filter error as of $\hat{n} - n$, and Lyapunov function V in FDE

trajectories of parameter estimates $\hat{\omega}_1$ and $\hat{\omega}_2$, noise error which is defined as

$$-\tilde{y} = y - \hat{y} = \hat{n} - n,$$

and Lyapunov function V which is defined as in (2) for PAE and (11) for FDE. The simulation results show clearly that for both cases, the FDE leads to asymptotic convergence

of $\hat{\omega}$, \tilde{y} , V and $\tilde{y}_{n\epsilon}$. For PAE, none of these variables converges.

The ability that FDE can filter out biased measurement or noise is extremely useful in practical applications. In the on-line measurement of dynamic systems, unlike the unbiased measurement uncertainty which is always unavoidable and restricted by measurement precision, measurement offset often means a quality problem and it is important that it can be detected on-line without perturbing the normal process of the plant.

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