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Robust L_1 Filtering with Pole Constraint in A Disk Via Parameter-Dependent Lyapunov Functions

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Abstract—This paper discusses the problem of robust L_1 filtering with pole constraint in a disk for linear continuous polytopic uncertain systems. The attention is focused on the design of full-order and reduced-order filters with suitable dynamic behavior via parameter-dependent Lyapunov functions such that the filtering error system remains robustly stable, and has a guaranteed L_1 performance constraint and pole constraint in a disk. The admissible filter with suitable dynamic behavior can be obtained from the solution of convex optimization problems.

I. Introduction

Filtering is a very important issue in systems diagnosis, surveillance and control. When peak bounded input signals are considered, the L_1 norm appears to be more adequate as performance criterion for the filter design, dealing with the minimization of peak-to-peak gain [1]. In addition, regional constraints on the closed-loop poles can be imposed in order to improve the filter dynamics by constraining the poles of system to lie inside some particular region of the left-half complex plane [2].

This paper is concerned with the problem of robust L_1 filtering with pole placement in a disk for linear continuous-time polytopic uncertain systems. We first introduce a new parameter-dependent stability result [3] to the multiobjective robust filtering, and derive new L_1 performance criteria and a new pole assignment condition, which has the potential to reduce the conservativeness of the quadratic framework. Then, the admissible full-order and reduced-order filters with suitable dynamic behavior are obtained from the solution of convex optimization problems.

II. Problem Formulation

Consider the linear continuous-time uncertain system

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bw(t)\\ y(t) &= Cx(t) + Dw(t)\\ z(t) &= Lx(t)\end{aligned} \tag{1}$$

where $x(t)\in \mathrm{R}^n$ is the state vector, $w(t)\in \mathrm{R}^q$ is magnitude-bounded noise signal vector, $y(t)\in \mathrm{R}^m$ is the measurement output vector, $z(t)\in \mathrm{R}^p$ is the signal to be estimated. The system matrices are assumed to unknown (uncertain), but belong to a known convex compact set of polytopic type

$$\Xi \doteq (A,B,C,D,L)\in \Re \tag{2}$$

$$\Re \cong \left\{(A,B,C,D,L)\left|(A,B,C,D,L)=\sum_{i=1}^{h}\tau_i(A_i,B_i,C_i,D_i,L_i);\ \tau_i\ge 0;\ \sum_{i=1}^{h}\tau_i=1\right.\right\}$$

We are interested in designing filters of order l described by

$$\begin{aligned}\dot{\hat{x}}(t) &= A_f\hat{x}(t) + B_f y(t)\\ \hat{z}(t) &= C_f\hat{x}(t)\end{aligned} \tag{3}$$

where $\hat{x}(t)\in \mathrm{R}^l$ (l=n for full-order filtering, and $1\le l<n$ for reduced-order filtering). The matrices $A_f\in R^{l\times l}$, $B_f\in R^{l\times m}$, $C_f\in R^{p\times l}$ are to be determined.

By defining the state vector $\bar{x}(t)=\left[x(t)^T,\hat{x}(t)^T\right]^T$, error output $\bar{z}(t)=z(t)-\hat{z}(t)$, the filtering error system can be described by

$$\begin{aligned}\dot{\bar{x}}(t) &= \bar{A}\bar{x}(t) + \bar{B}w(t)\\ \bar{z}(t) &= \bar{C}\bar{x}(t)\end{aligned} \tag{4}$$

where $\bar{A}=\begin{bmatrix}A & 0\\ B_fC & A_f\end{bmatrix}$, $\bar{B}=\begin{bmatrix}B\\ B_fD\end{bmatrix}$, $\bar{C}=\begin{bmatrix}L & -C_f\end{bmatrix}$.

define 1: The L_1 norm of the transfer function $T_{\bar{z}w}(s)$ is defined as

$$\left\|T_{\bar{z}w}(s)\right\|_{L_1} \doteq \sup_{0\ne w\in L_\infty}\frac{\|\bar{z}\|_{L_\infty}}{\|w\|_{L_\infty}} = \sup_{0\ne w\in L_\infty}\left(\sup_t\|\bar{z}(t)\|_2\Big/\sup_t\|w(t)\|_2\right)$$

define 2: Define the following disk region

$$C(\eta,\rho)\doteq\left\{\lambda\in \mathrm{C}^- : |\lambda+\eta|<\rho\right\}\quad (\eta,\ \rho\in\mathrm{R} \text{ and } \eta>\rho>0)$$

where C^- denotes the left-half complex plane, $C(\eta,\rho)$ denotes any disk centered in $-\eta$ with radius ρ in C^-.

Our purpose is to design filter in (3) for system (1), which guarantee the filtering error system (4) to be asymptotically stable and $\left\|T_{\bar{z}w}(s)\right\|_{L_1}<\gamma$ ($\gamma>0$), $\lambda(\bar{A})\in C(\eta,\rho)$.

III. Main Results

In this section, we will derive new robust L_1 performace criteria and disk pole constraint condition with parameter-dependent Lyapunov functions, and develop linear filter of form (3) assuring robust L_1 performance and pole placement condition for continuous system (1).

Theorem 1: Considering the system (1) with $\Xi\in\Re$ standing for uncertain system matrix and let $\gamma>0$ be given constants. Then, the following conditions can be obtained

1) The filtering error system (4) is asymptotically stable and $\left\|T_{\bar{z}w}(s)\right\|_{L_1}<\gamma$, $\lambda(\bar{A})\in C(\eta,\rho)$ if there exist scalars $\alpha\in\mathrm{R}^+$, $\mu\in\mathrm{R}$ and matrix $0<P=P^T\in\mathrm{R}^{(n+l)\times(n+l)}$ satisfying

$$\begin{bmatrix}\bar{A}_iP+P\bar{A}_i^T+\alpha P & \bar{B}_i\\ \bar{B}_i^T & -\mu I\end{bmatrix}<0,\quad \begin{bmatrix}\alpha P & 0 & P\bar{C}_i^T\\ 0 & (\gamma-\mu)I & 0\\ \bar{C}_iP & 0 & \gamma I\end{bmatrix}>0,$$

$$\begin{bmatrix} \bar{A}_i P_2 + P_2 \bar{A}_i^T & P_2 & \bar{A}_i P_2 \\ P_2 & -\dfrac{\eta}{\eta^2-\rho^2} P_2 & 0 \\ P_2 \bar{A}_i^T & 0 & -\eta P_2 \end{bmatrix} < 0 \tag{5}$$

2) The filtering error system (4) is asymptotically stable and $\|T_{\tilde{z}w}(s)\|_{L_1} < \gamma$, $\lambda(\bar{A}) \in C(\eta, \rho)$ if there exist scalars $\alpha \in \mathrm{R}^+$, $\mu \in \mathrm{R}$ and matrices $G \in \mathrm{R}^{(n+l)\times(n+l)}$, $0 < Y_{1i} = Y_{1i}^T \in \mathrm{R}^{(n+l)\times(n+l)}$, $0 < Y_{2i} = Y_{2i}^T \in \mathrm{R}^{(n+l)\times(n+l)}$ satisfying

$$\begin{bmatrix} Y_{1i} & -Y_{1i} & Y_{1i} & 0 \\ -Y_{1i} & 0 & 0 & 0 \\ Y_{1i} & 0 & -\alpha^{-1} Y_{1i} & 0 \\ 0 & 0 & 0 & -\mu I \end{bmatrix} + \mathrm{K_e}\left\{ \begin{bmatrix} I \\ -I \\ I \\ 0 \end{bmatrix} G^T \begin{bmatrix} \bar{A}_i - \frac{1}{2} I & I & 0 & \bar{B}_i \end{bmatrix} \right\} < 0 \tag{6}$$

$$\begin{bmatrix} \alpha Y_{1i} & 0 & \bar{C}_i^T \\ 0 & (\gamma-\mu) I & 0 \\ \bar{C}_i & 0 & \gamma I \end{bmatrix} > 0 \tag{7}$$

$$\begin{bmatrix} Y_{2i} & -Y_{2i} & Y_{2i} & \frac{1}{2} Y_{2i} \\ -Y_{2i} & 0 & 0 & -Y_{2i} \\ Y_{2i} & 0 & -\nu_1 Y_{2i} & 0 \\ \frac{1}{2} Y_{2i} & -Y_{2i} & 0 & -\nu_2 Y_{2i} \end{bmatrix} + \mathrm{K_e}\left\{ \begin{bmatrix} G^T(\bar{A}_i - \frac{1}{2} I) \\ G^T \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} I & -I & I & \frac{1}{2} I \end{bmatrix} \right\} < 0 \tag{8}$$

where $(\bar{A}_i, \bar{B}_i, \bar{C}_i)$ denotes the matrices of the system (4) at each of the vertices of the polytope, $\alpha \in (0, -2\max R_e(\lambda(A)))$, $\nu_1 \doteq \dfrac{\eta}{\eta^2 - \rho^2}$, $\nu_2 \doteq \eta$, $\forall i = 1, \cdots, h$. Note that $\mathrm{K_e}[A]$ is a shorthand notation for $A + A^T$.

The proof of Theorem 1 is based on Lemma 1 in [3] and the intrinsic characteristic of the convex polytopic uncertainty system [4], and then omitted here.

Remark 1: The minimization of the upper bound γ depends on the appropriate choice of α, that is $\|T_{\tilde{z}w}(s)\|_{L_1} < \gamma(\alpha)$, and a line search on α must be performed in order to obtain tighter bounds.

Remark 2: The condition 1) provides robust L_1 performance criteria and disk pole placement condition for the filtering error system (4) based on the notion of traditional quadratic stability. While the multiobjective performance criteria in the condition 2) is based on parameter-dependent Lyapunov functions and exhibit a kind of decoupling between the Lyapunov variables and system matrices. Furthermore, according to conventional approach, minimizing γ subject to the LMIs in (5) need to force a common Lyapunov variable *P*, while according to new approach, minimizing γ subject to (6)-(8) need to force a common auxiliary variable *G*. Therefore, the proposed new robust multiobjective performance conditions will lead to potentially less conservative results.

The following theorem provides sufficient conditions for the existence of robust L_1 filter with suitable dynamic behavior.

Theorem 2: Considering the system (1) with $\Xi \in \Re$ standing for uncertain system matrix and let $\gamma > 0$, $\alpha > 0$ and $\eta > \rho > 0$ be given constants. Then, the filtering error system (4) is asymptotically stable and $\|T_{\tilde{z}w}(s)\|_{L_1} < \gamma(\alpha)$, $\lambda(\bar{A}) \in C(\eta, \rho)$ if there exist matrices $0 < X_i = X_i^T \in \mathrm{R}^{n\times n}$, $0 < V_i = V_i^T \in \mathrm{R}^{l\times l}$, $Z_i \in \mathrm{R}^{n\times l}$, $0 < H_i = H_i^T \in \mathrm{R}^{n\times n}$, $0 < K_i = K_i^T \in \mathrm{R}^{l\times l}$, $Q_i \in \mathrm{R}^{n\times l}$, $R \in \mathrm{R}^{n\times n}$, $F \in \mathrm{R}^{l\times l}$, $U \in \mathrm{R}^{n\times l}$, $M \in \mathrm{R}^{l\times l}$, $N \in \mathrm{R}^{l\times m}$, $T \in \mathrm{R}^{p\times l}$ and scalar $\mu \in \mathrm{R}$ satisfying

$$\begin{bmatrix} \Psi_{2i} + \Psi_{5i} + \Psi_{5i}^T & * & * & * \\ -\Psi_{2i} + \Psi_{4i} - \Psi_{5i} & -\Psi_{4i} - \Psi_{4i}^T & * & * \\ \Psi_{2i} + \Psi_{5i} & \Psi_{4i}^T & -\alpha^{-1}\Psi_{2i} & * \\ \Psi_{6i} & -\Psi_{6i} & \Psi_{6i} & -\mu I \end{bmatrix} < 0 \tag{9}$$

$$\begin{bmatrix} \alpha \Psi_{2i} & * & * \\ 0 & (\gamma-\mu) I & * \\ \Psi_{1i} & 0 & \gamma I \end{bmatrix} > 0 \tag{10}$$

$$\begin{bmatrix} \Psi_{3i} + \Psi_{5i} + \Psi_{5i}^T & * & * & * \\ -\Psi_{3i} + \Psi_{4i}^T - \Psi_{5i}^T & -\Psi_{4i} - \Psi_{4i}^T & * & * \\ \Psi_{3i} + \Psi_{5i}^T & \Psi_{4i} & -\upsilon_1 \Psi_{3i} & * \\ \frac{1}{2}\Psi_{3i} + \frac{1}{2}\Psi_{5i}^T & -\Psi_{3i} + \frac{1}{2}\Psi_{4i} & 0 & -\upsilon_2 \Psi_{3i} \end{bmatrix} < 0 \tag{11}$$

If the above LMIs have unique a set of solution, an admissible filter (3) can be given by

$$A_f = F^{-1} M, \quad B_f = F^{-1} N, \quad C_f = T \tag{12}$$

where $\Delta_i = R^T A_i - \frac{1}{2} R^T + ENC_i$, $\Phi_i = U^T A_i - \frac{1}{2} U^T + NC_i$,

$$E \doteq \begin{bmatrix} I_{l\times l} & 0_{l\times(n-l)} \end{bmatrix}^T, \ \Psi_{1i} = \begin{bmatrix} L_i & -T \end{bmatrix}, \ \Psi_{2i} = \begin{bmatrix} X_i & * \\ Z_i^T & V_i \end{bmatrix},$$

$$\Psi_{3i} = \begin{bmatrix} H_i & * \\ Q_i^T & K_i \end{bmatrix}, \ \Psi_{4i} = \begin{bmatrix} R & U \\ F^T E^T & F^T \end{bmatrix}, \ \Psi_{5i} = \begin{bmatrix} \Delta_i & EM - \frac{1}{2} EF \\ \Phi_i & M - \frac{1}{2} F \end{bmatrix},$$

$$\Psi_{6i} = \begin{bmatrix} B_i^T R + D_i^T N^T E^T & B_i^T U + D_i^T N^T \end{bmatrix}, \ \forall i = 1, \cdots, h.$$

Corollary 1: The optimal L_1 guaranteed cost filter with pole placement inside a disk $C(\eta, \rho)$ for system (1) can be stated as follows.

$$\underset{X_i, V_i, Z_i, H_i, K_i, Q_i, R, F, M, N, T, U, \gamma, \mu, \alpha}{\text{minimize}} \ \gamma \quad \text{subject to} \quad (9)-(11)$$

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