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Energy-to-Peak Filtering for Markov Jump Systems

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Abstract

The energy-to-peak filtering problem of Markov jump systems is investigated in this paper. Based on the definition of generalized H_2 gain index extended from deterministic systems, the existence condition and design method for filter are presented to achieve a prespecified peak level of estimation error against all bounded-energy noises. Moreover, by minimizing the peak value, an optimal energy-to-peak guaranteed gain index is obtained. Applying matrix transformation and variable substitution, the main results are provided by LMI form.

1 Introduction

For deterministic linear systems, there are extensive literatures dealing with the estimation and filtering problems. In the case where the power spectral density of the noise input is known, the H_2 filtering design is employed, by using Riccati-based approaches [1] or linear matrix inequalities (LMI) approaches [2], and where the statistical information of the noise input is insufficient, the H_∞ filtering design is taken into account [3], [4]. Also some mixed H_2/H_∞ filtering problems have been developed [5]. Recent years, the so-called energy-to-peak gain filtering problem has been presented [6], [7], which uses the peak instead of the energy to bound the level of estimation error.

On the other hand, for many practical plants with randomicity, the linear Markov jump systems have been used to model its stochastic dynamic mechanism, such as component failures or repairs, sudden environmental disturbance, interconnections changing, and operating in different point of a nonlinear plant [8], [9]. Recently, some applications lead to a great interest in filtering problems of jump systems. But, up to now, almost all the filter design focus on H_∞ gain [10], [11].

This paper discusses the filter design for Markov jump systems with energy-to-peak gain performance. Based on the definition of generalized H_2 norm under state-space representation, the existence condition is obtained by using stochastic Lyapunov functional. Assume that the mode of jump systems is available, a kind of mode-dependent filter is proposed via LMI technique.

2 Problem Statement

Consider a class of Markov jump systems described as

$$\begin{cases} \dot{x}(t) = A(r_t)x(t) + B(r_t)w(t) \\ y(t) = C(r_t)x(t) + D(r_t)w(t) \end{cases} \tag{1}$$
$$x(t) = x_0, r_t = r_0, t = 0$$

where $x(t) \in R^n$ is the state vector, $y(t) \in R^m$ is measured output vector, $w(t) \in L_2^q[0,\infty]$ is the noise input vector. $A(r_t)$, $B(r_t)$, $C(r_t)$ and $D(r_t)$ are mode-dependent matrices with appropriate dimensions, and r_t represents a continuous-time discrete state Markov process with values in a finite set $\Omega = \{1,2,\cdots,N\}$. While $r_t = i$ the matrices $A(r_t)$, $B(r_t)$, $C(r_t)$ and $D(r_t)$ are represented by A_i, B_i, C_i, D_i. Let $\Pi = [\pi_{ij}], i, j \in \Omega$ denote the transition rate matrix, where π_{ij} is the transition rate from mode i to mode j satisfying $\pi_{ij} \geq 0$ for $i \neq j$ with $\sum_{j=1, j\neq i}^{N} \pi_{ij} = -\pi_{ii}$.

The jump system (1) with $w \equiv 0$, is said to be stochastically stable, if for all initial state x_0 and mode r_0, following relation holds

$$\lim_{T\to\infty} \mathrm{E}\left\{\int_0^T \|x(t, x_0, r_0)\|^2 \mid x_0, r_0\right\} < \infty . \tag{2}$$

Introducing linear time-variant mode-dependent filter as follows

$$\dot{x}_F(t) = F_A(r_t)x_F(t) + F_B(r_t)y(t) . \tag{3}$$

Assume that the mode r_t is available at time t and the order of the filter is equal to the order of the systems (1), i.e. $x_F(t) \in R^n$. $F_A(r_t)$, $F_B(r_t)$ are filtering gain matrices, and also are represented by F_{Ai}, F_{Bi} when $r_t = i$. The state estimation error vector is given by $e(t) = x(t) - x_F(t)$, and error dynamical equation follows

$$\begin{cases} \dot{e}(t) = F_{Ai}e(t) + (A_i - F_{Ai} - F_{Bi}C_i)x(t) + (B_i - F_{Bi}D_i)w(t) \\ z(t) = L_i e(t) \end{cases} \tag{4}$$

where L_i is the selected gain matrix of error state outputs, generally, let $L_i = L$ for $i \in \Omega$. Defining augmented state $\bar{x}(t) = \begin{bmatrix} x^T(t) & e^T(t) \end{bmatrix}^T$, following filtering error dynamics is obtained by mixing (1) and (4),

$$\begin{cases} \dot{\bar{x}}(t) = \bar{A}_i \bar{x}(t) + \bar{D}_i w(t) \\ z(t) = \bar{C}_i \bar{x}(t) \end{cases} \quad (5)$$

where

$$\bar{A}_i = \begin{bmatrix} A_i & 0 \\ A_i - F_{Ai} - F_{Bi}C_i & F_{Ai} \end{bmatrix},$$

$$\bar{D}_i = \begin{bmatrix} B_i \\ B_i - F_{Bi}D_i \end{bmatrix}, \ \bar{C}_i = \begin{bmatrix} 0 & L_i \end{bmatrix}. \quad (6)$$

For error dynamics (5), the energy-to-peak gain index is defined by

$$J = \sup\{\mathrm{E}\{\|z(T)\|\} : \int_0^T \|w(t)\|^2 dt \le 1, \ x_0 = 0, \ T \ge 0\}, \quad (7)$$

which is measured by the peak value of the error outputs $z(t)$, in response to arbitrary but bounded-energy exogenous inputs $w(t)$.

Definition 1 For jump systems (1), if filtering error dynamics (5) remains stochastically stable and ensures a prespecified energy-to-peak gain level $J < \gamma$, the filter (3) is said to be an energy-to-peak filter.

3 Main Results

This section aims to design an admissible internally stabilizing filter such that the disturbance attenuation of augmented jump system (5), from L_2 to L_∞, is less than a specified level. In fact, this gives an answer to the problem that how to minimize the peak amplitude of the filtering error over any unit-energy input signals.

Theorem 1 The filtering error dynamics (5) is stochastically stable and satisfies $J < \gamma$, if there exists a set of positive-definite symmetric matrices P_i, such that $P_i - \gamma^{-2}\bar{C}_i^T \bar{C}_i > 0$ and $\bar{A}_i^T P_i + P_i \bar{A}_i + \sum_{j=1}^{N} \pi_{ij} P_j + P_i \bar{D}_i \bar{D}_i^T P_i < 0$ for all $i \in \Omega$.

Based on above analysis result, the next theorem gives a parameterized realization of admissible energy-to-peak filter.

Theorem 2 For jump systems (1), there exists an energy-to-peak filter (3) with the mode-dependent gain matrices given by

$$F_{Ai} = P_{2i}^{-1} Q_{1i}, \ \ F_{Bi} = P_{2i}^{-1} Q_{2i}, \quad (8)$$

if following coupled LMIs (9) and (10) are feasible in the variable set of positive-definite symmetric matrices P_{1i}, P_{2i} and matrices Q_{1i}, Q_{2i} for all $i \in \Omega$.

$$\begin{bmatrix} S_{11} & S_{21}^T & S_{31}^T \\ S_{21} & S_{22} & S_{32}^T \\ S_{31} & S_{32} & S_{33} \end{bmatrix} < 0 \quad (9)$$

$$\begin{bmatrix} P_{1i} & 0 & 0 \\ 0 & P_{2i} & L_i^T \\ 0 & L_i & \gamma^2 I \end{bmatrix} > 0 \quad (10)$$

where

$$S_{11} = A_i^T P_{1i} + P_{1i} A_i + \sum_{j=1}^{N} \pi_{ij} P_{1j}, \ S_{22} = Q_{1i}^T + Q_{1i} + \sum_{j=1}^{N} \pi_{ij} P_{2j},$$

$$S_{21} = P_{2i} A_i - Q_{1i} - Q_{2i} C_i, \ \ S_{33} = -I,$$

$$S_{31} = B_i^T P_{1i}, \ \ S_{32} = B_i^T P_{2i} - D_i^T Q_{2i}^T.$$

Let $\rho = \gamma^2$, the energy-to-peak filter (3) with gain matrices given by (8), which minimize the peak amplitude against input noises, can be obtained by solving following optimization problem

$$\min_{P_{1i}, P_{2i}, Q_{1i}, Q_{2i}, \rho} \rho \quad \text{s.t. (9),(10).}$$

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