

# Motion and Multimode Vibration Control of A Flexible Transport System

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**Abstract**— This paper deals with transversal motion and vibration control for a flexible tower-like transport system that uses an elevator to lift payloads vertically. Applying a lumped modeling method presented by Seto and a LQI-based control system, the controller to achieve good performance is designed. The purpose of this research is to control motion and vibration under moving flexible transport systems. It is demonstrated that this transport system moves without vibration from one side to another at a high speed, and that the multimode-vibration of the flexible structure is controlled without spillover caused by neglected higher modes.

## I. INTRODUCTION

IN recent years, factories and warehouses around the world have tended towards automation to reduce the cost of employees and to cut down production time. Thus, the existence of a transport system that carries goods, instead of them being carried by employees, is indispensable for factory and warehouse automation. Higher speeds and higher positioning accuracy are increasingly demanded for transport system.[1],[2]. As is well known, strong demands for energy savings have also been made in recent years [3]. To meet these demands, transport systems are constructed with a lightweight structure, and are relatively flexible [4]. As a result, the transport system suffers a decline in its natural frequency because of its flexibility. High acceleration during operation induces vibration, that decreases positioning accuracy. The influence of the vibration occurring in positioning is not negligible for reducing setting time [5].

The purpose of this research is to control motion and multi-modes of vibration of flexible transport systems for moving equipment and goods from one place to another. Then, a method called “reduced order physical modeling method” is used for making a lumped mass model of the flexible transport systems [6],[7]. This method has the characteristic that the control model is produced in physical

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coordinates according to the information obtained from the mode shape of the control object.

The transport system is composed of a horizontally moving part driven by an AC servo motor, a vertically moving part with a movable table that can move up and down and a controller with sensors. At the stage of designing the controller, it is necessary to consider the uncertainty of the movable rack (elevator) caused by the changing of the weight of the carrier. Therefore, as a control technique to support the parameter change, LQI control theory, which uniformly designs positioning and vibration control, is used. It is demonstrated that this transport system moves without vibration from one side to another at a high speed and in addition that the 1st and 2nd vibration modes of the flexible structure are considered

## II. COMPOSED TRANSPORTATION SYSTEM

### A. Basic construction of transport system

A transport system considered in this paper is shown in Fig.1. The transport system is composed of a transport table driven by a servomotor through a feed drive mechanism, a flexible tower structure equipped with an elevator on the transport table and a control system with sensor and amplifier. The flexible tower structure is made of two flat plate structures arranged in parallel and is 1000 mm in height and 180 mm in width as shown in Fig.2. In order to move a carrier on the elevator from one position to another position, two driving systems are installed in the horizontal and vertical directions.

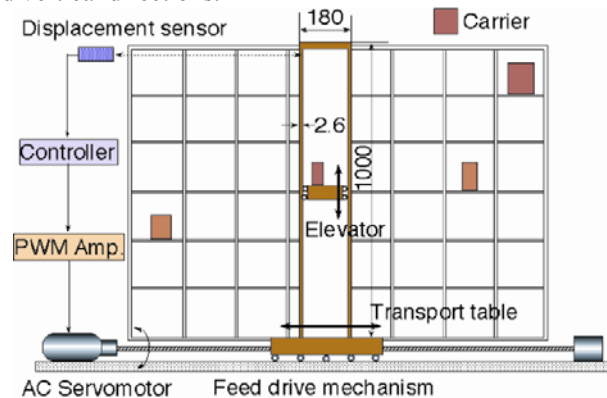


Fig. 1 Basic construction of transport system

Since the horizontal driving system is mainly important for positioning control with high speed and accuracy, the motion and vibration control of the flexible tower structure is studied in this paper.

### B. Control purpose

The purposes of control in this research are the simultaneous execution of motion and vibration control of the flexible tower structure and the execution of robust vibration control for changing variable parameters caused by moving of the elevator. Therefore, the purposes are to control the position in the vertical direction of the tower structure and the vibration caused by positioning control of the structure. Important vibration modes of the control object are the first and second modes. The control input in positioning and vibration control is only one of the inputs forwarded to a PWM amp from the controller.

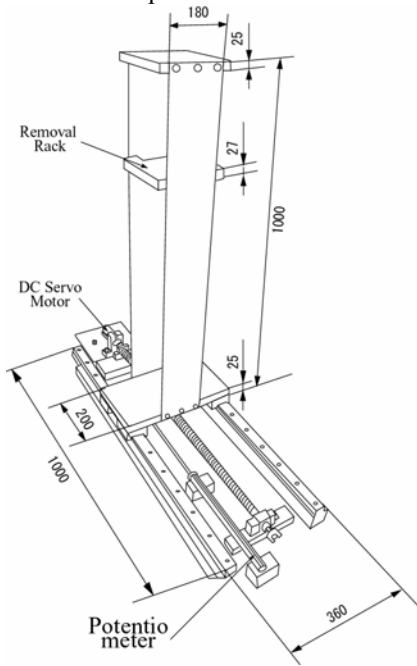


Fig.2 Construction of experimental setup

The flexible tower structure is composed of two flat plates of 1.5 mm and 2.6 mm in thickness. When two flat plates with the same shape and stiffness are used, the vibration mode called “drumming mode” appears [8]. The drumming mode occurs when two flat plates symmetrically vibrate at the opposite phase on the same frequency. Thus it is impossible to control this vibration mode by using only one control system like this research. So this tower structure has two flat plates with different thickness in order to avoid the drumming mode.

The first step in making a control model is to make a “reduced order physical model”, which is a control model for vibration control of the structure. The second step is modeling of the range to base part of the structure of the PWM amp. The third step is to interlace the two models to

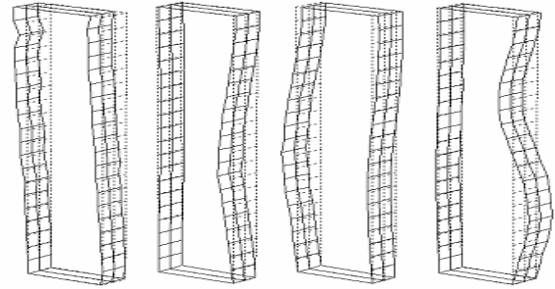
make a control model for motion and vibration control of a structure.

## III. MAKING A CONTROL MODEL

### A. Vibration characteristic

Fig.3 shows the vibration mode shapes from 1<sup>st</sup> to 4<sup>th</sup> analyzed by use of a ME Scoop. The natural frequencies of the vibration modes are 2.41 Hz, 9.53 Hz, 16.40 Hz, and 25.23 Hz., respectively.

In this paper, the vibration control of the flexible tower structure is in respect to the 1st and 2nd modes, because the higher modes only slightly influence for the setting time.



1st mode 2nd mode 3rd mode 4th mode  
3.41[Hz] 9.53[Hz] 16.40[Hz] 25.23[Hz]

Fig.3 Vibration mode shapes

### B. Brief introduction of Seto's procedure [6]

This procedure is presented using two-degrees-of-freedom (2DOF) model as an example [9]. Generally, the mass matrix  $\mathbf{M}$  and the stiffness matrix  $\mathbf{K}$  on the physical domain are shown in the following equations.

$$\mathbf{M} = (\Phi\Phi^T)^{-1} \quad (1)$$

$$\mathbf{K} = (\Phi^T)^{-1} \Omega^2 \Phi^{-1} \quad (2)$$

Here,  $\Phi$  is a normalized modal matrix, and  $\Omega$  is a diagonal matrix of the natural frequencies of each mode. Because a lumped mass model is not obtained at this stage, the exact value of  $\Phi$  is an unknown quantity. Therefore, a temporary normalized modal matrix is constructed from mode shapes.

$$\Phi = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \quad (3)$$

Because these elements are given from the mode shapes of the distributed parameter system, the temporary normalized modal matrix constructed from the mode shapes is not guaranteed to satisfy the equation of the mass matrix, where both sides should be diagonal matrices.

$$\Phi\Phi^T = \begin{bmatrix} \phi_{11}^2 + \phi_{12}^2 & \phi_{11}\phi_{21} + \phi_{12}\phi_{22} \\ \phi_{11}\phi_{21} + \phi_{12}\phi_{22} & \phi_{21}^2 + \phi_{22}^2 \end{bmatrix} \neq \begin{bmatrix} \frac{1}{M_1} & 0 \\ 0 & \frac{1}{M_2} \end{bmatrix} \quad (4)$$

By using this modification vector and iterating the nondiagonal elements in Eq.(4), tends toward zero. So, the expected normalized modal matrix  $\Phi$  is obtained. The mass matrix  $\mathbf{M}$  and Stiffness matrix  $\mathbf{K}$  in the physical domain are then determined.

### C. Modeling for motion and vibration

The control model is composed of two parts, one is of a rigid body to express translation motion of a flexible structure and other is of a model of a mass, a spring and a damper to express vibration of a flexible structure. Thus, at first the lumped mass model for vibration control of a control object structure is located at the node of the 3rd mode using the reduced order physical modeling method. When the modeling point of the lumped mass is set at the node of 3<sup>rd</sup> mode, a spillover caused by the neglected higher modes is prevented in terms of uncontrollable and unobservable points [10]-[12].

The important points for making a vibration control model with 2DOF are the two points of the node of the 3<sup>rd</sup> vibration mode shown in Fig.4. These two points are called as mass1 and mass2, at the upper side and lower side, respectively.

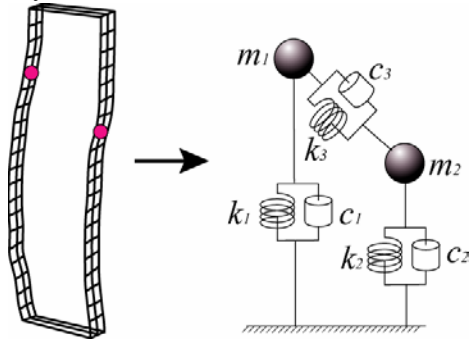


Fig.4 Modeling points

The temporary normalized modal matrix obtained from the vibration mode shapes shown in Fig.3 is modified through our proposed method and the normalized modal matrix is obtained as follows,

$$\Phi = \begin{bmatrix} 0.5926 & -0.2182 \\ 0.4067 & 1.1044 \end{bmatrix} \quad (5)$$

The parameter of the mass and spring elements of a control model with 2DOF is worked out next.

$$\mathbf{M} = (\Phi\Phi^T)^{-1} = \begin{bmatrix} 2.5074 & 0 \\ 0 & 0.7219 \end{bmatrix} \quad (6)$$

$$\mathbf{K} = \left\{ \Phi(\Omega^2)^{-1}\Phi^T \right\}^{-1} = \begin{bmatrix} 0.1665 & -0.1556 \\ -0.1556 & 0.2439 \end{bmatrix} \times 10^4 \quad (7)$$

Here  $\Omega$  is the matrix that is the natural frequency. Therefore the physical parameters of the lumped mass model of the control object corresponding to the 1st and 2nd modes of structure were deferring as:

$$m_1 = 2.5074[\text{kg}], \quad m_2 = 0.7219[\text{kg}]$$

$$k_1 = 1.5557 \times 10^3 [\text{N/m}], \quad k_2 = 0.88307 \times 10^3 [\text{N/m}],$$

$$k_3 = 0.1091 \times 10^3 [\text{N/m}]$$

$$c_1 = 1.34 [\text{Ns/m}], \quad c_2 = 0.0 [\text{Ns/m}]$$

$$c_3 = 2.32 [\text{Ns/m}],$$

The damping parameter is obtained by comparison of the sympathetic vibration peak of this model and the actual system.

A comparison of the model and the actual system is as in Fig.5-6. The figure shows that the two responses are the same, and that the control model enables us to exactly express the 1st mode and 2nd modes of the structure. Figs.5 and 6 show the comparison of calculated and measured frequency responses observed at mass1 and mass2, respectively. Then an excitation point is at mass1. It is clear that the model system expresses well the dynamic behavior of the actual system.

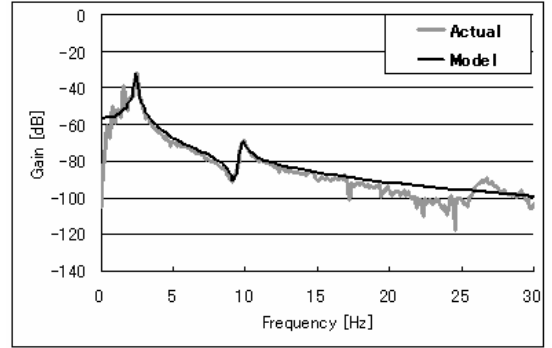


Fig.5 Comparison of model and actual systems by frequency responses observed at mass1

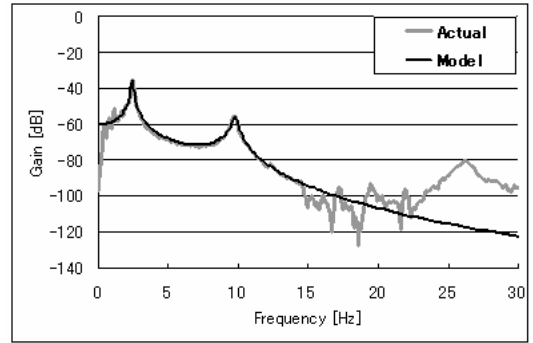


Fig.6 Comparison of model and actual systems by frequency responses observed at mass2

The model of the motion is made by an approximation of transfer function. Fig.7 is the comparison of the model of the motion and actual motion of the experimental device. We used the following approximate expression of the transfer function.

$$G = \frac{k\omega_n^2}{s(s^2 + 2\zeta\omega_n + \omega_n^2)} \quad (8)$$

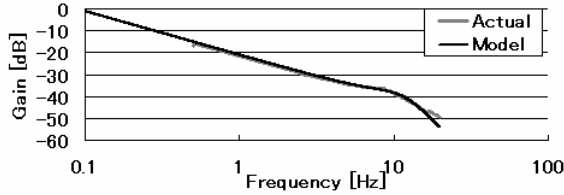


Fig.7 Comparison of model and actual systems

Therefore the motion parameter of the control object was that the 1st and 2nd modes of the structure were worked out using.

$$\zeta = 0.40$$

$$\omega_n = 2\pi \times 12 \text{ [rad / s]}$$

$$k = 0.55$$

#### IV. CONTROL SYSTEM DESIGN

##### A. State equation of the control model

The state equation of the control model is expressed with state variables consisting of the velocity and displacement of each mass, and the acceleration, velocity and displacement of the table. Note that.  $x_1$  is the displacement of mass1,  $x_2$  is the displacement of mass2 and  $x_3$  is the displacement of table, the state vector and state equation are expressed as,

$$\mathbf{X} = \{\dot{x}_1 \quad \dot{x}_2 \quad x_1 \quad x_2 \quad \ddot{x}_3 \quad \dot{x}_3 \quad x_3\}^T \quad (9)$$

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{B}u \quad (10)$$

$$y = \mathbf{C}\mathbf{X}$$

where matrices A, B and C are shown as.

$$\mathbf{A} = \begin{bmatrix} \frac{c_1+c_3}{m_1} & \frac{c_1}{m_1} & \frac{k_1+k_3}{m_1} & \frac{k_1}{m_1} & 0 & \frac{c_3}{m_1} & \frac{k_3}{m_1} \\ \frac{c_1}{m_2} & \frac{c_2+c_2}{m_2} & \frac{k_1}{m_2} & \frac{k_1+k_2}{m_2} & 0 & \frac{c_2}{m_2} & \frac{k_2}{m_2} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2\zeta\omega_n & -\omega_n^2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{B} = [0 \quad 0 \quad 0 \quad 0 \quad k_p k \omega_n^2 \quad 0 \quad 0]^T$$

$$\mathbf{C} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1]$$

Note that each parameter has been presented above.

##### B. LQI control

In this research, the LQI control theory that enables LQ control theory to cope with servo problems is applied. Figure 8 shows a block diagram of the LQI control system.

The feedback gains vectors  $K$  and  $K_I$  are simultaneously obtained by applying LQ control theory to the augmented system.

According to Fig.8, the state equation and output equation are shown as.

$$\dot{\mathbf{X}}_{cc} = \mathbf{A}_{cc}\mathbf{X}_{cc} + \mathbf{B}_{cc}\mathbf{U}_{cc} \quad (11) \quad (11)$$

$$\mathbf{Y}_{cc} = \mathbf{C}_{cc}\mathbf{X}_{cc} \quad (12) \quad (12)$$

Here,

$$\mathbf{X}_{cc} = \begin{Bmatrix} \mathbf{X} \\ \mathbf{X}_e \end{Bmatrix}, \mathbf{A}_{cc} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ -\mathbf{C} & \mathbf{0} \end{bmatrix}, \mathbf{B}_{cc} = [\mathbf{B} \quad \mathbf{0}]^T$$

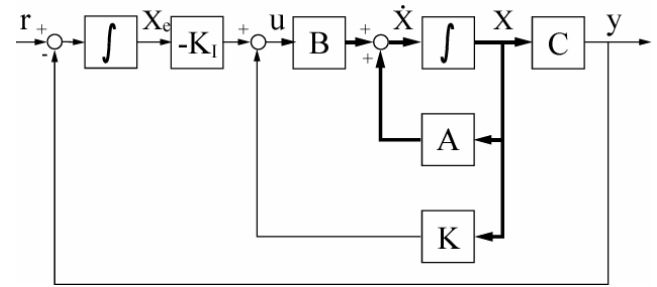


Fig.8 Block diagram of LQI control

#### V. SIMULATION

Simulation results are calculated using MATLAB. Figure 9 shows the time responses observed at mass1, mass2 under LQI control, where the upper part is uncontrolled and the lower one is controlled. The step input target of the table is 0.05 m.

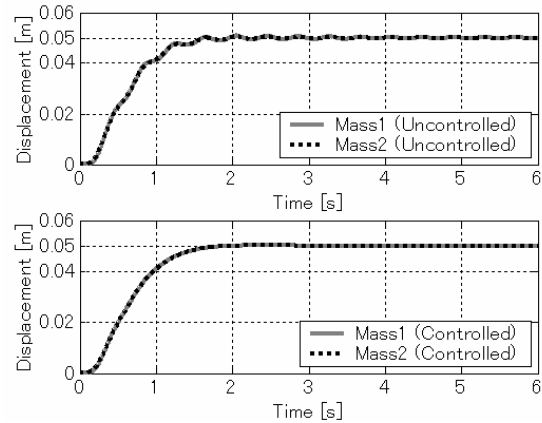


Fig.9 Simulation results of time response by LQI control

Both masses 1 and 2, which express the flexible structure, don't vibrate using LQI control and attain the target displacement. Fig.9 shows that controlled system is more effective than the uncontrolled one. These results show that this research can design a suitable control system.

## VI. EXPERIMENT

In order to demonstrate the effectiveness of the modeling and control approaches, experimental studies were done for two cases. One is a nominal case fixed to the lower part of the tower structure. The other is the case of a time-varying parameter by moving the elevator with carriers vertically.

### A. Experimental setup

Figure 10 shows the schematic view of the completed experimental setup and control system. In this research, the control object is to move the carrier from one point to another point quickly without vibration. The table with the flexible tower structure is moved by a DC servomotor driven by a PWM amplifier through gears and the feed screw. Since this control system uses a state feedback control, it is necessary to measure the rotational angle of the servomotor, the displacement and velocity of the table and to detect the vibration of the structure at the mass points. The equipment for the measurement of the displacement of the table is a straight-line potentiometer. The velocity value of the table is calculated from time differentials of the displacement. For measuring the rotational angle of the servomotor, a rotary encoder is used. The equipment for measuring the displacement of the mass point is a laser sensor. The velocity values of the mass point are also obtained by time differential calculations of the laser sensor. Since the 2DOF model is used for controlling the 1<sup>st</sup> to 2<sup>nd</sup> vibration modes of the flexible structure, this research needs to observe two mass points using two laser sensors. However in reality the displacement of mass2 is obtained using a “state observer”. A personal computer as the controller is used; so the operating system of this controller is thus MS-DOS.

### B. Experiment based on LQI control theory (Case 1)

First the control experiment was carried out by the case1

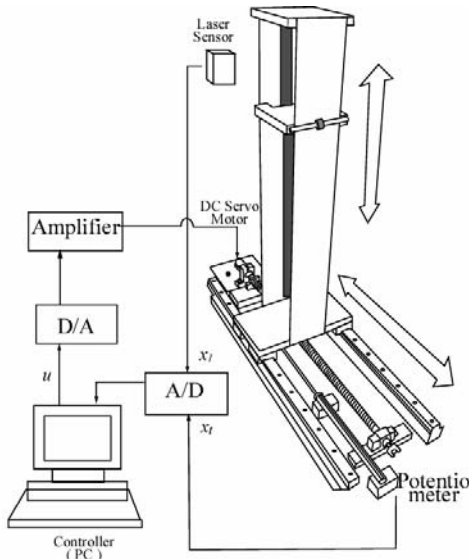


Fig.10 Experimental setup constructed in this research

with the elevator (removable rack) fixed at lower part of the table. Positioning accuracy and the time response in this experiment is undertaken using LQI control theory.

Figure 11 shows that the step responses of the flexible structure. The upper figures measured at mass1 are compared with and without control. It shows that residual vibration of the two mass points from the positioning control is well controlled. The lower figures measured at mass2 is also well controlled.

Figure 12 shows the frequency responses observed at the mass1. It shows that the resonance peaks of the 1st and the 2nd modes are well suppressed by both the positioning control and the vibration control. As a result, the control system effectively works for controlling the 1st and 2<sup>nd</sup> modes.

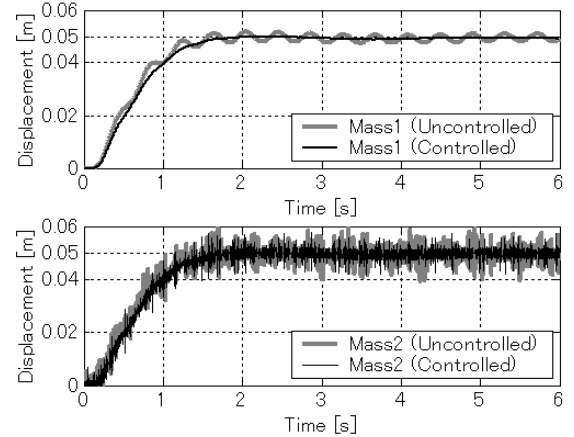


Fig.11 Experimental results of time responses

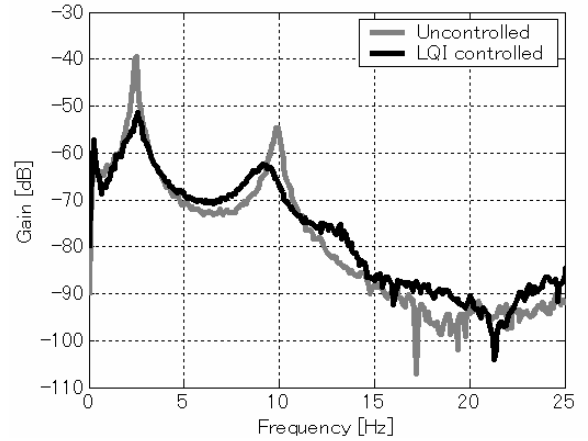


Fig.12 Experimental results of frequency responses

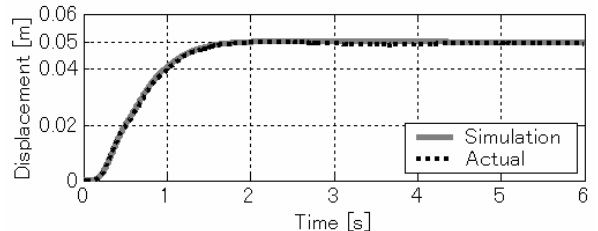


Fig.13 Comparison of simulation and experimental results

vibration modes of the actual experiment setup. These responses show that the influence of the 3rd mode doesn't appear if we put the two virtual masses at the nodes of the 3rd mode.

A comparison of the simulation and experiment results measured at mass 1 is shown in Fig.13. It shows that the experiment results of the time responses of mass 1 agree well with the simulation results. This result shows that the control model can accurately represent the actual experiment setup.

### C. Experiment based on LQI control theory (Case 2)

An experiment with the time-varying parameter is considered, and thus the robust stability in the control system is demonstrated for case 1. In case 2, the removable rack is moving in a perpendicular direction. We can expect that these actions are carried out to reduce time in an actual transport system. The elevator moves from the lower part of the structure to the upper part at a speed of 0.17 m/s, using a constant voltage of 15V. Positioning control horizontally plus vibration control of the structure as in case 1 is carried out during the movement of the elevator.

Figure 12 shows this experiment result, and also shows that the time response of the mass point that expresses vibration of the structure by LQI control. Here the target displacement is 0.15 m because the elevator needs a long time to move.

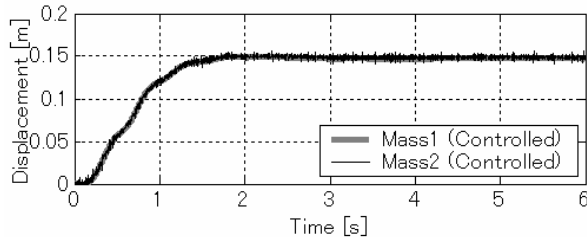


Fig.14 Experimental results of time responses

Fig.14 shows that the structure can be accurately moved to a target location, and the vibration from that positioning likewise controlled. Thus, LQI control theory consists of feedback that can realize robust stability. It is also considered that the “reduced order physical modeling method” that this research proposed combines well with LQI control theory. From the results above, we demonstrated robust stability of our control system even considering a time-varying parameter.

## VII. CONCLUSIONS

The goal of this research was to control motion and multi-modes of vibration of flexible transport systems for moving the elevator with carriers from one place to another. This was a robust control problem for a typical time variant system. The problem was solved by combining

the effective modeling method with LQI control. Thus, the following knowledge has been obtained.

(1) The frequency response of the control model was in agreement with experiment results using our modeling method.

(2). Experiment results obtained using LQI control theory shows that remnant vibrations didn't occur and that positioning enabled slaved tracking. The experiment results showed a good control effect.

(3). We carried out a robust experiment in which physical parameters kept changing, and could demonstrate that robust control was possible according to the use of LQI control theory and use of our “reduced order physical modeling method”.

In this research we consider as a motion control object of only a single spool translation direction, but this modeling, the “reduced order physical modeling method”, can be applied to a more complex motion utilizing the same approach. Higher vibration modes can be considered in the same manner.

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