Stability and Control Synthesis of Switched Systems Subject to Actuator Saturation

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Abstract— : **This paper presents sufficient conditions for the stabilization of switched discrete-time linear systems subject to actuator saturations. The results obtained are formulated in terms of LMIs. These conditions are obtained by using a state feedback control law. A numerical example is used to illustrate the technique by using a linear optimization problem subject to LMI constraints.**

Key-words: Switched systems, Actuator saturations, invariant sets, Lyapunov functions, LMIs.

I. INTRODUCTION

Switched systems are a class of hybrid systems encountered in many systems practical situations which involve switching between several subsystems dependening on various factors. Generally, a switching system consists of a family of continuous-time subsystems and a rule that supervises the switching between them. This class of systems have numerous applications in the control of mechanical systems, the automotive industry, aircraft and air traffic control [17], switching power converters and many other fields. Two main problems are widely studied in the literature: The first one deals with the obtention of testable conditions that guarantee the asymptotic stability of a switched system under arbitrary switching rules, while the second is to determine a switching sequence that renders the switched system asymptotically stable (see [12] and the reference therein).

A main problem which is always inherent to all dynamical systems is the presence of actuators saturations. Even for linear systems this problem has been an active area of research for many years. Two main approaches have been developed in the literature: The first is the so-called positive invariance approach which is based on the design of controllers which work inside a region of linear behavior where saturations do not occur (see [1], [2], [5] and the references therein). The second approach however allows saturations to take effect while guaranteeing asymptotic stability (see [13]- [15] and the references therein). The first approach was also applied to a class of hybrid systems involving jumping parameters [6]. The main challenge in these two approaches is to obtain a large enough domain of initial states which ensures asymptotic stability for the system despite the presence of saturations [3], [4], [11], [13].

The objective of this paper is to study a switched system subject to actuator saturations by extending the results obtained for unsaturated switched systems by [9], [8], [10] and [16] . This extension relies on the use of a lemma rewriting the saturation function as a certain convex combination ([13]-[15]).

With this technique, sufficient conditions of asymptotic stability are obtained for switched systems subject to actuator saturations. Furthermore, these conditions are presented in the form of an LMI.

The paper is organized as follows: The second section deals with the problem formulation while the third section presents some preliminary results. The main results of this work together with a numerical example are presented in the fourth and fifth section respectively.

II. PROBLEM FORMULATION

In this section, we give a more precise problem statement for the class of systems under consideration, namely, discrete-time switched linear systems with input saturation and state feedback. An equivalent description of such systems, based on the indicator function is also used in this work ([9], [8]).

Thus we consider systems described by:

$$x_{k+1} \quad = \quad A_\alpha x_k + B_\alpha sat(u_k) \qquad (1)$$

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where $x_k \in \mathbb{R}^n$ is the state, $u_k \in \mathbb{R}^m$ is the control, $sat(.)$ is the standard saturation function and α a switching rule which takes its values in the finite set $\mathcal{I} := \{1, \ldots, N\}$. The saturation function is assumed here to be normalized, ***i. e.***, $(|sat(u_k)| = min\{1, |u_k|\})$.

It is assumed that:

(AS):The switching rule α is not known ***a priori*** but its value is available at each sampling period, that is each k. As reported by the cited references, assumption (H) corresponds to practical implementations where the switched system is supervised by a discrete-event system or operator allowing for the value of α to be known at only each sampling period in real time.

In this work, we are interested by the synthesis of a stabilizing controller for this class of hybrid systems subject to actuator saturation. We use a state feedback control law:

$$u_k = F_\alpha x_k, \tag{2}$$

and write the closed-loop system as

$$x_{k+1} = A_\alpha x_k + B_\alpha sat(F_\alpha x_k). \tag{3}$$

Upon introducing the indicator function ([9]):

$$\xi(k) = [\xi_1(k), \ldots, \xi_N(k)]^T \tag{4}$$

where $\xi_i(k) = 1$ if the switched system is in mode i and $\xi_i(k) = 0$ if it is in a different mode, one can write the closed-loop system (3) as follows:

$$x_{k+1} = \sum_{i=1}^{N} \xi_i(k)[A_i x_k + B_i sat(F_i x_k)]. \tag{5}$$

III. PRELIMINARY RESULTS

In this section, we recall two results on which our work is based, namely, a condition of stability of a linear system subject to actuator saturation of [13] and a condition of stability of switched system without saturation of [9].

Let α be fixed. Then System (3) becomes a linear time-invariant system with input saturation given by:

$$x_{k+1} = Ax_k + Bsat(Fx_k) \tag{6}$$

Define the following subsets of $\mathbb{R}^n$:

$$\varepsilon(P, \rho) = \{x \in \mathbb{R}^n / x^T P x \le \rho, \rho > 0\}, \tag{7}$$

$$\mathcal{L}(F) = \{x \in \mathbb{R}^n / |f_l x| \le 1,\, l = 1, \ldots, m\}, \tag{8}$$

with P a positive definite matrix and f_l the lth row of the matrix $F \in \mathbb{R}^{m \times n}$. Thus $\varepsilon(P, \rho)$ is an ellipsoid while $\mathcal{L}(F)$ is a polyhedral consisting of states for which the saturation does not occur.

Theorem 3.1: [13] Given an ellipsoid $\varepsilon(P, \rho)$, if there exists a matrix $H \in \mathbb{R}^{m \times n}$ such that:

$$[A + B(D_s F + D_s^- H)]^T P [A + B(D_s F + D_s^- H)] - P < 0, \forall s \in [1, 2^m], \tag{9}$$

and $\varepsilon(P, \rho) \subset \mathcal{L}(H)$, then $\varepsilon(P, \rho)$ is a contractively invariant set for the closed-loop system with saturation (6).

Here, D_s is an m by m diagonal matrix with elements either 1 or 0 and $D_s^- = \mathbb{I}_m - D_s$. There are 2^m possible such matrices.

The matrices D_s and D_s^- were introduced by [13] to model the saturation function as a linear function by using the following lemma:

Lemma 3.1: [13] For all $u \in \mathbb{R}^m$ and $v \in \mathbb{R}^m$ such that $|v_l| < 1, l \in [1, m]$

$$sat(u) \in co\{D_s u + D_s^- v, s \in [1, \eta]\} \;;\quad \eta = 2^m \tag{10}$$

where co denotes the convex hull.

Consequently, there exist $\delta_1 \ge 0, \ldots, \delta_\eta \ge 0$ with $\sum_{s=1}^{\eta} \delta_s = 1$ such that,

$$sat(u) = \sum_{s=1}^{\eta} \delta_s [D_s u + D_s^- v] \tag{11}$$

We now recall a useful stability result for switched systems with no input saturations presented by many authors (see [9], [8],[10] and [16]).

Consider the following switched system:

$$x_{k+1} = \sum_{i=1}^{N} \xi_i(k) A_i x_k \tag{12}$$

Theorem 3.2: The closed-loop switched system (12) is asymptotically stable at the origin if there exist N symmetric matrices $P_1, \ldots, P_N$ satisfying,

$$\begin{bmatrix} P_i & A_i^T P_j \\ * & P_j \end{bmatrix} > 0 \;\;,\;\; \forall (i, j) \in \mathcal{I} \times \mathcal{I} \tag{13}$$

where $*$ denotes the transpose of the off diagonal element of the LMI. A corresponding Lyapunov function for the system is then given by:

$$V(k, x) = x^T (\sum_{i=1}^{N} \xi_i(k) P_i) x \tag{14}$$

Subsequently, we will need the following equivalent LMI representation of (13):

$$\begin{bmatrix} X_i & X_i A_i^T \\ * & X_j \end{bmatrix} > 0 \;\;,\;\; \forall (i, j) \in \mathcal{I} \times \mathcal{I} \tag{15}$$

where $X_i = P_i^{-1}$.

It is worth nothing that condition (13) is to be satisfied $\forall (i, j) \in \mathcal{I} \times \mathcal{I}$, in particular for $i = j$. This means that each mode is necessarily asymptotically stable and $V(k, x) = x^T P_i x$ is the associated Lyapunov function.

Recall that the level sets of the Lyapunov function $V(x)$ given by (14) and associated to the switched system (12) is given by the following:

$$\varepsilon(P, \rho) = \{x \in \mathbb{R}^n / \sum_{i=1}^{N} \xi_i x^T P_i x \le \rho, \rho > 0\}. \tag{16}$$

Ascertaining that such a set is a region of stability is, in general, not easy. However, a simple estimate of $\varepsilon(P,\rho)$ is given by the intersection of all the ellipsoids [7]:

$$\Omega = \bigcap_{i=1}^{N} \varepsilon(P_i, \rho_i) \tag{17}$$

Obviously, this region could be very conservative. This conservatism is often encountered in hybrid systems with saturations [6]. To overcome this, one can use the technique proposed by [7] to enlarge the set Ω by introducing a prescribed set as a reference shape. This idea is characterized by a simple LMI corresponding to a special optimization problem.

IV. MAIN RESULT

In this section, the design of a stabilizing controller for the class of switched system with actuator saturation is presented by extending the results of Theorem 3.1 using Lemma 3.1.

Theorem 4.1: For given positive scalars $\rho_1,\ldots,\rho_N$, if there exist symmetric $n\times n$ matrices $P_1,\ldots,P_N$ and $m\times n$ matrices $H_1,\ldots,H_N$ such that,

$$\begin{bmatrix} P_i & [A_i + B_i(D_{is}F_i + D_{is}^{-}H_i)]^T P_j \\ * & P_j \end{bmatrix} > 0 \quad , \tag{18}$$
$$\forall (i,j) \in \mathcal{I}\times\mathcal{I}, \forall s \in [1,\eta]$$

and,

$$\varepsilon(P_i,\rho_i) \subset \mathcal{L}(H_i), \forall i \in \mathcal{I} \tag{19}$$

then, the closed-loop switched system (3) is asymptotically stable at the origin $\forall x_0 \in \Omega$ and for all sequences of switching $\alpha(k)$.

Proof: Assume there exist N matrices $H_1,\ldots,H_N$ and N symmetric matrices $P_1,\ldots,P_N$ such that condition (18) and (19) are satisfied. Using the expression in (11) and rewriting System (3) as in (5) yields that:

$$sat(F_i x_k) = \sum_{s=1}^{\eta} \delta_s(k)[D_{is}F_i + D_{is}^{-}H_i]x_k \ ; \tag{20}$$

$$\delta_s(k) \geq 0, \ \sum_{s=1}^{\eta}\delta_s(k) = 1 \tag{21}$$

and, subsequently :

$$\begin{aligned} x_{k+1} &= \sum_{s=1}^{\eta}\sum_{i=1}^{N}\xi_i(k)\delta_s(k)Ac_{is}x_k; \\ Ac_{is} &:= A_i + B_i(D_{is}F_i + D_{is}^{-}H_i), \quad s\in[1,\eta] \end{aligned} \tag{22}$$

The rate of change of the Lyapunov function candidate (14) along the trajectories of (22) is given by:

$$\begin{aligned} \Delta V(k,x_k) &= x_{k+1}^T(\sum_{j=1}^{N}\xi_j(k+1)P_j)x_{k+1} - \\ &\qquad x_k^T(\sum_{i=1}^{N}\xi_i(k)P_i)x_k \\ &= x_k^T\left[\sum_{s=1}^{\eta}\sum_{i=1}^{N}\xi_i(k)\delta_s(k)Ac_{is}\right]^T(\sum_{j=1}^{N}\xi_j(k+1)P_j) \\ &\quad \left[\sum_{s=1}^{\eta}\sum_{i=1}^{N}\xi_i(k)\delta_s(k)Ac_{is}\right]x_k - x_k^T\sum_{i=1}^{N}\xi_i(k)P_i x_k \end{aligned} \tag{23}$$

In light of the definition of the indicator function ξ, the above expression, at any given instant k, coincides with

$$\Delta V(k,x_k) = x_k^T\left\{\left[\sum_{s=1}^{\eta}\delta_s(k)Ac_{is}\right]^T(P_j)\left[\sum_{s=1}^{\eta}\delta_s(k)Ac_{is}\right] - P_i\right\}x_k \tag{24}$$

for some $i,\ j = 1,...,N$. Since Assumption (18) implies that the P_i's are necessarily positive, it follows from the convexity of the quadratic functions $x^T P_i x$ that

$$\Delta V(k,x_k) \leq \sum_{s=1}^{\eta}\delta_s(k)(Ac_{is}x_k)^T(P_j)\sum_{s=1}^{\eta}\delta_s(k)(Ac_{is}x_k) - x_k^T P_i x_k$$

That is,

$$\Delta V(k,x_k) \leq \max_s x_k^T\left\{Ac_{is}^T P_j Ac_{is} - P_i\right\}x_k \tag{25}$$

The use of Condition (18) and the Schur complement implies that

$$Ac_{is}^T P_j Ac_{is} - P_i < 0 \tag{26}$$

for all i, j and s. Letting λ be the largest eigenvalue among all the above matrices, we obtain that

$$\Delta V(k,x_k) \leq \lambda x_k^T x_k < 0. \tag{27}$$

which ensures the desired result. Noting that condition (18) is also satisfied for $i=j$, this implies that each set $\varepsilon(P_i,\rho_i)$ is a contractively invariant set for the corresponding mode. Further, by taking account of condition (19), one can guarantees that for every $x_o \in \varepsilon(P_i,\rho_i) \subset \mathcal{L}(H_i)$, each mode is asymptotically stable at the origin. However, a switch can occur at any time to handle the state outside the set $\varepsilon(P_j,\rho_j), j\neq i$ leading to the instability of the switched system. This problem is avoided by initializing the switched system inside the intersection of all the ellipsoids Ω. Note that this region is invariant for each mode.

This stability result can be used for control synthesis as follows.

Theorem 4.2: For given scalars $\rho_1,\ldots,\rho_N$, if there exists N symmetric matrices $X_1,\ldots,X_N$ and $2N$ matrices $Y_1,\ldots,Y_N$ and $Z_1,\ldots,Z_N$ solutions of the following

LMI's:

$$\left[\begin{array}{cc} X_i & (A_iX_i + B_iD_{is}Y_i + B_iD_{is}^-Z_i)^T \\ * & X_j \end{array}\right] > 0, \tag{28}$$

$$\left[\begin{array}{cc} \mu_i & z_{li} \\ * & X_i \end{array}\right] > 0, \tag{29}$$

$$X_i > 0,$$

$$\forall (i,j) \in \mathcal{I} \times \mathcal{I}, \forall s \in [1,\eta], \forall l \in [1,m]$$

where $\mu_i = 1/\rho_i$, z_{li} is the lth row of matrix Z_i; then the switched system with saturations in closed-loop (3), with,

$$F_i = Y_iX_i^{-1} \tag{30}$$

$$H_i = Z_iX_i^{-1} \tag{31}$$

$$P_i = X_i^{-1} \tag{32}$$

is asymptotically stable at the origin $\forall x_0 \in \Omega$ and for any sequences of switching $\alpha(k)$.

Proof: The LMI (18) can be transformed equivalently by Schur complement to the following:

$$P_j\left[A_i + B_i(D_{is}F_i + D_{is}^-H_i)\right]P_i^{-1} \left[A_i + B_i(D_{is}F_i + D_{is}^-H_i)\right]^T P_j - P_j < 0 \tag{33}$$

By post-multiplying and pre-multiplying the latter by P_j^{-1}, it follows:

$$\left[A_i + B_i(D_{is}F_i + D_{is}^-H_i)\right]P_i^{-1} \left[A_i + B_i(D_{is}F_i + D_{is}^-H_i)\right]^T - P_j^{-1} < 0 \tag{34}$$

Pose $X_i = P_i^{-1}$, inequality (34) can then be rewritten as,

$$\left[A_i + B_i(D_{is}F_i + D_{is}^-H_i)\right]X_i \left[A_i + B_i(D_{is}F_i + D_{is}^-H_i)\right]^T - X_j < 0$$

The use of the Schur complement a second time leads to obtain:

$$\left[\begin{array}{cc} X_i & (A_iX_i + B_iD_{is}F_iX_i + B_iD_{is}^-H_iX_i)^T \\ * & X_j \end{array}\right] > 0 \tag{35}$$

By letting $K_iX_i = Y_i$ and $H_iX_i = Z_i$, the LMI (28) follows readily. Using [13], the inclusion condition (19) can also be transformed to the equivalent LMI (29). □

It is worth noting that the resolution of this LMI can be extended to the case where the scalars ρ_i are also taken as design variables. Thus, one can use as main variables $X_i = P_i^{-1}/\rho_i$ and solve the LMI's (28)-(29) with $\mu = 1$.

The authors who work on this type of saturated problem are usually interested to obtain larger ellipsoid domains $\varepsilon(P_i, \rho_i)$. To attempt this, even if the system with saturations, in our case, is of switching type and with an augmented number of LMI's, we can apply one of the two following optimization problem:

$$(Pb.1) : \left\{ \begin{array}{c} \inf_{(X_i,Y_i,Z_i)}(\mu_i) \\ s.t.\ (28),(29), \\ i = 1,\ldots,N \end{array}\right.$$

This optimization problem, when is feasible, can help to enlarge the ellipsoids $\varepsilon(P_i, \rho_i)$ by maximizing the scalars ρ_i. A second way to obtain larger sets of invariance and contractivity is to solve the following optimization problem:

$$(Pb.2) : \left\{ \begin{array}{c} \sup_{(X_i,Y_i,Z_i)} Trace(X_i) \\ s.t.\ (28),(29), \\ i = 1,\ldots,N \end{array}\right.$$

When this optimization problem is feasible, the volume of the obtained ellipsoids are maximum with respect to the data of the system. Note that the scalars $\rho_i, i = 1,\ldots,N$ can be *a priori* fixed.

V. EXAMPLE

The application of the previous results on a practical example is under study. Thus, consider a numerical switching discrete-time system with saturation specified by the two modes:

$$A_1 = \left[\begin{array}{cc} -0.7 & 1 \\ -0.5 & -1.5 \end{array}\right]; B_1 = \left[\begin{array}{c} 1 \\ 0 \end{array}\right];$$

$$A_2 = \left[\begin{array}{cc} 0.9 & -1 \\ 1.7 & -1.5 \end{array}\right]; B_2 = \left[\begin{array}{c} 0 \\ -1 \end{array}\right].$$

For this example $n = 2, m = 1$ and $N = 2$. We have to solve 12 LMI's with 6 variables. The use of the Matlab LMI Toolbox to check our conditions leads to the following results.

$$\begin{array}{rcl} P_1 & = & \left[\begin{array}{cc} 0.035 & 0.088 \\ 0.088 & 0.281 \end{array}\right]; \\ H_1 & = & \left[\begin{array}{cc} 1.334 & 0.993 \end{array}\right]; \\ F_1 & = & \left[\begin{array}{cc} 1.3555 & 0.977 \end{array}\right]; \\ Ac_1 & = & \left[\begin{array}{cc} 0.655 & 1.977 \\ -0.500 & -1.500 \end{array}\right]; \\ P_2 & = & \left[\begin{array}{cc} 0.019 & -0.018 \\ -0.018 & 0.037 \end{array}\right]; \\ H_2 & = & \left[\begin{array}{cc} 1.776 & -1.591 \end{array}\right]; \\ F2 & = & \left[\begin{array}{cc} 1.781 & -1.587 \end{array}\right]; \\ Ac_2 & = & \left[\begin{array}{cc} 0.900 & -1.000 \\ -0.081 & 0.087 \end{array}\right]; \\ \rho_1 & = & 0.0042; \rho_2 = 0.0039. \end{array}$$

Figure 1 presents the two ellipsoid sets of invariance and contractivity $\varepsilon(P_i, \rho_i)$ together with the sets of saturation $\mathcal{L}(H_i)$ computed without any optimization program. Using

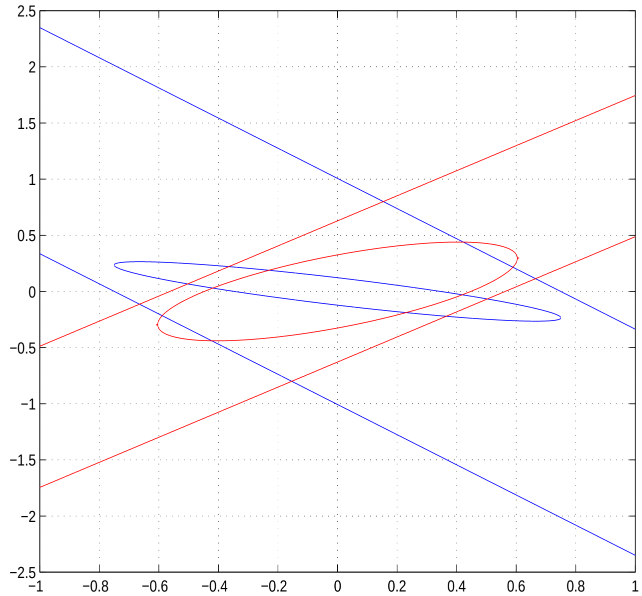

Fig. 1. The ellipsoid sets of invariance and contractivity for the switched discrete-time linear system computed with (28)-(29).

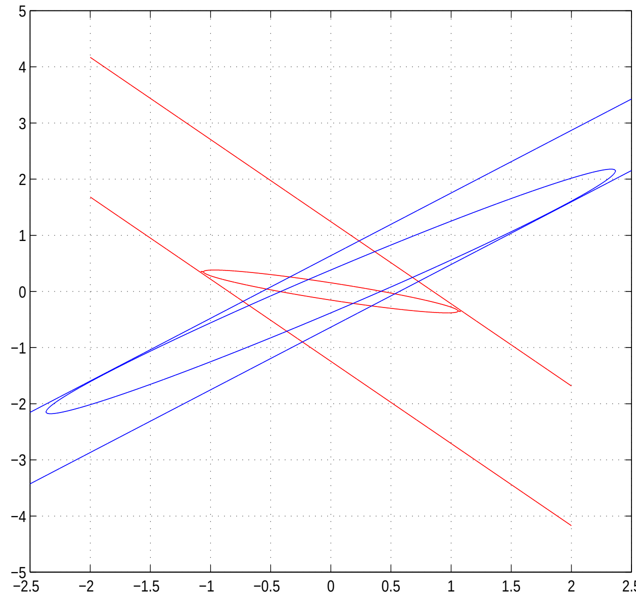

Fig. 2. The ellipsoid sets of invariance and contractivity for the switched discrete-time linear system computed with $(Pb.1)$.

the optimization problem $(Pb.1)$ one obtains:

$$
\begin{aligned}
P_1 &= \begin{bmatrix} 83.2184 & 211.1949 \\ 211.1949 & 640.4167 \end{bmatrix};\\
H_1 &= \begin{bmatrix} 1.1764 & 0.8039 \end{bmatrix};\\
F_1 &= \begin{bmatrix} 1.2864 & 0.7620 \end{bmatrix};\\
Ac_1 &= \begin{bmatrix} 0.5864 & 1.7620 \\ -0.5000 & -1.5000 \end{bmatrix};\\
P_2 &= \begin{bmatrix} 51.9386 & -55.5496 \\ -55.5496 & 61.2969 \end{bmatrix};\\
H_2 &= \begin{bmatrix} 1.7527 & -1.5703 \end{bmatrix};\\
F_2 &= \begin{bmatrix} 1.7632 & -1.5703 \end{bmatrix};\\
Ac_2 &= \begin{bmatrix} 0.9000 & -1.0000 \\ -0.0632 & 0.0703 \end{bmatrix};\\
(\rho_1)_{opt} &= 15.2030; (\rho_2)_{opt} = 8.9446.
\end{aligned}
$$

Figure 2 presents the two ellipsoid sets of invariance and contractivity $\varepsilon(P_i, \rho_i)$ together with the set of saturations $\mathcal{L}(H_i)$ computed with $(Pb.1)$.

Finally, the use of the optimization problem $(Pb.2)$ yields:

$$
\begin{aligned}
P_1 &= 1.0e-006 \begin{bmatrix} 0.0160 & 0.0393 \\ 0.0393 & 0.1224 \end{bmatrix};\\
H_1 &= \begin{bmatrix} 1.2534 & 0.8569 \end{bmatrix};\\
F_1 &= \begin{bmatrix} 1.2641 & 0.8526 \end{bmatrix};\\
Ac_1 &= \begin{bmatrix} 0.5641 & 1.8526 \\ -0.5000 & -1.5000 \end{bmatrix};\\
P_2 &= 1.0e-07 \begin{bmatrix} 0.0943 & -0.0916 \\ -0.0916 & 0.1375 \end{bmatrix};\\
H_2 &= \begin{bmatrix} 1.7693 & -1.5845 \end{bmatrix};\\
F2 &= \begin{bmatrix} 1.7697 & -1.5836 \end{bmatrix};\\
Ac_2 &= \begin{bmatrix} 0.9000 & -1.0000 \\ -0.0697 & 0.0836 \end{bmatrix};\\
\mu_1 &= 4.6270e+8; \mu_2 = 4.7934e+8
\end{aligned}
$$

Note that the optimal values of this optimization problem are given by $Trace(P_1^{-1}) = 3.3826e + 8$ and $Trace(P_2^{-1}) = 5.0584e{+}8$. Figure 3 presents the two ellipsoid sets of invariance and contractivity $\varepsilon(P_i, \rho_i)$ together with the set of saturations $\mathcal{L}(H_i)$ computed with $(Pb.2)$.

It is worth noting that the sets obtained with the optimization problem $(Pb.2)$ are slightly larger that the one obtained with $(Pb.1)$. Also, notice that the matrices H_i are very closed up to matrices F_i, that is, $\mathcal{L}(H_i) \cong \mathcal{L}(F_i)$. This means that in case of $m = 1$, the saturations allowed by this technique are not very important. Recall that inside the sets $\mathcal{L}(F_i)$ no saturations occur.

VI. CONCLUSION

In this work, sufficient condition of asymptotic stability are obtained for switched discrete-time linear systems sub-

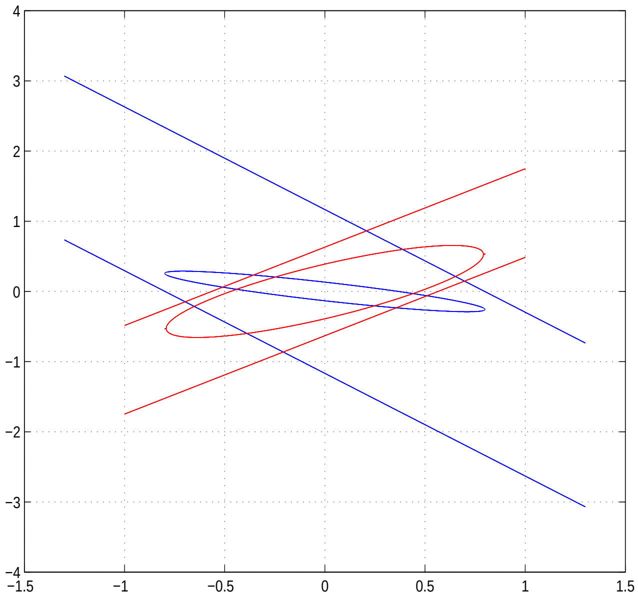

Fig. 3. The ellipsoids sets of invariance and contractivity for the switched discrete-time linear system computed with $(Pb.2)$.

ject to actuator saturations by using the idea of Lemma 3.1 which rewrites the saturation function under a combination of 2^m elements. These results are the direct extensions of the conditions obtained by [9] and [10] for unsaturated switched systems based on the use of the indicator function. The main results of this paper are given under LMI's formulation leading to the design of the stabilizing state feedback controllers of the system. An illustrative example is also studied by using the direct resolution of the LMI's (28)-(29) and the two linear optimization problems $(Pb.1)$ and $(Pb.2)$.

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