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Robust control using incremental sliding mode for underactuated systems with mismatched uncertainties

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Abstract—**A new robust controller using sliding mode control method for a class of underactuated mechanical systems with mismatched uncertainties is proposed in this paper. Two state variables of the underactuated system are chosen to construct the first-layer sliding surface. The first-layer sliding surface and one of the left state variables are used to construct the second-layer sliding surface. This process continues till the last sliding surface is constructed. And a distributed compensator is added to the sliding mode surfaces. We design a new sliding mode control law to guarantee that every sliding surface can converge rapidly to zero. For an underactuated system, which consists of 2n state variables, the controller has the (2n-1)-layer structure. Using Lyapunov law, we prove the stability of all the sliding surfaces theoretically. The simulation results show the validity of this method.**

I. Introduction

In recent years, there has been an increasing interest in underactuated systems. These systems are characterized by the fact that they have fewer actuators than the degrees of freedom to be controlled. Designing a common sliding mode surface for underactuated systems is not appropriate. Because underactuated systems usually consist of several subsystems and the state variables have no obvious differential relationship among these subsystems, the parameters of the common sliding mode surface can't be obtained directly according to the Hurwitz condition.

In the last few decades, sliding mode control has become a very popular control strategy for trajectory tracking and stabilization of dynamical systems. The reason for the popularity of sliding mode controller is robustness and system order reduction. As a kind of highly robust variable structural control method, the sliding-mode controller (SMC) can respond quickly, invariant to systemic parameters and external disturbance. Usually, SMC laws include two parts: switching control law and equivalent control law. The switching control law is used to drive the system's states towards a specific sliding surface and the equivalent control law guarantees the system states to stay on the sliding surface and converge to zero along the sliding surface. Many papers about the control of underactuated mechanical systems models were published [1]-[7] in the last few years. Fantoni, Lozano, and Spong [6] solved the control of an underactuated two-link robot called the Pendubot based on an energy approach and the passivity properties of the system. Wang [5] presented a stable hierarchical sliding mode control method for a class of second-order underactuated systems, which had two subsystems. Yi [7] designed a cascade sliding mode controller for large-scale underactuated systems, where the asymptotic stability of all the sliding surfaces was proved theoretically. In order to proving all sliding surface are stable, Yi [7] gave a strict limit to the underactuated system's state variables. Voytsekhovsky and Hirschorn [8] achieved the stabilization of single-input nonlinear systems using higher order compensating sliding mode control. Laghrouche, Plestan and Glumineau [9] presented a practical higher order sliding mode controller for multi-input multi-output nonlinear systems. Hirschorn [10] designed an incremental sliding mode control of the Ball and Beam. Lin and Mon [11] presented a hierarchical fuzzy sliding mode controller, which only guaranteed the second level sliding mode surface was asymptotically stable.

In this paper, a new robust controller using sliding mode control method for a class of underactuated mechanical systems with mismatched uncertainties is proposed. Moreover, the total control can guarantee every state variable to follow its own sliding surface to zero by choosing proper parameters of the controller. At the same time, the whole system's sliding surfaces are asymptotic stable. The simulations for the double inverted pendulum system show the validity of this method

II. Problem formulation

The norm expression of a class of underactuated systems can be given in the following form [1]:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f_1(X) + b_1(X)u(t) + d_1(t) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= f_2(X) + b_2(X)u(t) + d_2(t) \\
&\vdots \\
\dot{x}_{2n-1} &= x_{2n} \\
\dot{x}_{2n} &= f_n(X) + b_n(X)u(t) + d_n(t)
\end{aligned} \tag{1}$$

where $X=[x_1, x_2, \ldots, x_{2n-1}, x_{2n}]^T$ is the state variables; u is the input of the system; $f_n(x)$ and $b_n(x)$ are bounded nominal

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nonlinear functions. d_1(t), d_2(t)…, d_n(t) are the lumped disturbances, which include the system uncertainties and external disturbances. It's assumed that they are bounded by $|d_i(t)| \leq d_{i\max}$, where $d_{i\max}$ are upper boundary, i=1, 2, …, n. This large system is a typical single-input multi-output nonlinear coupled system. The objective of control is to design a single input u, which simultaneously controls the state variables $X=[x_1, \ldots, x_{2n}]^T$ to zero. In order to design stable sliding mode controller, we make the assumptions as following for plant(1): $f_i(x) \leqslant M_i$, $b_i(x) \leqslant B_i$, , $X \in \Omega_d$ (i=1,2, …,n), where M_i, B_i are definite positive constants, and Ω_d is a set given as $\Omega_d = \{X \mid \|X - X_0\| \leq \Delta\}$. Here Δ is a positive constant which denotes all state variables' boundaries. $X_0 \in R^{2n}$ is a fixed point.

In the following, we will design an incremental sliding surface.

First, we can choose two state variables to construct a sliding surface as the first-level. For example: the state variables (x_1, x_2) are chosen.

$$s_1 = c_1 x_1 + c_2 x_2 \tag{2}$$

where c_1 and c_2 are constants which have the same sign. The equivalent control law is

$$u_{eq(1)} = -\frac{c_2 f_1 + c_1 x_2}{c_2 b_1} \tag{3}$$

Then, the first-layer surface s_1 can be considered as a general state variable. We can use it and one of the left state variable to construct the second-layer surface s_2:

$$s_2 = c_3 x_3 + s_1 \tag{4}$$

where c_3 is a constant.

Similarly, the ith-layer surface s_i can be defined as:

$$s_i = c_{i+1} x_{i+1} + s_{i-1} \tag{5}$$

where c_{i+1} is a constant.

From (5), we can obtain the derivative of the i-th layer sliding surface.

$$\dot{s}_i = \sum_{j=1}^{m} c_{2j-1} x_{2j} + \sum_{j=1}^{m} c_{2j}(f_j + b_j u + d_j) \tag{6}$$

where

$$m = \begin{cases} (i+1)/2, & i \text{ is odd number} \\ i/2, & i \text{ is even number} \end{cases} \tag{7}$$

This process continues till the entire state variables are included in the sliding mode surfaces. The structure of the sliding surface is shown as Fig. 1. For the underactueated system (1), which has $2n$ state variables, the last sliding surface is s_{2n-1}. This incremental structure makes the i-th layer sliding mode controller have the information from the (i-1)th layer.

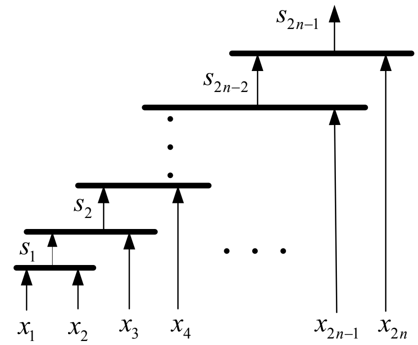

Fig. 1. The structure of the sliding surface

The state variables of underactuated system (1) have no obvious differential relationship. Excepting c_1 and c_2, the other parameters of the sliding mode surface can't be obtained directly according to the Hurwitz condition. So we propose a new incremental sliding mode controller to guarantee every layer to converge to zero.

In the following description, we define the errors of the system's state variables as $X=[x_1, \ldots, x_{2n}]^T$, and present the sliding control method to drive asymptotically the errors to zero for an initial state.

For the underactuated system (1), the Lyapunov function can be defined as: $V_{2n-1} = \frac{1}{2} s_{2n-1}{}^2$. Hence,

$$\dot{V}_{2n-1} = s_{2n-1} \cdot \dot{s}_{2n-1} \tag{8}$$

From (5), we can know that

$$\begin{aligned} \dot{V}_{2n-1} &= s_{2n-1} \cdot \dot{s}_{2n-1} \\ &= s_{2n-1} \cdot [\sum_{j=1}^{n} c_{2j-1} x_{2j} + \sum_{j=1}^{n} c_{2j}(f_j + b_j u + d_j)] \\ &= s_{2n-1} \cdot [\sum_{j=1}^{n} c_{2j-1} x_{2j} + \sum_{j=1}^{n} c_{2j} f_j + \sum_{j=1}^{n} c_{2j} b_j u + \sum_{j=1}^{n} c_{2j} d_j] \end{aligned} \tag{9}$$

Let d_i = 0, Equation (1) can be treated as the nomial form of the SIMO underactuated system. The incremental sliding mode controller can be assumed as:

$$u_{sm(i)} = u_{eq(i)} + u_{sw(i)} + u'_{sw(i)} \tag{10}$$

$u_{eq(i)}$ is the equivalent control law.

$u_{sw(i)}$ is the switch control law for every layer sliding surface.

$u'_{sw(i)}$ is the switch control law for the last layer sliding surface.

The equivalent control law of the ith layer sliding surface can be defined as:

$$u_{eq(i)} = -\frac{\sum_{j=1}^{m} c_{2j-1} x_{2j} + \sum_{j=1}^{m} c_{2j} f_j}{\sum_{j=1}^{m} c_{2j} b_j} \tag{11}$$

The switch control law is defined as:

$$u_{sw(i)}=\begin{cases}0 & i=1\\ \sum_{j=1}^{i}\eta_j\,\mathrm{sgn}(s_j)/den(i) & i>1\end{cases} \tag{12}$$

where η_1 is a positive constant,

$$\eta_j=2\eta_{j-1} \tag{13}$$

$$den(i)=c_2b_1+\sum_{j=2}^{m}(c_{2j}\cdot b_j\cdot\mathrm{sgn}(s_{2j-1})) \tag{14}$$

and

$$u'_{sw}=-k\cdot s_{2n-1}/\sum_{i=1}^{n}c_{2n}b_n \tag{15}$$

s_{2n-1} denotes the last layer of the sliding mode surface. k is a positive constant.

The switch control law $u_{sw(i)}$ and $u'_{sw(i)}$ can improve the response time. When the last-layer sliding surface s_{2n-1} converges to zero, the control law $u'_{sw(2n-1)}$ becomes zero. The expressions of $u_{eq(i)}$ and $u_{sw(i)}$ degenerate respectively, which become the equivalent control law and switch control law for the (2n-2)th layer sliding surface respectively. Similarly, when the sliding surface s_i (i>1) converges to zero, the state variables x_i ($i=3,4,\cdots,2n$) converges to zero. When the sliding surface s_i degenerates to s_1, the equivalent control law becomes $u_{eq(1)}=-(c_2f_1+c_1x_2)/c_2b$, which satisfies the reachable and stable condition of the sliding mode control.

For the matched uncertainties, the above incremental sliding mode controller can resist them because of the invariant characteristic of the sliding mode. For the mismatched uncertainties, we will design a sliding mode compensator to resist them.

For this incremental sliding mode surface, there are two methods to design a compensator. One is to design a distributed compensator and compensate the mismatched uncertainties at every layer sliding surface. The other method is to design a lumped compensator and compensate the mismatched uncertainties at the last layer. For the former, the distributed compensator makes the control accurate, and it could guarantee the stability, such as [11]. For the latter, the compensator simplifies the control design, but it is difficult to guarantee the stability.

Based on the above viewpoints, we design a distributed compensator. The total controller for the underactuated system (1) can define as:

$$u_{(i)}=u_{sm(i)}+u_{com(i)} \tag{16}$$

where $u_{com(i)}$ is the distributed compensator, and $u_{sm(i)}$ is the incremental sliding mode controller.

For the i-th layer sliding surface, $u_{com(i)}$ is given by:

$$u_{com(i)}=-\frac{\sum_{j=1}^{m}C_{2j}\bar{d}_j|\mathrm{sgn}(s_{2j-1})|}{\sum_{j=1}^{m}c_{2j}b_j} \tag{17}$$

Here $\bar{d}_i=d_{i\max}$, the parameters $C_i=|c_i|$. The mismatched uncertainties are compensated by the distributed sliding mode compensator at the sliding surfaces.

III. Stability Analysis

In this section, we will prove the asymptotical stability of the entire sliding mode surfaces with Barbalat's lemma.

Theorem 1: Consider a class of the underactuated system with mismatched uncertainties as (1). If the sliding mode control law is defined as (10), (11), (12) and (15), the distributed compensator is defined by (16) and (17), and let the assumptions for plant (1) be true, the entire sliding mode surfaces are asymptotically stable.

Proof:

Both sides of (8) is integrated,

$$\begin{aligned}\int_0^t\dot{V}_{2n-1}d\tau=\int_0^t(-k\cdot s_{2n-1}^2-s_{2n-1}\cdot\sum_{i=1}^{2n-1}\eta_i\cdot\mathrm{sgn}(s_i)\\-\sum_{i=1}^{n}(C_{2i}\bar{d}_i-c_{2i}d_i)\,|\,\mathrm{sgn}(s_{2i-1})\,|)d\tau\end{aligned} \tag{18}$$

$$\begin{aligned}V_{2n-1}(t)=V_{2n-1}(0)-\int_0^t(ks_{2n-1}^2+s_{2n-1}\cdot\sum_{i=1}^{2n-1}\eta_i\cdot\mathrm{sgn}(s_i)\\+\sum_{i=1}^{n}(C_{2i}\bar{d}_i-c_{2i}d_i)\,|\,\mathrm{sgn}(s_{2i-1})\,|)d_\tau<\infty\end{aligned} \tag{19}$$

Because the sliding mode parameter η_i is define as (13), we can know that

$$\mathrm{sgn}(\sum_{i=1}^{2n-1}\eta_i\cdot\mathrm{sgn}(s_i))=\mathrm{sgn}(\eta_{2n-1}\cdot\mathrm{sgn}(s_{2n-1})) \tag{20}$$

which means $s_{2n-1}\cdot\sum_{i=1}^{2n-1}\eta_i\cdot\mathrm{sgn}(s_i)\geq 0$. At the same time, from the definition of the (17), we also know that

$$\sum_{i=1}^{n}(C_{2i}\bar{d}_i-c_{2i}d_i)\,|\,\mathrm{sgn}(s_{2i-1})\,|\geq 0 \tag{21}$$

Then, we can obtain

$$\begin{aligned}\lim_{t\to\infty}\int_0^t(ks_{2n-1}^2+s_{2n-1}\sum_{i=1}^{2n-1}\eta_i\,\mathrm{sgn}(s_i)\\+\sum_{i=1}^{n}(C_{2i}\bar{d}_i-c_{2i}d_i)\,|\,\mathrm{sgn}(s_{2i-1})\,|)d\tau\leq V_{2n-1}(0)<\infty\end{aligned} \tag{22}$$

According to Barbalat's lemma, we can know when $t\to\infty$,

$$ks_{2n-1}^2+s_{2n-1}\sum_{i=1}^{2n-1}\eta_i\,\mathrm{sgn}(s_i)+\sum_{i=1}^{n}(C_{2i}\bar{d}_i-c_{2i}d_i)\,|\,\mathrm{sgn}(s_{2i-1})\,|\to 0$$

,which means $\lim_{t\to\infty}s_{2i-1}=0$, i=1, 2, 3, …, n. When i=n, the last layer sliding mode surface is asymptotically stable.

For the ith layer sliding surface, the Lyapunov function is $V_i = \frac{1}{2}s_i^2$. Differentiating $V_i(t)$ with respect to time t obtains

$$\dot{V}_i = s_i \dot{s}_i = s_i\left(\sum_{j=1}^{m} c_{2j-1}x_{2j} + \sum_{j=1}^{m} c_{2j}(f_j + b_j u + d_j)\right) \quad (23)$$

$$m = \begin{cases} (i+1)/2\ , & i \text{ is odd number} \\ i/2\ , & i \text{ is even number} \end{cases}$$

From (11), (12) and (17), we have

$$\dot{V}_i = -s_i \cdot \sum_{j=1}^{i} \eta_i \cdot \mathrm{sgn}(s_j) - \sum_{j=1}^{m} (C_{2j}\bar{d}_j - c_{2j}d_j)\,|\,\mathrm{sgn}(s_{2j-1})\,| \le 0 \quad (24)$$

From (13), we can know that $\sum_{j=1}^{i} \eta_j \,\mathrm{sgn}(s_j)$ and $\eta_i \,\mathrm{sgn}(s_i)$ are the same sign. The following inequation can be obtained:

$$|\sum_{j=1}^{i} \eta_j \,\mathrm{sgn}(s_j)\,| < 2\eta_i \,|\,\mathrm{sgn}(s_i)\,| < \infty \quad (25)$$

Defines $\sum_{j=1}^{i} \eta_j \,\mathrm{sgn}(s_j) = \eta_i' \,\mathrm{sgn}(s_i)$, where η_i' is a positive constant and from (13), we can know $\eta_i' < 2\eta_i$. Consequently, (24) becomes

$$\dot{V}_i = -\eta_i'\,|\,s_i\,| - \sum_{j=1}^{m} (C_{2j}\bar{d}_j - c_{2j}d_j)\,|\,\mathrm{sgn}(s_{2j-1})\,| \le 0 \quad (26)$$

Integrating both sides of (26),

$$\int_0^t \dot{V}_i d\tau = \int_0^t -\eta_i'\,|\,s_i\,| - \sum_{j=1}^{m} (C_{2j}\bar{d}_j - c_{2j}d_j)\,|\,\mathrm{sgn}(s_{2j-1})\,|\,d\tau \quad (27)$$

$$V_i(t) = V_i(0) - \int_0^t \eta_i'\,|\,s_i\,|\,d\tau - \sum_{j=1}^{m} (C_{2j}\bar{d}_j - c_{2j}d_j)\,|\,\mathrm{sgn}(s_{2j-1})\,| < \infty \quad (28)$$

$$\lim_{t\to\infty} \int_0^t \eta_i'\,|\,s_i\,| - \sum_{j=1}^{m} (C_{2j}\bar{d}_j - c_{2j}d_j)\,|\,\mathrm{sgn}(s_{2j-1})\,|\,d\tau \le V_i(0) < \infty \quad (29)$$

From (22) and (29), and using the Barbalat's lemma, there is $\lim_{t\to\infty} s_i = 0$, ($i = 1,2,...,2n-2$). That is to say, all the sliding surfaces are asymptotically stable.

IV. Simulation results

In this section, we will demonstrate the robust control strategy. This roubust controller is applicable to a double inverted pendulum system. The structure of double inverted pendulum is shown in Fig.2.

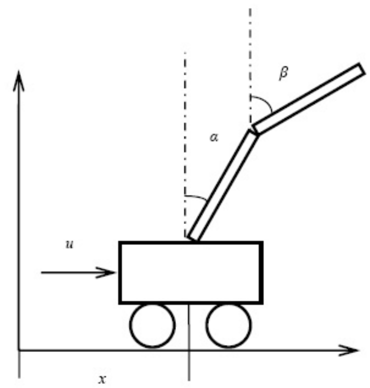

Fig.2. Structure of the double inverted pendulum system

There are three subsystems: the lower pendulum, the upper pendulum, and the cart. From (1), let $n = 3$. The state space expression of the double inverted pendulum system can be described by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f_1(X) + b_1(X)u + d_1 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= f_2(X) + b_2(X)u + d_2 \\ \dot{x}_5 &= x_6 \\ \dot{x}_6 &= f_3(X) + b_3(X)u + d_3 \\ y(t) &= [x_1, x_2, x_3, x_4, x_5, x_6]^T \end{aligned} \quad (30)$$

where $X = [x_1, x_2, x_3, x_4, x_5, x_6]^T$ is state variable vector, $f_i(x)$ and $b_i(x)$ (i=1,2,3) are the nonlinear function of the state variables. $d_i(x)$(i=1,2,3) is the mismatched uncertain term whose bound is known. And the system variables $x_1 = \theta_1$ is the lower pendulum angle with respect to the vertical line; $x_2 = \dot{\theta}_1$ is the angular velocity of the lower pendulum angle; $x_3 = \theta_2$ is the upper pendulum angle with respect to the vertical line; $x_4 = \dot{\theta}_2$ is the angular velocity of the upper pendulum angle; $x_5 = x$ is the cart position; $x_6 = \dot{x}$ is the cart velocity; u is the applied force to move the cart.

For simulative comparison, the parameters of the double inverted pendulum system are chosen as that the cart mass is 1kg, the lower pendulum mass is 0.1kg, the upper pendulum mass is 0.1kg; the lower pendulum length is 0.1m, the upper pendulum length is 0.1m, the gravitational accelerating is 9.81 m/s^2 ,which are same to the simulation model in [12].

The mismatched uncertain terms of the system are assumed as follows:

$d_1 = 0.1+0.5p$, $d_2 = 0.1+0.5\,p$, $d_3 = 0.5\,p$,

Here p is a random number whose range is from -1 to 1. Thus, the upper boundary of the mismatched uncertain terms $d_{1\max} = 0.6$, $d_{2\max} = 0.6$ and $d_{3\max} = 0.5$.

The parameters of the incremental sliding mode controller are selected as $c_1 = -0.01$, $c_2 = -2.0$, $c_3 = 3.0$, $c_4 =$

0.2, $c_5 = 0.2$, $c_6 = 0.2$, $\eta_1 = 0.4$ and $k = 0.8$. The control objective is from the initial conditions $[\pi/6, 0, \pi/9, 0, 0, 0]^T$ of the inverted pendulum system to the desired state $[0, 0, 0, 0, 0]^T$ of the double inverted pendulum system. The simulation results are shown as follows.

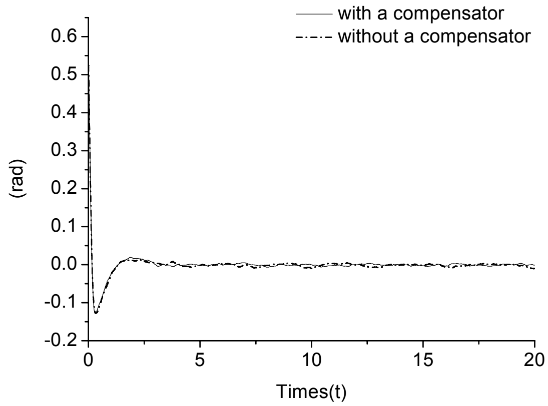

Fig.3. Lower pendulum angle

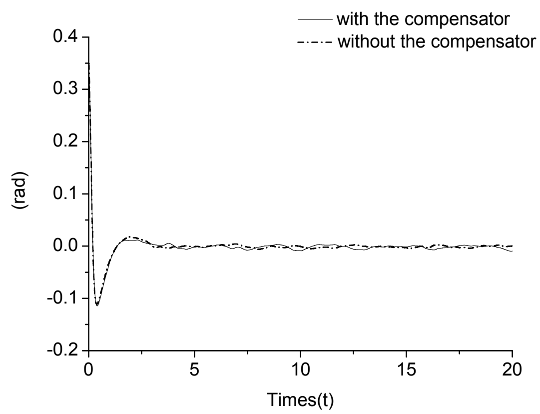

Fig.4. Upper pendulum angle

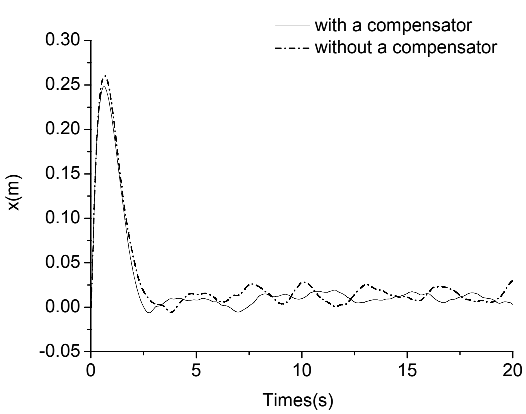

Fig.5. Cart position

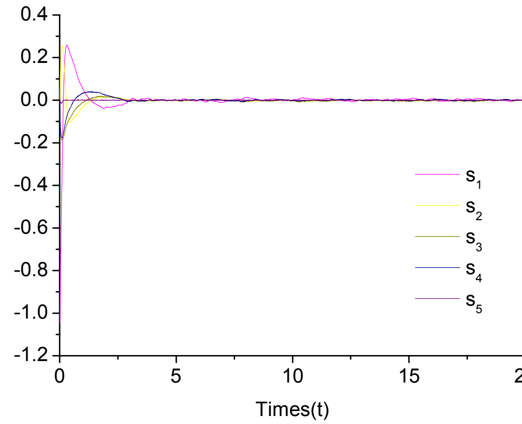

Fig.6. Sliding surface of the ISMC

From Fig.3-5, we know that, by using the incremental sliding mode control with distributed compensator, all state variables can converge to the desired states with favorable system performance. Fig.6 shows the entire sliding surfaces. By this control method, all the sliding surfaces are asymptotically stable.

Comparing with the method in [11], which the control objective was only to make the double pendulums upright without considering the cart position, our objective is more difficult. Comparing with the method in [12], the initial state of our objective is more difficulty, the curves are smoother and the response time is shorter.

V. CONCLUSIONS

An incremental sliding mode controller with distributed compensator has been proposed to achieve decoupling performance for a class of underactuated systems with mismatched uncertainties. For a class of underactuated systems, which consists of 2n state variables, the controller has the (2n-1)-layer sliding surfaces. The paper has shown that all the sliding surfaces are asymptotically stable. The simulation results also show the validity of this incremental sliding mode controller.

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