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Resilient Delay-Dependent Observer-based Stabilization of Continuous-time Symmetric Composite Systems

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Abstract— **The objective of this paper is to propose an approach to decentralized resilient robust observer-based stabilization with delayed feedback for uncertain delayed systems which are composed of identical nominal subsystems and symmetric nominal interconnections. The proposed method employs the structural properties of the system to construct a low order control design model and the convex optimization approach. It is shown how this methodology can lead to a reduced-order control design with delayed feedback when considering a delay-dependent approach and additive gain perturbations in the gain matrices to obtain a quadratically stabilized global closed-loop system.**

I. INTRODUCTION

Time delays, uncertainties, information structure constraints, and dimensionality belong often to important issues in large scale complex systems. To cope effectively with control designs for such systems means to deal with decentralization of the design problems. Time-delayed systems represent a class of infinite-dimensional systems largely used to describe transport or heredity phenomena. Their stability is a problem of recurring interest since the existence of a delay in a system model may induce instability, oscillations, or bad performance. This feature motivates the study of dynamic systems with time delays over the past decades. The various criteria for the stability and the stabilization of delayed systems have been developed. The relevant methods belong to two categories according to their measure of the influence of the delays in the corresponding stability conditions. Delay-independent criteria and delay-dependent criteria have been distinguished. The majority of the proposed methods present delay-independent criteria. These criteria are generally more conservative than the delay-dependent ones when considering small time delays.

A standard assumption on the controller design is that they can be implemented exactly on real world systems. In practice, controllers are implemented imprecisely because of various reasons determined by digital controller properties or the need for additional tuning of parameters. Even small implementation changes in the controller parameters may either destabilize the closed-loop system or cause a deterioration some of its performance indices. While robustness relates to uncertainties in the plant, fragility relates to uncertainties or inaccuracies in the implementation of a controller. The need to have a certain degree of freedom in controller parameters, i.e. the robustness of stability against perturbations in controller parameters, in the feedback loop is underlined when considering the implementation of low cost local controllers resulting from the control design of large scale complex systems. Similar composite systems represent an important class of these systems.

1.1 Prior Work

The motivation for studying symmetric composite systems arises in very different real world systems. Systems with identical subsystems and symmetric coupling can be found in parallel systems such as flow splitting parallel reactors [1], paper machines control [2], electric power systems operating in parallel [3], [4], industrial manipulators [5], mechanic systems [6], space crystal furnace [7], homogeneous interconnected systems such as seismic cables [8], or in the problem of formations of vehicles in cyclic pursuit solved using circulant matrices [9]. More complete survey presents [1].

Several control design methods have been developed for symmetric composite systems [1], [10], [11], [12], [13]. Their extension to uncertain systems is presented in [14] and [15]. Both delay-independent and delay-dependent control design methods are surveyed in [16] including several methods for uncertain systems with time delays. However, their extension to uncertain similar composite systems with time delays is unusual. [17] and [18] proposed a state space approach to cope with this class of systems with time delays. A more exhaustive presentation is discussed in this context in the survey paper [19]. All previous results deal with a delay-independent approach and low order control design models.

Delay-dependent criteria have been used for control design with delayed feedback by [20], [21], [22] for the centralized case, while [23] deals with the decentralized case.

The motivating reasons for introducing the notion of fragility, practical examples, and the design methods are surveyed in [24] and [25] for centralized systems. Decentralized resilient control design for uncertain interconnected systems with time delays is developed in [26] by using the delay-independent approach. A resilient stabilization method

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is proposed in [18] for state delayed uncertain symmetric composite systems.

The paper extends the results by [21] and [25] into resilient decentralized setting by using the reduced-order control design when considering the delay-dependent approach within the framework of the LMI constraints by [27].

To the author's best knowledge, the problem of resilient decentralized robust observer-based controller design with delayed feedback has not been solved up to now for this class of similar composite systems.

1.2 Outline of the Paper

This paper presents a sufficient condition for the reduced-order design of resilient decentralized observer-based controller with delayed feedback in the convex optimization context for stabilization of a class of uncertain delayed similar composite systems. This controller requires the construction of a low order design model as well as the selection of the gain matrices for this model. These gain matrices guarantee the quadratic stability of the global closed-loop system when implemented into the global system. A delay-dependent approach and additive gain perturbations are used.

II. Problem Formulation

Consider an uncertain linear similar composite system consisting of N subsystems, where the ith subsystem is described as follows

$$\begin{aligned}\dot{x}_i(t) &= (A + \Delta A_i(t))x_i(t)\\ &\quad + (A_d + \Delta A_{di}(t))x_i(t-d)\\ &\quad + Bu_i(t) + s_{zi}(t)\\ y_i(t) &= Cx_i(t) \qquad x_i(t) = \Phi_i(t) \qquad \forall t \in [-d, 0]\\ & \qquad\qquad i = 1, \ldots, N \qquad N \geq 2\end{aligned} \tag{1}$$

x_i, u_i, s_{zi}, y_i are $n-, m-, p_s-, p_y-$dimensional vectors of the subsystem states, control inputs, interconnection inputs and measured outputs, respectively. Given initial functions $\Phi_i(t)\ :\ [-d, 0] \rightarrow R^n$ are absolutely continuous vector functions satisfying $x_i(0) = \Phi_i(0)$ for all i. d denotes a point time delay. Interconnections are described in the form

$$s_{zi} = \sum_{j=1}^{N}(L_{ij}y_{zj} + L_{dij}y_{dzj}) \tag{2}$$

where y_{zj} is the p_z-dimensional vector of the interconnection output from the subsystem j which is related to the state vector in the form

$$y_{zj} = C_z x_j \qquad y_{dzj} = C_{dz}x_j(t-d) \tag{3}$$

The interconnection matrices L_{ij}, L_{dij} have the structure as follows

$$\begin{aligned}L_{ii} &= 0 \qquad L_{ij} = L_q + \Delta L_{qij}(t)\\ L_{dii} &= 0 \qquad L_{dij} = L_{dq} + \Delta L_{dqij}(t), \quad (i \neq j)\end{aligned} \tag{4}$$

A, A_d, B, C, C_z and L_q, L_{dq} are constant nominal matrices. $\Delta A_i(t), \Delta A_{di}(t), \Delta B_i(t), \Delta C_i(t)$, and $\Delta L_{qij}(t), \Delta L_{dqij}(t)$ are norm bounded uncertainties which admit the following structure

$$\begin{aligned}\Delta A_i(t) &= D_A F_{Ai}(t) E_A\\ \Delta A_{di}(t) &= D_{dA} F_{dAi}(t) E_{dA}\\ \Delta L_{qij}(t) &= D_L F_{Lij}(t) E_L\\ \Delta L_{dqij}(t) &= D_{dL} F_{dLij}(t) E_{dL}\end{aligned} \tag{5}$$

$F_{Ai}(t), F_{dAi}(t), F_{Lij}(t), F_{dLij}$ are unknown time-varying real $p_A \times q_A, p_{dA} \times q_{dA}, p_L \times q_L, p_{dL} \times q_{dL}$ matrices with Lebesgue measurable elements, respectively. $F_{(*)}(t)$ satisfy $F^T_{(*)}(t)F_{(*)}(t) \leq I$ for all $t \geq 0$. I denotes a unit matrix of appropriate dimensions. $D_A, ..., E_{dL}$ are known real constant matrices of appropriate dimensions.

Supposing that all states are not available, we seek for a decentralized controller–observer scheme stabilizing the system (1)–(5) for $t \geq 0$. We propose this scheme to be composed of N decentralized resilient controllers with delayed feedback and full state observers of the form

$$\begin{aligned}\dot{\hat{x}}_i(t) &= (A_r + \Delta A_{ri})\hat{x}_i(t)\\ &\quad + (A_{dr} + \Delta A_{dri})\hat{x}_i(t-d) + Bu_i(t)\\ &\quad + (K_o + \Delta K_{oi})(y_i(t) - C_{ri}\hat{x}_i(t))\\ u_i(t) &= (K_c + \Delta K_{ci})(\hat{x}_i(t) + \int_{t-d}^{t} A_{dr}\hat{x}_i(s)ds)\\ &\qquad \hat{x}_i(t) = 0 \qquad \forall t \in [-d, 0] \qquad i = 1, \ldots, N\end{aligned} \tag{6}$$

where $\hat{x}_i(t)$ is the $n-$dimensional observer state of the subsystem i. A_{ri} and A_{dri} are the observer state nominal matrices. K_o and K_c are the observer gain matrices and the feedback gain matrices, respectively. The uncertainties in the state matrices as well as the additive gain perturbations are defined as follows

$$\begin{aligned}\Delta A_{ri}(t) &= D_r F_{ri}(t) E_r\\ \Delta A_{dri}(t) &= D_{dr} F_{dri}(t) E_{dr}\\ \Delta K_{oi}(t) &= D_o F_{oi}(t) E_o\\ \Delta K_{ri}(t) &= D_c F_{ci}(t) E_c\end{aligned} \tag{7}$$

$F_{ri}(t), F_{dri}(t), F_{oi}(t), F_{ci}(t)$ are unknown time-varying real $p_r \times q_r, p_{dr} \times q_{dr}, p_o \times q_o, p_c \times q_c$ matrix functions with Lebesgue measurable elements, respectively. They all satisfy $F^T_{(*)}(t)F_{(*)}(t) \leq I$ for all $t \geq 0$. I denotes a unit matrix of appropriate dimensions. $D_r, ..., E_c$ are known real constant matrices of the corresponding dimensions.

The controller-observer parameters of (6) to be determined are the system matrices A_r, A_{dr} and the gain matrices K_o, K_c.

The global system description of the system (1)–(5) has the form

$$\begin{aligned}\dot{x}(t) &= (\overline{A} + \Delta\overline{A}(t))x(t) + (\overline{A}_d + \Delta\overline{A}_d(t))x(t-d)\\ &\quad + \overline{B}u(t)\\ y(t) &= \overline{C}x(t) \qquad\qquad x(t) = \Phi_o(t) \qquad \forall t \in [-d, 0]\end{aligned} \tag{8}$$

where x, u, y are $nN-, mN-, pN-$dimensional vectors of the system states, control inputs and measured outputs, respectively. $\Phi_o(t)$ is a given initial function. Further $x_d(t) = x(t-d)$. The nominal matrices are defined as

$$\begin{aligned}
&\overline{A} = (\overline{A}_{ij}) && \overline{A}_{ii} = A && \overline{A}_{ij} = L_{ij}C_z \\
&\overline{A}_d = (\overline{A}_{dij}) && \overline{A}_{dii} = A_d, && \overline{A}_{dij} = L_{dij}C_{dz} \\
&\overline{B} = \text{diag}(B, ..., B) \\
&\overline{C} = \text{diag}(C, ..., C) && && i = 1, \dots, N
\end{aligned} \tag{9}$$

The uncertainty terms have the form

$$\begin{aligned}
\Delta\overline{A}(t) &= \overline{D}_A \overline{F}_A(t) \overline{E}_A \\
\Delta\overline{A}_d(t) &= \overline{D}_{dA} \overline{F}_{dA}(t) \overline{E}_{dA} \\
\Delta\overline{B}(t) &= \overline{D}_B \overline{F}_B(t) \overline{E}_B \\
\Delta\overline{C}(t) &= \overline{D}_C \overline{F}_C(t) \overline{E}_C
\end{aligned} \tag{10}$$

The constant matrices are defined as follows

$$\begin{aligned}
\overline{D}_A =&\text{diag}(\overline{D}_1, ..., \overline{D}_N) \\
&\overline{D}_i = (D_L...D_L \quad D_A \quad D_L...D_L) \\
\overline{E}_A =&\text{diag}(\overline{E}_1 ..., \overline{E}_N) \\
&\overline{E}_i = (E_L...E_L \quad E_A \quad E_L...E_L) \\
\overline{D}_{dA} =&\text{diag}(\overline{D}_{d1}, ..., \overline{D}_{dN}) \\
&\overline{D}_{di} = (D_{dL}...D_{dL} \quad D_{dA} \quad D_{dL}...D_{dL}) \\
\overline{E}_{dA} =&\text{diag}(\overline{E}_{d1} ..., \overline{E}_{dN}) \\
&\overline{E}_{di} = (E_{dL}...E_{dL} \quad E_{dA} \quad E_{dL}...E_{dL})
\end{aligned} \tag{11}$$

while the remaining matrices have the form

$$\begin{aligned}
\overline{D}_B =&\text{diag}(D_B, ..., D_B) && \overline{E}_B = && \text{diag}(E_B, ..., E_B) \\
\overline{D}_C =&\text{diag}(D_C, ..., D_C) && \overline{E}_C = && \text{diag}(E_C, ..., E_C)
\end{aligned} \tag{12}$$

D_A is located at the i-th position in $\overline{D}_i$. Each $\overline{D}_i$ has N elements $D_{(*)}$. Symbols with an analogous meaning are used for the locations of E_A, D_{dA}, E_{dA} in $\overline{E}_i$, $\overline{D}_{di}$, $\overline{E}_{di}$, respectively. The uncertainty structure is lumped in uncertainty functions in the form

$$\begin{aligned}
\overline{F}_A(t) =&\text{diag}(F_{A1}, ..., F_{AN}) \\
&F_{Ai} = \text{diag}(F_{Li1}, ... \\
&\quad ..., F_{Li\,i-1}, F_{Ai}, F_{Li\,i+1}, ..., F_{LiN}) \\
\overline{F}_{dA}(t) =&\text{diag}(F_{dA1}, ..., F_{dAN}) \\
&F_{dAi} = \text{diag}(F_{dLi1}, ... \\
&\quad ..., F_{dLi\,i-1}, F_{dAi}, F_{dLi\,i+1}, ..., F_{dLiN}) \\
\overline{F}_B(t) =&\text{diag}(F_{B1}, ..., F_{BN}) \\
\overline{F}_C(t) =&\text{diag}(F_C, ..., F_{CN})
\end{aligned} \tag{13}$$

Denote the overall decentralized controller (6) rewritten into a compact form as follows

$$\begin{aligned}
\dot{\hat{x}}(t) &= (\overline{A}_r + \Delta\overline{A}_r)\hat{x}(t) + (\overline{A}_{dr} + \Delta\overline{A}_{dr})\hat{x}(t-d) \\
&\quad + \overline{B}u(t) + (\overline{K}_o + \Delta\overline{K}_o)(y(t) - \overline{C}\hat{x}(t)) \\
u(t) &= (\overline{K}_c + \Delta\overline{K}_c)(\hat{x}(t) + \int_{t-d}^{t} \overline{A}_d \hat{x}(s) ds)
\end{aligned} \tag{14}$$

where all matrices have a diagonal form of the corresponding dimensions. Note only that $\overline{K}_o = \text{diag}(K_o, ..., K_o)$ and $\overline{K}_c = \text{diag}(K_c, ..., K_c)$.

2.1 The Problem

The goal is to derive a procedure reducing the control design complexity of a resilient observer-based output feedback decentralized controller with delayed feedback (6)–(7) for the system (1)–(5) such that the closed–loop system (8)–(13), (14) is quadratically stable for all admissible uncertainties. Solve the problem by using a delay-dependent approach.

Remark 1. The notion of fragility means the sensitivity of controller to parameters perturbations. An attempt is made to construct non-fragile controllers, i.e. controller which ensure robustness of stability against parameter perturbations of the controller in the feedback loop. More precisely, the formulated problem considers norm bounded additive uncertainties in the controller (6) for the system (1). When all controller uncertainties disappear, The problem reduces on a robust decentralized control design problem without any fragility issues.

III. MAIN RESULTS

The solution of the problem requires finding the system matrices of the controller as well as its gain matrices. Let us divide the solution into two steps. The first part consists of the construction of the control design model which includes the required system parameters of the controller. The second part presents the method of the design of the required gain matrices.

The structural properties of the system (8)–(13) are employed to obtain the control design model. Consider the transformation of the states

$$\tilde{x}(t) = Sx(t) \tag{15}$$

by using the transformation $T = S^{-1}$. Suppose a real snxsn matrix $T(n, s)$ in the form

$$\begin{aligned}
T(n,1) &= I \\
T(n,s) &= \begin{pmatrix} I & 0 & \dots & 0 & I \\ 0 & I & \dots & 0 & I \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & I & I \\ -I & -I & \dots & -I & I \end{pmatrix} \qquad s > 1
\end{aligned} \tag{16}$$

where I denotes here nxn identical matrix. Then T is defined as

$$\begin{aligned}
&T(i) = \text{diag}[T(n, N-i),\ I, ..., I] \quad i = 0, ..., N-1 \\
&\quad T = T(0)\ \ T(1) \quad ... \quad T(N-1)
\end{aligned} \tag{17}$$

The constructive way how to use this transformation is presented the subsequent lemma by [14]. First, define the nominal matrices

$$\begin{aligned}
A_s &= A - L_q C_z && A_c = A_s + NL_q C_z \\
A_{ds} &= A_d - L_{dq} C_{dz} && A_{dc} = A_{ds} + NL_{dq} C_{dz}
\end{aligned} \tag{18}$$

Lemma 1. Consider the matrix $\overline{A}$ in the system (8)–(13) and any given $J = \text{diag}[J_o, ..., J_o]$, where J, J_o are $nN\text{x}nN$, $n\text{x}n$ matrices. Then, the following equalities hold

$$\begin{aligned} &T^{-1}\overline{A}T = \text{diag}(A_s, ..., A_s,\ A_c) \\ &T^T\overline{A}T = \text{diag}(2A_s,\ 6A_s, ..., N(N-1)A_s,\ NA_c) \\ &T^{-1}J(T^{-1})^T = \text{diag}(\frac{1}{2}J_o,\ \frac{1}{6}J_o, ..., \frac{1}{N(N-1)}J_o) \\ &T^TJT = \text{diag}(2J_o, ..., N(N-1)J_o,\ NJ_o) \end{aligned} \tag{19}$$

The same relations hold when applying Lemma 1 on the matrix $\overline{A}_d$. It leads to analogous matrices A_{ds} and A_{dc} for the delayed terms.
The terms NL_qC_z and $NL_{dq}C_{dz}$ can be decomposed into the nominal parts and the uncertain parts. The nominal part has the form

$$\frac{N}{2}L_qC_z = DE \qquad \frac{N}{2}L_{dq}C_{dz} = D_dE_d \tag{20}$$

where D, E, D_d, E_d are constant matrices. Note only that such decomposition is not unique. The uncertain part has the standard form of norm bounded uncertainties

$$\Delta A_v = DF(t)E \qquad \Delta A_{dv} = D_dF_d(t)E_d \tag{21}$$

Define the n-dimensional system using the decomposition (20), (21) as follows

$$\begin{aligned} \dot{x}_r(t) &= (A_r + \Delta A_r(t))x_r(t) \\ &\quad + (A_{dr} + \Delta A_{dr}(t))x_r(t-d) + Bu_r(t) \\ y_r(t) &= Cx_r(t) \end{aligned} \tag{22}$$

The nominal matrices in (22) are defined as

$$A_r = A + (\frac{N}{2} - 1)L_qC_z \qquad A_{dr} = A_d + (\frac{N}{2} - 1)L_{dq}C_{dz} \tag{23}$$

while the uncertainties are given as follows

$$\begin{aligned} \Delta A_r(t) &= D_AF_{rA}(t)E_A \\ &\quad + (\frac{N}{2} - 1)D_LF_{rL}(t)E_L + DF(t)E \\ &= D_rF_{rAm}(t)E_r \\ \Delta A_{dr}(t) &= D_{dA}F_{drA}(t)E_{dA} \\ &\quad + (\frac{N}{2} - 1)D_{dL}F_{drL}(t)E_{dL} + D_dF_d(t)E_d \\ &= D_{dr}F_{dr}(t)E_{dr} \end{aligned} \tag{24}$$

Consider an observer-based output resilient controller with delayed feedback for the control design model (22) in the following form

$$\begin{aligned} \dot{\hat{x}}_r(t) &= (A_r + \Delta A_r)\hat{x}_r(t) \\ &\quad + (A_{dr} + \Delta A_{dr})\hat{x}_r(t-d) + Bu_r(t) \\ &\quad + (K_o + \Delta K_{or})(y_r(t) - C_{rr}\hat{x}_r(t)) \\ u_r(t) &= (K_c + \Delta K_{cr})(\hat{x}_r(t) + \int_{t-d}^{t} A_{dr}\hat{x}_r(s)ds) \\ \hat{x}_r(t) &= 0 \qquad \forall t \in [-d, 0] \end{aligned} \tag{25}$$

where $\Delta K_{or} = D_oF_o(t)E_o$ and $\Delta K_{cr} = D_cF_c(t)E_c$.

The selection of the gain matrices K_o, K_c in (25) must guarantee the quadratic stability of the closed-loop system (22), (25). Denote the error vector $e_r(t) = x_r(t) - \hat{x}_r(t)$).

The quadratic stability of the closed-loop system can be established when using the neutral transformation in the state and the error vectors by [21] or [16] in the form

$$\begin{aligned} \mathcal{D}(x_r) &= x_r(t) + \int_{t-d}^{t} A_{dr}x_r(s)ds \\ \mathcal{D}(e_r) &= e_r(t) + \int_{t-d}^{t} A_{dr}e_r(s)ds \end{aligned} \tag{26}$$

for the system (22), (25). Consider the Lyapunov function candidate as follows

$$V(t) = V_1(x_r) + V_2(e_r) \tag{27}$$

where

$$\begin{aligned} V_1(x_r) &= \mathcal{D}^T(x_r)P_x\mathcal{D}(x_r) + \int_{t-d}^{t}\int_{s}^{t} x_r^T(u)Q_xx_r(u)duds \\ &\quad + \int_{t-d}^{t} x_r^T(s)T_xx_r(s)ds \end{aligned} \tag{28}$$

$$V_2(e_r) = \mathcal{D}^T(e_r)P_e\mathcal{D}(e_r) + \int_{t-d}^{t}\int_{s}^{t} e_r^T(u)Q_ee_r(u)duds \tag{29}$$

P_x, Q_x, T_x, P_e, Q_e are positive definite matrices of appropriate dimensions.

The time derivative of $V(t)$ leads after tedious manipulations to the sufficient condition for the existence of stabilizing gain matrices. This condition is invoked in the following theorem which is stated in terms of the LMIs when introducing the linearization variables $X_x = P_x^{-1}$, $Y_x = K_cX_x$, $W_x = T_x^{-1}$, $R_x = dQ_x^{-1}$, $R_e = d^{-1}Q_e$, $G_e = K_oP_e$.

Theorem 1: Given the system (22) and the controller (25). K_o, K_c are unknown gain matrices to be determined in (25). For any given time delay $d > 0$, consider the following inequalities

$$M1(A_r) < 0 \qquad M2(A_r) < 0 \qquad M3(A_r) < 0 \tag{30}$$

where

$$M1(A_r) = \begin{pmatrix} \Phi_1 & \Phi_{12} & 0 & dX_x & X_x & \Phi_{16} & \Phi_{17} \\ \bullet & -R_x & \Phi_{23} & \Phi_{24} & \Phi_{25} & \Phi_{26} & 0 \\ \bullet & \bullet & -\varepsilon_4 I & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & -R_x & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -W_x & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \Phi_6 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \Phi_7 \end{pmatrix} \tag{31}$$

$$M2(A_r) = \begin{pmatrix} -W_x & W_xE_d^T & W_xE_d^T \\ \bullet & -\varepsilon_3 I & 0 \\ \bullet & \bullet & -I \end{pmatrix} \tag{32}$$

$$M3(A_r) = \begin{pmatrix} \Pi_1 & \Pi_2 & dR_e & \Pi_3 \\ \bullet & -R_e + C^T E_o^T E_o C & -dA_{dr}^T R_e & 0 \\ \bullet & \bullet & -R_e & 0 \\ \bullet & \bullet & \bullet & \Pi_4 \end{pmatrix} \tag{33}$$

and

$$\begin{aligned} A_{or} &= A_r + A_{dr} \\ \Phi_1 &= A_{or}X_x + X_x A_{or}^T + BB^T + BY_x + Y_x^T B^T \\ &\quad + (\varepsilon_1 + \varepsilon_4)D_A D_A^T + (1+\varepsilon_2)D_c D_c^T + \varepsilon_3 D_d D_d^T \\ \Phi_{12} &= -A_{or}A_{dr}R_x \\ \Phi_{16} &= (X_x E_A^T,\ X_x E_o^T) \\ \Phi_{17} &= (X_x E_A^T,\ Y_x^T E_c^T,\ Y_x^T E_c^T) \\ \Phi_{23} &= R_x A_{dr}^T E_d^T \\ \Phi_{24} &= -dR_x A_{dr}^T \\ \Phi_{25} &= -R_x A_{dr}^T \\ \Phi_6 &= \text{diag}(I,\ I) \\ \Phi_7 &= \text{diag}(\varepsilon_1 I,\ \varepsilon_2 I,\ I) \\ \Pi_1 &= P_e A_{or} + A_{or}^T P_e - G_e C - C^T G_e^T \\ &\quad + (1+\varepsilon_5)X_x^{-1}Y_x^T E_c^T E_c Y_x X_x^{-1} \\ \Pi_2 &= -P_e A_{or} + G_e C A_{dr} + C^T E_o^T E_o C \\ \Pi_3 &= (P_e D_A,\ P_e D_A,\ P_e D_c,\ P_e D_c,\ G_e D_o) \\ \Pi_4 &= \text{diag}(-I,\ -I,\ -I,\ -\varepsilon_5 I,\ -I) \end{aligned} \tag{34}$$

If there exist positive definite matrices X_x, P_e, W_x, R_x, R_e, constants $\varepsilon_i > 0$ for $i = 1, ..., 5$, and the matrices Y_x, G_e satisfying the inequalities (30), then the system (22), (25) is quadratically stabilized with the gain matrices

$$K_c = Y_x X_x^{-1} \qquad\qquad K_o = P_e^{-1} G_e \tag{35}$$

Remark 2. We solve first the LMIs (31), (32) to obtain the matrices X_x and Y_x. Then, the LMI (33) is solved by using the matrices X_x and Y_x to obtain the solutions G_e and P_e.
Remark 3. Theorem 1 is an extension of Theorem 3.1 in [21] for centralized systems. In particular, the extension concerns the inclusion of additive gain perturbations in the controller when considering the nominal input and output matrices of the system. Thus, the resilient controller design is performed.
Remark 4. Theorem 1 may hold for some given constant time delay $d > 0$. However, the stabilizing solution satisfying the inequalities (30) for such a d cannot be considered as an upper-bound in the sense that all time delays less than d belong to an admissible delay interval which guarantees the stabilizing solution with the same gain matrices [21].
The following theorem states the main result.

Theorem 2: Given the symmetric composite system (1)–(5). Construct the reduced control design system (22)–(24). Select the observer gain matrix K_o and the controller matrix K_c (35) in the controller (25) for the system (22)–(24) satisfying the inequalities (30) for any given time delay $d > 0$. Implement the matrices K_o, K_c into (6). Then, the closed-loop overall system (8)–(13), (14) is quadratically stable.
The proof is given in the Appendix.
Remark 5. To get a deeper insight into this result, note that Lemma 1 uses the structural properties of this class of systems transforming the original $nN \times nN$ system into two $n \times n$systems with the state matrices defined by (18). This is a particular simultaneous stabilization problem which can be further reduced to a single robust control design problem for the $n \times n$ system (22) by using the construction (23)-(24) while keeping the dynamical properties of the original system. Theorem 2 surveys this derivation. Thus, the procedure essentially simplifies the control design complexity.

IV. Conclusion

This paper proposes a new scheme for the reduced-order control design of resilient decentralized robust observer-based controller with delayed feedback for nonlinear but nominally linear symmetric composite systems with state delay. Particular structural properties of the nominal part of this class of symmetric composite systems are used to construct a reduced order control design model with equivalent properties as those of the original system. A delay-dependent procedure and an additive gain perturbation approach in the controller parameters have been selected for the proper design of gain matrices by using the LMI constraints. These gain matrices, when implemented on the original system, guarantee the quadratic stability of the global closed–loop system.

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APPENDIX

Consider the global system (8)–(13) and the global controller (14) with the given delay d. Denote the error vector $e(t) = x(t) - \hat{x}(t))$ for this closed-loop system. Consider the neutral transformation in the state and the error vectors in the form

$$\begin{aligned}\overline{\mathcal{D}}(x) &= x(t) + \int_{t-d}^{t} \overline{A}_d x(s)ds \\ \overline{\mathcal{D}}(e) &= e(t) + \int_{t-d}^{t} \overline{A}_d e(s)ds\end{aligned} \tag{36}$$

for the system (8)–(14). Consider the Lyapunov function candidate

$$\overline{V}(t) = \overline{V}_1(x) + \overline{V}_2(e) \tag{37}$$

where

$$\begin{aligned}\overline{V}_1(x) = \overline{\mathcal{D}}^T(x)\overline{P}_x\overline{\mathcal{D}}(x) &+ \int_{t-d}^{t}\int_{s}^{t} x^T(u)\overline{Q}_x x(u)duds \\ &+ \int_{t-d}^{t} x^T(s)\overline{T}_x x(s)ds\end{aligned} \tag{38}$$

$$\overline{V}_2(e_r) = \overline{\mathcal{D}}^T(e)\overline{P}_e\overline{\mathcal{D}}(e) + \int_{t-d}^{t}\int_{s}^{t} e^T(u)\overline{Q}_e e(u)duds \tag{39}$$

The matrices in (38), (39) have the form

$$\begin{array}{ll}\overline{P}_x = \text{diag}(P_x, ..., \ P_x) & \overline{Q}_x = \text{diag}(Q_x, ..., \ Q_x) \\ \overline{T}_x = \text{diag}(T_x, ..., \ T_x) & \overline{P}_e = \text{diag}(P_e, ..., \ P_e) \\ \overline{Q}_e = \text{diag}(Q_e, ..., \ Q_e) & \end{array} \tag{40}$$

All these matrices are positive definite ones as follows from (28), (29).

The quadratic stability of the global closed-loop system (8)–(14) is proved by using the inequalities in Theorem 1 being appropriately modified to the global system when directly implementing the gain matrices $\overline{K}_c = \text{diag}(K_c, ..., \ K_c)$ and $\overline{K}_o = \text{diag}(K_o, ..., \ K_o)$. Note that K_c, K_o are obtained by using Theorem 1 for the closed–loop system (22)–(25). Then, the linearization matrices $\overline{X}_x = \overline{P}_x^{-1}$, $\overline{Y}_x = \overline{K}_c\overline{X}_x$, $\overline{W}_x = \overline{T}_x^{-1}$, $\overline{R}_x = d\overline{Q}_x^{-1}$, $\overline{R}_e = d^{-1}\overline{Q}_e$, $\overline{G}_e = \overline{K}_o\overline{P}_e$ are also available via (40) as well as constants $\varepsilon_i, i = 1, ..., 5$. Now, substitute the parameters of the control design system (22) by the global system (8). Substitute the controller (25) by the controller (14). Denote simply these changes with the replacements (22)→(8), (25)→(14) and implement them together with $\overline{X}_x, \overline{Y}_x, \overline{W}_x, \overline{R}_x, \overline{R}_e, \overline{G}_e$ into the inequalities (30). Denote the resulting matrices $\overline{M}1(\overline{A})$, $\overline{M}2(\overline{A})$, $\overline{M}3(\overline{A})$. Note they have the same structure as (31)–(33), but they are reformulated for the global system (8)–(14). Then, it remains to show that the matrices $\overline{M}1(\overline{A}) < 0$, $\overline{M}2(\overline{A}) < 0$, $\overline{M}3(\overline{A}) < 0$.

To simplify the discussion, consider only the matrix $\overline{M}1(\overline{A})$. Applying now the transformation of the states T by (15) and Lemma 1, we get the transformed system resulting in the relation

$$\begin{aligned}&P1^{-1}T1^{-1}\overline{M}1(\overline{A})T1P1 \\ &\quad = \text{diag}(M1(A_s), ..., \ M1(A_s), \ M1(A_c))\end{aligned} \tag{41}$$

where $P1$ is a convenient permutation matrix, $T1 = \text{diag}(T, \ ... \ , \ T)$. $P1$ and T are non-singular matrices. Analogous relations hold for $\overline{M}2(\overline{A}), \overline{M}3(\overline{A})$. If $M1(A_r) < 0$ by Theorem 1, then $M1(A_s) < 0$, $M1(A_c) < 0$ because the system (22)–(24) includes both systems with the matrices A_s, A_{ds} and A_c, A_{dc} by (19) as its special cases.

An analogous way of reasoning leads to the same conclusions when applying the transformation on the remaining matrices $\overline{M}2(\overline{A}), \overline{M}3(\overline{A})$ while taking into account the structure of (32), (33).

Thereby, the closed-loop system (1)–(5), (6) with the gain matrices K_o, K_c determined according to Theorem 1 by Eqs.(35) is quadratically stable. Q.E.D.