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Feedback Residual Generator with Decoupled Faults and Disturbances

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Abstract— We develop a new residual generator design which decouples the faults and disturbances in a linear time-invariant system. The decoupling is achieved using feedback to endow the residual generator with additional robustness to modeling errors and disturbances. We provide feasibility conditions for feedback decoupling. We illustrate the new approach using a design example.

I. Introduction

Fault detection is an important practical problem and several texts are already available on the subject [1], [2]. Approaches to fault detection include: the parity approach, observer-based methods [3], [4], [5], Kalman filters [6], parameter estimation [7], neural networks and fuzzy logic[8].

A drawback of many well known fault detection approaches is that they fail if the control input matrix is linearly dependent on the fault input matrix or the disturbance input matrix. For example, parity approaches that annihilate the effect of the control input typically annihilate faults with the same input matrix.

We present a new observer-based method that allows us to decouple faults and disturbances. The observer used is standard Luenberger observer. The columns of the control input matrix need not be linearly independent of those of the input matrices for the fault and the disturbance. The decoupling is achieved using a feedback residual generator that replaces the controller in feedback decoupling. Unlike the controller, the residual generator is on the output side of the plant but otherwise its design is similar to that of the controller. We use a synthesis approach to select the dynamics of the residual generator.

The input to the residual generator is the primary residual obtained by comparing the estimated output to the plant output. However, a transformed version of the output is used so that the overall system is square and decoupling is feasible.

The paper is organized as follows. In Section 2, we obtain the transfer function whose output is the primary residual. In Section 3, we derive the decoupling residual generator. In Section 4, we provide an example that illustrates the design methodology. Section 5 is the conclusion and suggestions for future work.

II. Observer-Based Fault Detection

Consider the continuous-time equations for a linear system with disturbance $d(t)$ and a fault $f(t)$ is

$$\dot{x}(t) = Ax(t) + B^u u(t) + B^w d(t) + B^f f(t) \qquad (1)$$

$$y(t) = Cx(t) \qquad (2)$$

where $x \in R^n$ is a state vector, $u \in R^m$, is the system input, and $y \in R^l$, is the system output vector, and A, B^u, B^w, B^f, C are matrices of appropriate dimensions. We assume that d is n_d by 1 and f is n_f by 1. From the state-space model, we can write the system transfer function relation

$$y(s) = \left[G_{pu} \;\vdots\; G_{pd} \;\vdots\; G_{pf} \right] \begin{bmatrix} u(s) \\ d(s) \\ f(s) \end{bmatrix} \qquad (3)$$

with the transfer functions

$$G_{pi} = C\left(sI_n - A\right)^{-1} B^i, i \in \{u, d, f\} \qquad (4)$$

Figure 1 shows a block diagram of the fault detection methodology proposed in this paper. We use an observer to estimate the output of the system and compare the actual output to the estimate to detect a fault. We include a feedback loop in our design rather than simply feeding the output to the observer. Thus, unlike traditional fault detection observers, the input to the observer is given by

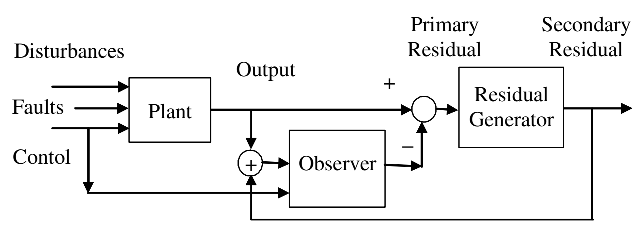

Figure 1. Closed-loop fault detection.

$$y_w(s) = y(s) + w(s) \qquad (5)$$

where $w(s)$ is the secondary residual output of a residual generator with transfer function $G_r(s)$.

The system observer is in the form

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + B^u u(t) + L\left(y_w(t) - C\hat{x}(t)\right) \\ &= A_o \hat{x}(t) + B^u u(t) + Ly(t) + Lw(t) \\ A_o &= A - LC \end{aligned} \qquad (6)$$

The estimate of the output is

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$$\hat{y}(s) = \left[G_{ou} + G_{oy} G_{pu} \;\vdots\; G_{oy} G_{pd} \;\vdots\; G_{oy} G_{pf} \right] \begin{bmatrix} u(s) \\ d(s) \\ f(s) \end{bmatrix} \quad (7)$$

$$+ G_{oy} w(s)$$

where

$$G_{ou} = C(sI_n - A_o)^{-1} B^u$$
$$G_{oy} = C(sI_n - A_o)^{-1} L \quad (8)$$

The primary residual is the difference between the estimate and the actual state/output of the system. The primary residual is

$$e_y = y - \hat{y} \quad (9)$$

We first prove the following lemma.

Lemma 1

For the closed-loop residual generator scheme, the control input is decoupled from the primary residual.

Proof

The component of the primary residual due to the control u is

$$e_y(s) = \{G_{pu} - (G_{ou} + G_{oy} G_{pu})\} u(s)$$
$$= \left[(I_l - G_{oy}) G_{pu} - G_{ou} \right] u(s)$$

We substitute to obtain

$$(I_l - G_{oy}) G_{pu}$$
$$= \{I - C(sI_n - A_o)^{-1} L\} C(sI_n - A)^{-1} B^u$$
$$= C(sI_n - A_o)^{-1} \{sI_n - A_o - LC\} (sI_n - A)^{-1} B^u$$
$$= C(sI_n - A_o)^{-1} B^u$$
$$= G_{ou}$$

Hence, the coefficient of u is zero.

•

The primary residual generator is given by

$$e_y = (I_l - G_{oy}) G_{pdf} \begin{bmatrix} d(s) \\ f(s) \end{bmatrix} - G_{oy} w(s) \quad (10)$$

$$G_{pdf} = \left[G_{pd} \;\vdots\; G_{pf} \right] \quad (11)$$
$$= C(sI_n - A)^{-1} \left[B^d \;\vdots\; B^f \right]$$

The secondary residual generator is given by

$$w(s) = G_r e_y(s) \quad (12)$$

where $G_r(s)$ is the transfer function of the residual generator. The secondary residual generator can now be expressed as

$$w(s) = (I_l + G_r G_{oy})^{-1} G_r (I_l - G_{oy}) G_{pdf} \begin{bmatrix} d(s) \\ f(s) \end{bmatrix} \quad (13)$$

The following lemma gives the transfer function of the closed-loop residual generator with the faults and disturbances as input.

Lemma 2

For the closed-loop residual generator scheme, the transfer function with the faults and disturbances as input and with the secondary residual generator w as output is given by

$$w(s) = (I_l + G_r G_{oy})^{-1} G_r G_{odf} \begin{bmatrix} d(s) \\ f(s) \end{bmatrix}$$
$$G_{odf} = C(sI_n - A_o)^{-1} B^{df}$$
$$B^{df} = \left[B^d \;\vdots\; B^f \right] \quad (14)$$

Proof

We simplify the term

$$(I_l - G_{oy}) G_{pdf}$$
$$= (I_l - C(sI_n - A_o)^{-1} L) C(sI_n - A)^{-1} B^{df}$$
$$= C(sI_n - A_o)^{-1} (sI_n - A_o - LC)(sI_n - A)^{-1} B^{df}$$
$$= C(sI_n - A_o)^{-1} B^{df}$$

•

Because our objective is to decouple the faults and disturbances, we require the system to be square. This is possible provided that the number of outputs exceeds the sum of the number of faults and disturbance. In other words, we can obtain a square system if we can satisfy the condition

$$l \geq n_f + n_d \quad (15)$$

If the above condition is satisfied with equality, the system is square. If the condition holds as a strict inequality, then pre-multiplication of the primary residual will "square down" the system.

III. Decoupling by Output Feedback

Consider the residual generator of (14) as represented by the block diagram of Figure 2.

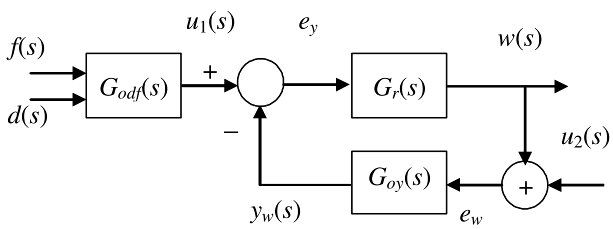

Figure 2. Block diagram of closed-loop fault detection.

The block diagram of the system of Figure 2 has the overall transfer function

$$\begin{bmatrix} w \\ y_u \end{bmatrix} = H_{yf} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$
$$H_{yf} = \begin{bmatrix} H_1 & H_2 \\ H_3 & H_4 \end{bmatrix} \tag{16}$$

where

$$\begin{aligned} H_1 &= \left(I + G_r G_{oy}\right)^{-1} G_r \\ &= G_r \left(I + G_{oy} G_r\right)^{-1} \\ H_2 &= -\left(I + G_r G_{oy}\right)^{-1} G_r G_{oy} \\ &= -G_r G_{oy} \left(I + G_r G_{oy}\right)^{-1} \\ H_3 &= \left(I + G_{oy} G_r\right)^{-1} G_{oy} G_r \\ &= G_{oy} G_r \left(I + G_{oy} G_r\right)^{-1} \\ H_4 &= \left(I + G_{oy} G_r\right)^{-1} G_{oy} \\ &= G_{oy} \left(I + G_r G_{oy}\right)^{-1} \end{aligned} \tag{17}$$

For the system to be internally stable, we must guarantee that all four transfer functions are stable. Clearly, a good observer design guarantees its stability. However, the residual generator transfer function G_r may or may not be stable. The following Lemma based on [7] provides sufficient conditions for stability.

Lemma 3

If H_1 and H_4 are stable, then the closed-loop residual generator in internally stable for any stable observer. If H_1 is stable and G_r is minimum phase, then the closed then the closed-loop residual generator in internally stable for any stable observer.

Proof

The lemma follows from the equalities

$$\begin{aligned} H_2 &= -H_1 G_{oy} \\ H_3 &= G_{oy} H_1 \\ H_4 &= G_r^{-1} H_1 G_{oy} \end{aligned}$$

•

Our design seeks to decouple the system of Figure 2 and yield a diagonal transfer function $H_1 G_{odf}$. Clearly, we assume that H_1 is square. The decoupled transfer function is in the form

$$\begin{aligned} \overline{H}_1 &= H_1(s) G_{odf}(s) \\ &= diag\{h_1(s), h_2(s), \cdots, h_l(s)\} \end{aligned} \tag{18}$$

We adopt a synthesis approach where we select the appropriate dynamics and then obtain the transfer function G_r to realize it. In terms of the desired transfer function, G_r is given by

$$\begin{aligned} G_r &= \left(G_{odf} - \overline{H}_1 G_{oy}\right)^{-1} \overline{H}_1 \\ &= \overline{H}_1 \left(G_{odf} - G_{oy} \overline{H}_1\right)^{-1} \end{aligned} \tag{19}$$

Since $\overline{H}_1$ is a diagonal matrix, it can be easily factorized as

$$\begin{aligned} \overline{H}_1 &= diag\left\{\frac{N_i}{D_i}\right\} \\ &= D_H^{-1} N_H = N_H D_H^{-1} \end{aligned}$$

The residual generator transfer function can then be simplified to

$$G_r = N_H \left(G_{odf} D_H - G_{oy} N_H\right)^{-1} \tag{20}$$

The residual generator plays the part of a, possibly unstable, plant in a feedback loop. We now need to examine conditions for decoupling the system using output feedback.

Lin [9] derived necessary and sufficient conditions for decoupling both for the case of distinct RHP poles as well as repeated RHP poles. However, these results are of no use in our decoupling problem since the presence of the prefilter converts it into a model matching problem. For model matching, we can either seek exact model matching using (20), or approximate model matching using H-infinity optimization as in [10]. For approximate model matching, we minimize the function

$$\begin{aligned} &\left\| T(s) - G_{oy} H_1 \right\| \\ &T(s) = G_{oy}^{-1} \overline{H}_1 G_{odf}^{-1}(s) \end{aligned} \tag{21}$$

For exact model matching the norm of (21) is zero.

We expand the desired transfer function as

$$T(s) = G_{oy}^{-1}\overline{H}_1 G_{odf}^{-1}(s)$$
$$= \begin{bmatrix} \mathbf{g}_{o1}^T \\ \mathbf{g}_{o2}^T \\ \vdots \\ \mathbf{g}_{ol}^T \end{bmatrix} \begin{bmatrix} h_1\mathbf{g}_{df1} & h_2\mathbf{g}_{df2} & \cdots & h_l\mathbf{g}_{dfl} \end{bmatrix}$$
$$= \begin{bmatrix} \mathbf{g}_{oi}^T h_j \mathbf{g}_{dfj} \end{bmatrix}$$

For a proper controller and desired transfer function, we must satisfy the condition

$$pzd\{h_j(s)\} + pzd\{\mathbf{g}_{dfj}\} + pzd\{\mathbf{g}_{oi}\} \geq 0, i, j = 1,\cdots,l$$

where $pzd\{.\}$ denotes the pole-zero difference or number of poles minus number of zeros and $\mathbf{g}_{df,j}$ is the jth row of the inverse of G_{odf}.

IV. Example

Consider the plant with state-space model

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -384 & -304 & -112 & -8 \end{bmatrix}$$
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$
$$B^d = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad B^f = \begin{bmatrix} 0 \\ 0 \\ 10 \\ 0 \end{bmatrix} \qquad d = 0$$

Let the desired observer poles be $\{-4\pm j4, -12, -12\}$, then the observer gain L and observer state matrix A_o are given by

$$L = \begin{bmatrix} 11 & 1 \\ -12 & 12 \\ 80 & -80 \\ 240 & -624 \end{bmatrix}$$
$$A_o = \begin{bmatrix} -12 & 0 & 0 & 0 \\ 0 & -12 & 1 & 0 \\ 0 & 80 & 0 & 1 \\ 0 & 320 & -112 & -8 \end{bmatrix}$$

The transfer function of the plant for residual generator design is

$$G_{oy} = \begin{bmatrix} g_{o1} & g_{o2} \\ g_{o3} & g_{o4} \end{bmatrix}$$
$$g_{o1} = g_{o2} = \frac{11}{s+12}$$
$$g_{o3} = -\frac{s^2 - 72s + 112}{s^3 + 20s^2 + 128s + 384}$$
$$g_{o3} = -\frac{13s^2 + 24s + 112}{s^3 + 20s^2 + 128s + 384}$$

Its inverse has all entries with pole-zero difference equal to −1.

The prefilter transfer function of Figure 2 is inverted to give

$$G_{odf}^{-1} = \begin{bmatrix} g_{-1} & g_{-2} \\ g_{-3} & g_{-4} \end{bmatrix}$$
$$g_{-1} = s + 12$$
$$g_{-2} = 0$$
$$g_{-3} = \frac{2s^2 + 16s + 144}{s^3 + 20s^2 + 128s + 384}$$
$$g_{-4} = \frac{10s + 80}{s^3 + 20s^2 + 128s + 384}$$

The inverse of the prefilter transfer function is stable and has the first row with pole-zero difference equal to −1 and the second with pole-zero difference equal to 1. We select second order diagonal entries for the desired closed-loop transfer function for a realizable residual generator. We select the closed-loop transfer function

$$\overline{H}_1(s) = diag\{h_1(s), h_2(s))\}$$
$$h_1(s) = 1/\left(s^2 + 4s + 8\right)$$
$$h_2(s) = 2/\left(s^2 + 4s + 8\right)$$

The decoupling residual generator obtained using MATLAB© is given by

$$G_r = \frac{\begin{bmatrix} r_1 & r_2 \\ r_3 & r_4 \end{bmatrix}}{D_r(s)}$$

where

$$r_1 = s^4 + 21.48s^3 + 148s^2 + 464s + 499.2$$

$$r_2 = -\begin{pmatrix} 0.4s^5 + 9.6s^4 + 102.6s^3 + 666.4s^2 \\ +1770s + 3034 \end{pmatrix}$$

$$r_3 = 0.2s^3 + 4s^2 + 25.6s + 76.8$$

$$r_4 = 0.2s^5 + 4.8s^4 + 41s^3 + 167.2s^2 + 230.4s - 230.4$$

$$D_r = s^5 + 13.8s^4 + 74.6s^3 + 199.2s^2 + 187.2s + 128$$

The system was simulated using the MATLAB© toolbox SIMULIMK©. The outputs of the residual generator are shown in Figures 3 and 4. The simulation diagram is shown in Figure 5.

The simulation includes a sinusoidal disturbance and a step fault. The simulation results show that the closed-loop residual generator can identify the disturbance and the fault. Figure 3 shows the sinusoidal disturbance with no contribution from the fault while Figure 4 shows a step response due to the step fault with no contribution from the sinusoidal disturbance.

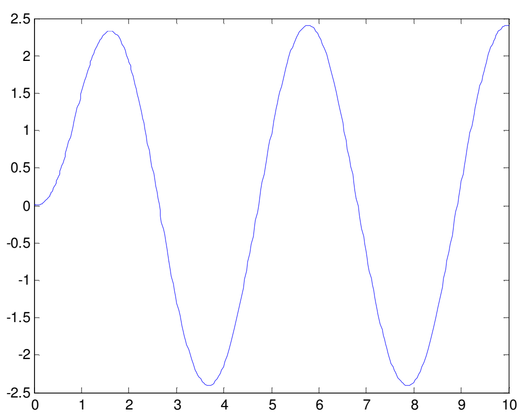

Figure 3. First output of the closed-loop residual generator.

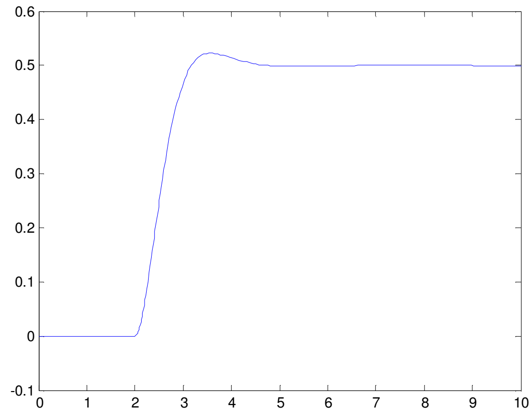

Figure 4. Second output of the closed-loop residual generator.

V. Conclusion

This paper presents a new closed-loop approach for fault detection and decoupling. The approach is valid for systems whose input matrix columns are linearly dependent on those of the fault and the disturbance. However, the fault input matrix and the disturbance input matrix are assumed to be linearly independent. The approach allows the decoupling of the faults and disturbances using a closed-loop residual generator. Thus, we are able to achieve fault diagnosis as well as detection in addition to disturbance detection. Although not explored in this paper, it is possible to design robust and fault tolerant systems that will adapt differently to different faults and can better reject disturbances. These new research directions will be the subject of future research.

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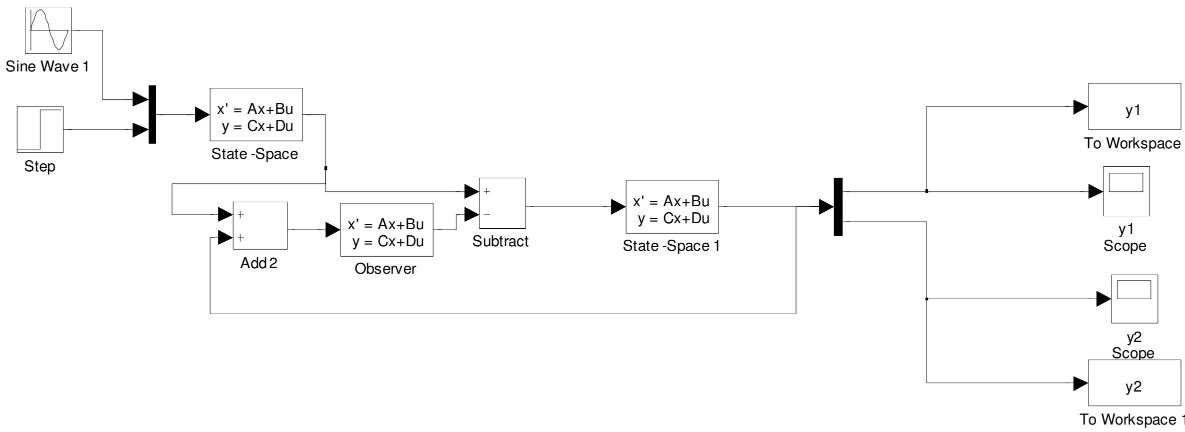

Figure 5. Simulation diagram for the closed-loop residual generator.