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A Fault Diagnosis Scheme for Time-varying Fault Using Output Probability Density Estimation

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***Abstract*—In this paper, a fault diagnosis scheme for a class of time-varying faults using output probability density estimation is presented. The system studied is a nonlinear system with time delays. The measured output is viewed as a stochastic process and its probability density function (PDF) is modeled, which leads to a deterministic dynamical model including nonlinearities, uncertainties. The fault considered in this paper is time-varying, piecewise continuous with finite discontinuous points. A new adaptive fault diagnosis algorithm is proposed. An ideal estimation of the fault and its modified form are analyzed. Simulation example is given to demonstrate the effectiveness of the proposed approaches.**

***Keywords*—Fault detection and diagnosis; time delays; observer design; B-spline expansions; probability density function (PDF).**

I. INTRODUCTION

As an important aspect for practical processes, such as large-scale chemical engineering processes, oil refining processes and aeronautical system processes, the safety and reliability problem of control systems has long been investigated [1-26]. For stochastic systems, the standard methodologies of the fault detection and diagnosis (FDD) or fault tolerant control (FTC) mainly include filter- or observer-based approaches, identification-based approaches, *et al*. [3-4], [17-20], [24]. Generally, the filter- or observer-based approaches suit systems with unknown input while identification-based approaches suit systems with unknown parameters [3-4], [17-20], [24] or their unpredictable change [29]. Up to now, most approaches concentrate on Gaussian systems. In fact, some processes exhibit asymmetric non-Gaussian distributions [17], the expectation/variance of the traditional Kalman filtering approach is obviously insufficient for characterizing such processes and hence the probability density functions (PDF) approach is needed. PDF approach is actually a shape control method. To approximate a kind of distributions, one way is to use statistical approaches, such as Monte-Carlo or particle filter approaches, where the Bayesian lemma and the likelihood method are used [27- 28]. Another way is function or functional approach, such as spline approach [17-20], [24], where a B-spline expansions technique is used.

In references [17-20], [24], a kind of general stochastic systems was investigated, where an output PDF approach via B-splines expansion technique was proposed. The B-spline bases represent different parts of an output distribution. In other words, one can determine details of the whole output through such an approach. The virtue of output PDF approach is that it transfers a stochastic system to a deterministic dynamical system, and hence the corresponding stochastic problem is transferred to a deterministic one .

Since the stochastic system can be modeled as a deterministic one by using PDF approach, the conventional linear or nonlinear filter can tackle the corresponding FDD or FTC problem. Wang and Lin [17] presented a linear spline functional approach and a fault detection threshold. Guo *et al*. [19, 20, 24] put up a nonlinear functional approach named square root B-spline functional approach to further investigate FDD and FTC filter problems, where time delays, nonlinearity and modeling uncertainties are considered and some optimization performances such as H_2 or H_∞ are applied [19], [24]. Their filter of FDD and FTC scheme consists of two parts, one is conventional systems filter and the other is fault filter. The fault filter is designed to perform faults mapping and faults measurement, which is the key of FDD and FTC schemes. A drawback of those algorithms is that only constant unknown input is considered.

This paper will continue our research on fault diagnosis to address a kind of time-varying faults. As a popular tool, linear matrix inequality (LMI) is used as a numerical method for its computing convenience. The remainder of this paper is organized as follows: In Section 2, some preliminaries on output PDF and related nonlinear system are introduced. The fault diagnosis problem is investigated in Section 3 where an adaptive filtering algorithm is provided. A fault tolerant controller is presented in Section 4 and to simulation examples show the effectiveness of the proposed methods in section 5.

II. SYSTEM DESCRIPTION

Consider a dynamical system

$$\dot{x}(t) = Ax(t) + A_d x(t-d) + Gg(x(t)) + Hu(t) + JF(t), \quad (1)$$

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$$y(t)=f\left(x(t),x(t-d),u(t),\xi(t),F(t)\right), \quad (2)$$

where $x(t)\in R^n$ is the system state, $u(t)\in R^m$ is the control input, $y(t)\in R$ is the measured output, $\xi(t)$ is the measurement noise or exogenous disturbance, $F(t)$ is the fault to be detected and diagnosed. *A, A_d, G, H* and *J* are constant matrices of appropriate dimensions. $g(x(t))$ is a continuous nonlinear function satisfying Lipschitz condition, that is, there exists known matrix U_1 such that

$$\left\|g(x_1(t))-g(x_2(t))\right\|\le\left\|U_1(x_1(t)-x_2(t))\right\| \quad (3)$$

for any $x_1(t)$ and $x_2(t)$.

The fault free system of (1) and (2) is a bounded input bounded output stable system. Assume that the measured output satisfies $y(t)\in[a,b]$. Based on the statistical information of sample data, the distribution function of the output sample can be obtained, and the corresponding probability density function (PDF) can be further studied. Of course, the output distribution law is usually complicated, which often results in the complexity of the output PDF. To obtain the output PDF, the B-spline approximation technique is often used. Under conditions of $u(t)$, $\xi(t)$ and $F(t)$,

$$P\{a\le y(t)<\xi\}=\int_a^{\xi}p(z,u,F)\,dz$$

defines a conditional probability on $[a,b]$, where $p(z,u(t),F)\ge 0$ is the corresponding conditional PDF. Let $\gamma(t,z,u,F)=\sqrt{p(t,z,u,F)}$, then $p(t,z,u,F)\ge 0$ is always true. We assume the model

$$\gamma(t,z,u,F)=\sum_{i=1}^{q}\beta_i(u,F)\phi_i(z)+\omega(z,u,F), \quad (4)$$

where $\phi_i(z)$, $i=1,2,\cdots,q$ are pre-specified basis functions $\beta_i(u,F)$, $i=1,2,\cdots,q$ are corresponding weighting functions. Assume that the model error ω satisfies $\left|\omega(z,u(t),F)\right|\le\delta$ for all $\{z,u(t),F\}$, where δ is a known positive constant. Due to

$$P\{a\le y(t)\le b\}=1 \;\Rightarrow \int_a^b\gamma^2(z,u,F)\,dz=1,$$

$b_i(z)$ $i=1,2,\cdots,q$ are not independent. Assuming that $\phi_q(z)$ can be described by $\phi_i(z)$ $i=1,2,\cdots,q-1$, $\beta_q(u,F)=h(\beta(t))$ is a function of $\beta_i(u,F)$ $i=1,2,\cdots,q-1$, that is

$$h(\beta(t))=\Lambda_3^{-1}\left(-\Lambda_2\beta(t)+\sqrt{\Lambda_3-\beta^T(t)\Lambda_0\beta(t)}\right) \quad (5)$$

where $\beta(t)=\left[\beta_1(u,F)\quad \beta_2(u,F)\quad\cdots\quad\beta_{q-1}(u,F)\right]^T$,

$$\Phi(z)=\left[\phi_1(z)\quad\phi_2(z)\quad\cdots\quad\phi_{q-1}(z)\right]^T,$$

$$\Lambda_1=\int_a^b\Phi(z)\Phi^T(z)\,dz,\ \Lambda_2=\int_a^b\Phi^T(z)\phi_q(z)\,dz,$$

$$\Lambda_3=\int_a^b\phi_q^2(z)\,dz\ne 0,\quad \Lambda_0=\Lambda_1\Lambda_3-\Lambda_2^T\Lambda_2.$$

Obviously, $h(\beta(t))$ satisfies Lipschitz condition, that is, there exists matrix U_2 such that

$$\left\|h(\beta_1(t))-h(\beta_2(t))\right\|\le\left\|U_2(\beta_1(t)-\beta_2(t))\right\| \quad (6)$$

for any $\beta_1(t)$ and $\beta_2(t)$. If $\beta(t)$ is known, the output PDF model is set up. We assume the model

$$\beta(t)=Ex(t), \quad (7)$$

where *E* is a known matrix. Let

$$\overline{\beta}(t)=\left[\beta^T(t)\quad\beta_q(t)\right]^T,\ \overline{\Phi}(z)=\left[\Phi^T(z)\quad\phi_q(z)\right]^T,$$

equation (4) can be further written as

$$\gamma(z,u,F)=\overline{\Phi}^T(z)\overline{\beta}(t)+\omega(z,u,F). \quad (8)$$

The deterministic equations (1), (7) and (8) describe a stochastic process, where (7) and (8) characterize the probability feature of the output.

As mentioned in section 1, only constant faults have been restricted in previous works [19-20], [24]. We will focus on a kind of time-varying faults in the following.

***Assumption 1*:** The following assumptions on fault are made in this paper

1) $F(t)=\left[F_1(t),F_2(t),\cdots,F_m(t)\right]^T$ ($t>0$) is piecewise continuous with finite discontinuous points at $t=t_i$, $0<t_1<t_2<\cdots<t_r$ and there exist $F(t_i)=F(t_i^+)=\lim_{t\to t_i^+}F(t)$ and $F(t_i^-)=\lim_{t\to t_i^-}F(t)$, $1\le i\le r$;

2) $F(t)$ ($t>0$) is differentiable except for $t=t_i, 1\le i\le r$ and its right-derivative $\dot{F}(t_i^+)$ exists at $t=t_i$, $1\le i\le r$, where

$$\dot{F}(t_i^+)=\lim_{\Delta t\to 0^+}\left(F(t_i+\Delta t)-F(t_i)\right)/\Delta t;$$

3) There exist scalars $M_1>0$ and $M_2>0$ such that $\left\|F(t)\right\|\le M_1$, $\left\|\dot{F}(t)\right\|\le M_2$ for $t>0$ and

$t \neq t_i$, $1 \leq i \leq r$ and $\left\| \dot{F}\left(t_i^+\right) \right\| \leq M_2$ for all $t = t_i$, $1 \leq i \leq r$.

Remark 1: In most present literature, the fault related is usually an unknown constant input or a bounded differentiable input with bounded derivative. Assumption 1 generalizes the case to the faults with discontinuities. In another aspect, the property of bounded derivative or right-derivative is very important, it follows that the filter of the fault in the following has bounded derivative or bounded right-derivative.

The system of (1), (7) and (8) being a deterministic formulation, the standard filter-based approach can be used to diagnose the fault. In the next section we will design an adaptive filter to estimate and to map the fault. Furthermore, we can obtain an ideal form of the fault by analyzing the fault dynamical equation.

III. Adaptive Fault Diagnosis

The following adaptive filter is applied

$$\dot{\hat{x}}(t) = A\hat{x}(t) + A_d\hat{x}(t-d) + Gg\left(\hat{x}(t)\right) + Hu(t) + L\varepsilon(t) + J\hat{F}(t), \quad (9)$$

$$\dot{\hat{F}}(t) = -\Upsilon_1\hat{F}(t) + \Upsilon_2\varepsilon(t), \quad (10)$$

$$\varepsilon(t) = \int_a^b \sigma(z)\left(\gamma(t,z,u,F) - \hat{\gamma}(t,z,u)\right)dz, \quad (11)$$

$$\hat{\gamma}(z,u) = \overline{\Phi}^T(z)\hat{\overline{\beta}}(t), \quad (12)$$

$$\hat{\beta}(t) = E\hat{x}(t) \quad (13)$$

where $\hat{x}(t)$ is the estimation of the state, $\hat{F}(t) = \left[\hat{F}_1(t), \cdots, \hat{F}_i(t), \cdots, \hat{F}_m(t)\right]^T$ is the estimation of $F(t)$, $L \in R^{n\times s}$ is the filter gain to be determined. Unlike the classical filtering methods, the residual signal $\varepsilon(t)$ is formulated as an integral of the difference between the measured PDF and the estimated one, where $\sigma(z) \in R$ is a pre-specified weighting vector defined on $[a,b]$. In fact, $\varepsilon(t)$ can be regarded as a generalized distance or difference of two PDFs. $\Upsilon_1 (>0)$ and Υ_2 are learning operators to be determined.

Let $\tilde{x}(t) = x(t) - \hat{x}(t)$, $g\left(\tilde{x}(t)\right) = g\left(x(t)\right) - g\left(\hat{x}(t)\right)$, $h\left(\tilde{x}(t)\right) = h\left(x(t)\right) - h\left(\hat{x}(t)\right)$, the estimation error system of (1) and (9) is formulated as (14),

$$\begin{aligned}\dot{\tilde{x}}(t) &= \left(A - L\Gamma_1\right)\tilde{x}(t) + A_d\tilde{x}(t-d) + Gg\left(\tilde{x}(t)\right) \\ &\quad - L\Gamma_2 - L\Gamma_2 h\left(E\tilde{x}(t)\right) - J\hat{F}(t) + JF(t) - L\Delta(t) \\ &:= \overline{A}X + JF(t) - L\Delta(t)\end{aligned} \quad (14)$$

where $\overline{A} = \left[A - L\Gamma_1 \quad A_d \quad -L\Gamma_2 \quad G \quad -J\right]$,

$$X^T = \left[\tilde{x}^T(t) \quad \tilde{x}^T(t-d) \quad h^T\left(E\tilde{x}(t)\right) \quad g^T\left(\tilde{x}(t)\right) \quad \hat{F}^T(t)\right],$$

and

$$\Gamma_1 = \int_a^b \sigma(z)\Phi^T(z)Edz, \quad \Gamma_2 = \int_a^b \sigma(z)\phi_q(z)dz,$$

$$\Delta(t) = \int_a^b \sigma(z)\omega(z,u,F)dz \quad (15)$$

It can be seen that

$$\varepsilon(t) = \Gamma_1\tilde{x}(t) + \Gamma_2 h\left(E\tilde{x}(t)\right) + \Delta(t). \quad (16)$$

Since $\left|\omega(z,u(t),F)\right| \leq \delta$, we have

$$\|\Delta(t)\| = \left\|\int_a^b \sigma(z)\omega(z,u,F)dz\right\| \leq \tilde{\delta}, \quad \tilde{\delta} = \delta\int_a^b \sigma(z)dz \quad (17)$$

Assume that $x(0)$ is known and $\tilde{x}(0)$ can be assumed to be zero or sufficiently small.

Theorem 1: The solution of error system (14) is bounded and satisfies $\left\|\left[\tilde{x}(t) \quad \hat{F}(t)\right]\right\|^2 \leq M$, if there exist matrices $P > 0$, $Q > 0$, R, L and constants $\mu > 0$, $\kappa > 0$ satisfying

$$\Psi = \begin{bmatrix} \Psi_{11} + \mu I & \Psi_{21}^T & \Psi_{31}^T \\ \Psi_{21} & -2\Upsilon_1 + \kappa I & 0 \\ \Psi_{31} & 0 & -I \end{bmatrix} < 0 \quad (18)$$

and $L = P^{-1}R$, for all $t \in [0, \infty)$, where $\tilde{\delta}$ is defined by (10), and

$$M = \max\{\mu^{-1}, \kappa^{-1}\}\left(\theta_1^{-2}\tilde{\delta}^2 + 2M_1\left\|\Upsilon_2\right\|\tilde{\delta} + \theta_2^{-2}\left\|J^TJ\right\|M_1^2\right),$$

$$\Psi_{11} = \begin{bmatrix} \Pi_1 & PA_d & -R\Gamma_2 & PG \\ A_d^TP & -Q & 0 & 0 \\ -\Gamma_2^TR^T & 0 & -\lambda_1^{-2}I & 0 \\ G^TP & 0 & 0 & -\lambda_2^{-2}I \end{bmatrix},$$

$$\Pi_1 = \left(PA - R\Gamma_1\right) + \left(PA - R\Gamma_1\right)^T + Q,$$

$$\Psi_{21} = \left[-J^TP + \Upsilon_2\Gamma_1 \quad 0 \quad \Upsilon_2\Gamma_2 \quad 0\right],$$

$$\Psi_{31} = \begin{bmatrix} \lambda_1^{-1} U_1 E & 0 & 0 & 0 \\ \lambda_2^{-1} U_2 & 0 & 0 & 0 \\ \theta_1 R^T & 0 & 0 & 0 \\ \theta_2 P & 0 & 0 & 0 \end{bmatrix},$$

for some λ_i, θ_i with $\lambda_i > 0, \ \theta_i > 0, i = 1,2$.

By Euler formula, for any sufficient small Δt, it follows from the fault filter (10) that

$$\hat{F}(t+\Delta t) = (I - \Upsilon_1 \Delta t)\hat{F}(t) + \Upsilon_2 \varepsilon(t)\Delta t.$$

Then,

$$\hat{F}(t+n\Delta t) = (I - \Upsilon_1 \Delta t)^n \hat{F}(t) + \sum_{i=0}^{n-1} (I - \Upsilon_1 \Delta t)^i \Upsilon_2 \varepsilon(t + (n-1-i)\Delta t)\Delta t.$$

With the diagnosis filter, we need $\hat{F}(t+n\Delta t) \to F(t)(\neq 0)$ and $\varepsilon(t+(n-1-i)\Delta t) \to \varepsilon(t)$ ($i = 0,1,\cdots,n-1$ and $\varepsilon(t)$ is small enough) when $\Delta t \to 0$. Then, for large enough t, we have

$$\hat{F}(t) = \Upsilon_1^{-1} \Upsilon_2 \varepsilon(t) \tag{19}$$

Equation (19) reflects the relationship between the residual signal $\varepsilon(t)$ and the fault estimation $\hat{F}(t)$. If the filter is so well designed that $\varepsilon(t)$ tends to zero when $t \to \infty$, $\|\Upsilon_1^{-1}\Upsilon_2\|$ should be sufficiently large for non-zero fault, which needs a sufficiently small solution of Υ_1. It means that a well effective diagnosis filter needs a small enough residual ε and a small enough learning rate Υ_1. Thus we have the following conclusion.

Theorem 2: Given the fault estimation $\dot{\hat{F}}(t) = -\Upsilon_1 \hat{F}(t) + \Upsilon_2 \varepsilon(t)$, where $\Upsilon_1 > 0$ and Υ_2 are designed learning rates, $\varepsilon(t)$ is the residual signal, then its ideal fault estimate is given by $\hat{F}(t) = \Upsilon_1^{-1} \Upsilon_2 \varepsilon(t)$.

Remark 2: We note that the ideal fault estimation $\hat{F}(t) = \Upsilon_1^{-1} \Upsilon_2 \varepsilon(t)$ is an analysis result under ideal modeling case, that is, the modeling is precise. The estimate precision depends on the learning rates Υ_1 and Υ_2. If the system is subject to process noise, exogenous disturbance and modeling error as usual, their effects may be amplified by $\Upsilon_1^{-1}\Upsilon_2$ through $\varepsilon(t)$.

Observe that in the residual formulation (16), the term $\Delta(t) = \int_a^b \sigma(z)\omega(z,u,F)dz$ relates to the modeling error $\omega(z,u,F)$, $\hat{F}(t) = \Upsilon_1^{-1}\Upsilon_2\varepsilon(t)$ is precise if and only if $\Delta(t) = 0$. Otherwise, when $\Delta(t) \neq 0$, the estimation should be modified

$$\hat{F}(t) = \Upsilon_1^{-1}\Upsilon_2 (\varepsilon(t) - \Delta(t)). \tag{20}$$

Theorem 3: Given the fault diagnosis filter (9-13), the fault estimate is given by $\hat{F}(t) = \Upsilon_1^{-1}\Upsilon_2(\varepsilon(t) - \Delta(t))$, where $0 < \Upsilon_1 \ll I$ and Υ_2 are determined by Theorem 1, $\varepsilon(t)$ is defined by (16) and $\Delta(t)$ is defined by (15).

IV. Simulation Example

A linear system on papermaking process is considered in reference [20],

$$\dot{x} = Ax + Hu + JF, \tag{21}$$

with output (3) and (6), where $u(t) = u_0(t) = Dx(t-2)$,

$$H = diag\{0.2, 0.2, \cdots, 0.2\} \in R^{9\times 9},$$

$$J = diag\{0 \ \ 0 \ \ 1.4 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 2.25 \ \ 0\}, \ D = E = I_9,$$

$$A = \begin{bmatrix} -0.83 & 0.25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.1 & -0.83 & 0.25 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.1 & -0.83 & 0.25 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.1 & -0.83 & 0.25 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.1 & -0.83 & 0.25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.1 & -0.83 & 0.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.1 & -0.83 & 0.25 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.1 & -0.83 & 0.25 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.1 & -0.83 \end{bmatrix}$$

$$\bar{\Phi}(z) = [\phi_1(z), \phi_2(z), \cdots, \phi_{10}(z)]^T,$$

$$\phi_i(z) = \exp\left(-(z - z_i)^2 \sigma_i^{-2}\right),$$

$$z_i = 0.003 + 0.006(i-1),$$
$$\sigma_i = 0.003,\ i = 1, 2, \cdots, 10.$$

It is assumed that the model is so precise that $|\omega(z,u,F)| \le 0.001$. Then $U_1 = 0.05 I_9 \in R^{9\times9}$ is reasonable due to (4). At the same time, $U_2 = 0 \in R^{9\times9}$ is natural for linear system. To diagnose the fault in (29), filter of (9)-(13) is adopted, where $\sigma(z) = 0.5$. The following results are obtained based on (15) and (17),

$$\Gamma_1 = [6.3\quad 7.5\quad 7.5\quad 7.5\quad 7.5\quad 7.5\quad 7.5\quad 7.4\quad 5.0]\times 10^{-3}$$
$$\Gamma_2 = 0.0063,\quad \Delta(t) \le 3\times10^{-5} = \tilde{\delta}.$$

In this case, the initial value of the filter is with $\hat{x}(0) \equiv 0 \in R^9$ for all $-2 \le t \le 0$ while the initial values of the plant (21) are

$$\begin{cases} x_1(t) = 2.05 + \exp(t-5) \\ x_2(t) = 1.05 + \exp(t-5) \\ x_5(t) = -2.05 + \exp(t-5) \quad (-2 \le t \le 0). \\ x_9(t) = 3.0 + \exp(t-5) \\ x_3 = x_4 = x_6 = x_7 = x_8 = 0 \end{cases}$$

In this example, the fault is assumed to occur at the 40th second with the values

$$F = \begin{cases} 0 & t < 40 \\ F_1 & 40 \le t < 120, \\ 1.3F & t \ge 120 \end{cases}$$

$$F_1 = [0, 0, 1.36 + 0.05\sin(\alpha t), 0, 0, 0, 0, 2.25 + 0.05\sin(\alpha t), 0]^T.$$

Choose $\lambda_1 = \lambda_2 = 0.55$, $\theta_1 = \theta_2 = 0.1$, by Theorem 1, we have the following results

$$L = [0.0058\quad 0.0081\quad 1.9074\quad -0.0042\quad -0.0010\quad -0.0068\quad -0.0576\quad 5.1125\quad 0.0229]^T,$$
$$\Upsilon_2 = [0\quad 0\quad 20.0609\quad 0\quad 0\quad 0\quad 0\quad 33.7806\quad 0]^T,$$
$$\mu = 4.4340\text{e-}004,\quad \kappa = 9.0085\text{e-}004,$$

$$\Upsilon_1 = \begin{bmatrix} 0.0097 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0097 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0172 & 0 & 0 & 0 & 0 & -0.0016 & 0 \\ 0 & 0 & 0 & 0.0097 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0097 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0097 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0097 & 0 & 0 \\ 0 & 0 & -0.0016 & 0 & 0 & 0 & 0 & 0.0157 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0097 \end{bmatrix}.$$

Figure 1 shows the 3-D plot of output probability information, there are 2 distinct changes in the 3-D plot at the 40th second and the 120th second, respectively. Figure 2 shows that the residual tends to zero vicinity rapidly. Figure 3 shows the fault and its estimate. From Figure 1 to Figure 3, it can be seen that the proposed fault diagnosis algorithm is effective for time-varying piecewise fault with finite discontinuous points.

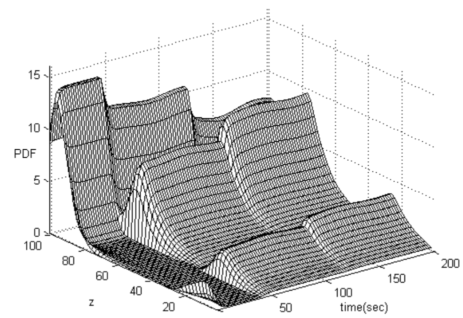

Fig.1 3-D Mesh Plot of Plant for FDD

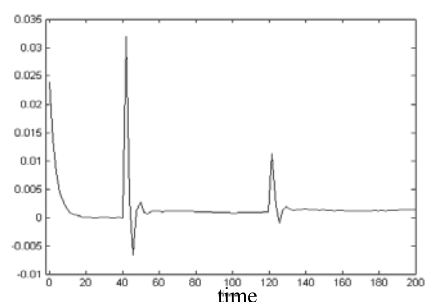

Fig.2 Residual response for FDD

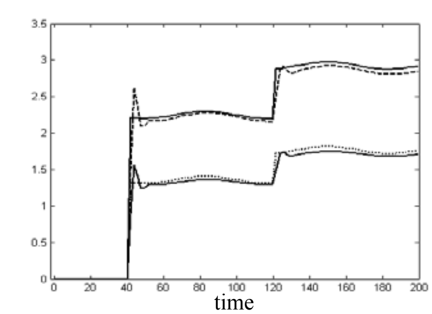

Fig.3 Fault and its Estimate

V. CONCLUSION

In this paper, a fault diagnosis scheme for a class of time-varying faults using output probability density estimation is presented. The system studied is a nonlinear system with time delays. The measured output is viewed as a stochastic process and its probability density function (PDF) is modeled, which leads to a deterministic dynamical model including nonlinearities, uncertainties. The fault considered in this paper is time-varying, piecewise continuous with finite discontinuous points. A new adaptive fault diagnosis algorithm is proposed. An ideal estimation of the fault and its modified form are analyzed. Simulation example is given to demonstrate the effectiveness of the proposed approaches.

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