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Simultaneous Design of Fault Tolerant Controller and Fault Detector for Linear Continuous-Time Systems with Actuator Outage Faults

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***Abstract*— The paper studies the problem of simultaneous design of fault tolerant controller and fault detector for a class of linear continuous-time systems with bounded disturbances and nonzero external constant inputs. A dynamic output feedback controller and a fault detector are designed simultaneously, where the output feedback controller stabilizes the closed-loop system for both normal and faulty cases and attenuates the effects of disturbances. By manipulating the steady-state values of the system states with the fault detector, a residual is then generated, through which the actuator outage faults can be detected effectively. An F-18 aircraft model is studied to illustrate the effectiveness of the proposed methods.**

I. INTRODUCTION

In the last decades, the well established H_∞ control theory has been widely used in fault detection, see [2][7][19][20][24] for example. In [17][18][28], the fault detection problem is formulated as an H_∞ fault detection filter design problem. And in [14], it is formulated as a mixed H_2/H_∞ filtering problem.

Most of the methods discussed above are considered using an open-loop model of the process, however, in many cases, the fault detection systems are closed-loop feedback systems. In such situation, faults may be covered by control actions and the early detection of process faults (low frequency faults) is clearly more difficult [6]. To solve this problem, the so-called integrated design of fault detection and control systems are proposed in the literature. In [10], the integrated approach to control and fault detection using the four parameter controller is proposed. In [5][15], the H_∞ optimization approach is used to minimize the fault estimation errors and attenuate the disturbance effects. In [4], an H_2 cost-function is introduced that involves traditional LQG cost terms and in addition fault estimation error component. In [13][25], the simultaneous fault detection and control problem is formulated as a mixed H_2/H_∞ optimization problem. In [23], the classical Youla parameterization technique is introduced. In [26], the authors make use of the extra degree of freedom in an H_∞-controller to optimize a diagnostic performance measured in H_2-norm.

In these papers, the magnitude of the fault should be big enough such that they can be detected, and actuator outage faults are not well investigated through these fault models. Note that it is quite common to include nonzero constant inputs in control systems, e.g., servo control systems with constant reference inputs [16][22]. For these systems, the steady-state values of their states do not converge to zero. This motivates the main approach proposed in this paper, using which the outage faults of actuators are detected effectively.

In this paper, the outage fault model are exploited [16][27], and new detection method is proposed to detect the actuator outage faults. Through satisfying certain performance indexes simultaneously, a dynamic output feedback controller and a detector are designed simultaneously to stabilize the closed-loop system with and without faults, attenuating the effects of disturbances and detecting the actuator outage faults. By the aid of the GKYP lemma proposed in [8], these performance indexes are converted to satisfy a set of inequalities, and through a two-step procedure, LMI conditions are then obtained. It should be pointed out that using the new methods proposed in this paper, the actuator outage faults can be detected effectively which is nontrivial in the literature.

Notation: For a matrix A, $A^T, A^\perp$ denote its transpose, and orthogonal complement, respectively. I denotes the identity matrix with an appropriate dimension. For a symmetric matrix A, $A > 0$ and $A < 0$ denote positive definiteness and negative definiteness, respectively. The Hermitian part of a square matrix M is denoted by $\mathrm{He}(M) := M + M^T$.The symbol $\star$ within a matrix represents the symmetric entries. $\sigma_{max}(G)$ and $\sigma_{min}(G)$ denote maximum and minimum singular values of the transfer matrix G, respectively.

II. PROBLEM FORMULATION

A. System model

Consider a linear time-invariant system model

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B_1 w(t) + B_2 u(t) + B_r r_0\\ z(t) &= C_1 x(t) + D_{12} u(t)\\ y(t) &= C_2 x(t) + D_{21} w(t)\end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input, $w(t) \in \mathbb{R}^{n_w}$ is the bounded disturbance input satisfying $w(t)^T w(t) \leq \bar{w}^2$, $r_0 \in \mathbb{R}^p$ is the external constant input,

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$z(t) \in \mathbb{R}^q$ denotes the controlled output, $y(t) \in \mathbb{R}^{n_y}$ denotes the measured output, and A, B_1, B_2, B_r, C_1, C_2, D_{12} and D_{21} are known constant matrices of appropriate dimensions.

Remark 1: The external input r_0 described in (1) is assumed to be a known constant, which is general [11], e.g., the reference input of a servo system [16][22].

B. Fault model

To formulate the simultaneous fault tolerant control and fault detection problem of this work, the following type fault model is adopted.

Definition 1 (Actuator outage fault): when outage faults occur, the input signals of systems are given by

$$u_{si}(t) = F_i u(t), i = 1, \ldots, N_s \tag{2}$$

where F_i's are diagonal matrices defined as

$$F_i = diag \begin{bmatrix} F_{i1} & F_{i2} & \ldots & F_{im} \end{bmatrix} \tag{3}$$

where $F_{ik} = 1$ if the kth actuator is fault free, and $F_{ik} = 0$ if the kth actuator is of outage.

C. Preliminaries

Consider the system model described by (1) with actuator outage faults given by (2), a full order dynamic output feedback controller $\mathcal{K}$ denoted as

$$\begin{aligned} \dot{x}_k(t) &= A_k x_k(t) + B_k y(t) \\ u(t) &= C_k x_k(t) \end{aligned} \tag{4}$$

is designed to stabilize the original system with or without faults. This yields the closed-loop system

$$\begin{aligned} \dot{\xi}(t) &= \bar{A}_n \xi(t) + \bar{B} w(t) + \bar{B}_n \\ z(t) &= \bar{C}_n \xi(t) \end{aligned} \tag{5}$$

for normal system model with $\xi(t)^T = \begin{bmatrix} x(t)^T & x_k(t)^T \end{bmatrix}$ and

$$\bar{A}_n = \begin{bmatrix} A & B_2 C_k \\ B_k C_2 & A_k \end{bmatrix}, \bar{B} = \begin{bmatrix} B_1 \\ B_k D_{21} \end{bmatrix}, \bar{B}_n = \begin{bmatrix} B_r r_0 \\ 0 \end{bmatrix},$$
$$\bar{C}_n = \begin{bmatrix} C_1 & D_{12} C_k \end{bmatrix}$$

and

$$\begin{aligned} \dot{\xi}(t) &= \bar{A}_{f_i} \xi(t) + \bar{B} w(t) + \bar{B}_{f_i} \\ z(t) &= \bar{C}_{f_i} \xi(t) \end{aligned} \tag{6}$$

for the ith fault model, with

$$\bar{A}_{f_i} = \begin{bmatrix} A & B_2 F_i C_k \\ B_k C_2 & A_k \end{bmatrix}, \bar{B}_{f_i} = \begin{bmatrix} B_r r_0 \\ 0 \end{bmatrix},$$
$$\bar{C}_{f_i} = \begin{bmatrix} C_1 & D_{12} F_i C_k \end{bmatrix}$$

The fault tolerant controller is designed as follows. Design a dynamic output feedback controller (4) such that the normal closed-loop system (5) and the faulty closed-loop system (6) for $i = 1, \ldots, N_s$ are all stable and satisfy the following performance indexes

$$\|G_{zw}(j\omega)\|_{peak} < \zeta_1 \tag{7}$$
$$\|G_{zwi}(j\omega)\|_{peak} < \zeta_3, \ i = 1, \ldots, N_s \tag{8}$$

where $G_{zw}(j\omega) = \bar{C}_n(j\omega I - \bar{A}_n)^{-1}\bar{B}, G_{zwi}(j\omega) = \bar{C}_{f_i}(j\omega I - \bar{A}_{f_i}))^{-1}\bar{B}$, and $\|G_{zw}\|_{peak}$ and $\|G_{zwi}\|_{peak}$ of are peak-to-peak gains of the transfer matrices from disturbance $w(t)$ to residual $z(t)$ for both the normal and faulty cases [21]. Solve the differential equations in (5) and (6) respectively, we have

$$\lim_{t\to\infty} \xi(t) = -\bar{A}_n^{-1}\bar{B}_n + \int_{t_0}^{\infty} e^{\bar{A}_n(t-\tau)}\bar{B}w(\tau)d\tau \tag{9}$$

for normal case, and

$$\lim_{t\to\infty} \xi(t) = -\bar{A}_{f_i}^{-1}\bar{B}_{f_i} + \int_{t_0}^{\infty} e^{\bar{A}_{f_i}(t-\tau)}\bar{B}w(\tau)d\tau \tag{10}$$

for the ith faulty model.

Then, the detector is designed as follows. Design a weighting matrix $V \in \mathbb{R}^{1\times n}$ such that the residual $r(t)$ is obtained as

$$r(t) = V x_k(t) = C_v \xi(t) \tag{11}$$

with $C_v = \begin{bmatrix} 0 & V \end{bmatrix}$.

Notice that for the normal system model

$$\begin{aligned} |C_v \lim_{t\to\infty} \xi(t)| &\le |C_v \bar{A}_n^{-1}\bar{B}_n| \\ &\quad + \lim_{t\to\infty} |C_v \int_{t_0}^{t} e^{\bar{A}_n(t-\tau)}\bar{B}w(\tau)d\tau| \\ &\le |C_v\bar{A}_n^{-1}\bar{B}_n| + \|G_{rw}\|_{peak}\bar{w} \end{aligned} \tag{12}$$

and for the ith faulty system model

$$\begin{aligned} |C_v \lim_{t\to\infty} \xi(t)| &\ge |C_v \bar{A}_{f_i}^{-1}\bar{B}_{f_i}| \\ &\quad - \lim_{t\to\infty} |C_v \int_{t_0}^{t} e^{\bar{A}_{f_i}(t-\tau)}\bar{B}w(\tau)d\tau| \\ &\ge |C_v\bar{A}_{f_i}^{-1}\bar{B}_{f_i}| - \|G_{rwi}\|_{peak}\bar{w} \end{aligned} \tag{13}$$

where $\|G_{rw}\|_{peak}$ and $\|G_{rwi}\|_{peak}$ of (12) and (13) are the peak-to-peak gains of the transfer matrices from disturbance $w(t)$ to residual $r(t)$ and

$$\begin{aligned} G_{rw}(j\omega) &= C_v(j\omega I - \bar{A}_n)^{-1}\bar{B}, \\ G_{rwi}(j\omega) &= C_v(j\omega I - \bar{A}_{f_i})^{-1}\bar{B} \end{aligned} \tag{14}$$

The bounds of the peak-to-peak gains are formulated as

$$\|G_{rw}(j\omega)\|_{peak} < \zeta_2 \tag{15}$$
$$\|G_{rwi}(j\omega)\|_{peak} < \zeta_4 \tag{16}$$

for both the normal and faulty cases.

To discriminate the normal and faulty system models, the following conditions should be satisfied

$$\begin{aligned} &|C_v\bar{A}_n^{-1}\bar{B}_n| + \|G_{rw}\|_{peak}\bar{w} + \|G_{rwi}\|_{peak}\bar{w} \\ &\qquad < |C_v\bar{A}_{f_i}^{-1}\bar{B}_{f_i}|, \ i = 1, \ldots, N_s \end{aligned} \tag{17}$$

which can be realized by satisfying the following performance indexes

$$\sigma_{max}(G_n(j\omega)) < \gamma_1, \text{ for } \omega = 0 \tag{18}$$
$$\sigma_{min}(G_{rf_i}(j\omega)) > \beta_1, \text{ for } \omega = 0, \ i = 1, \ldots, N_s \tag{19}$$
$$\|G_{rw}(j\omega)\|_{peak} < \zeta_2$$
$$\|G_{rwi}(j\omega)\|_{peak} < \zeta_4$$

where $G_n(j\omega) = C_v(j\omega I - \bar{A}_n)^{-1}\bar{B}_n, G_{rf_i}(j\omega) = C_v(j\omega I - \bar{A}_{f_i})^{-1}\bar{B}_{f_i}$.

D. Problem formulation

The problem of simultaneous design of fault tolerant controller and fault detector can be formulated as to find a dynamic output feedback controller $\mathcal{K}$ and a weighting matrix V such that the following conditions are satisfied

$$\|G_{zw}(j\omega)\|_{peak} < \zeta_1 \tag{20}$$
$$\|G_{rw}(j\omega)\|_{peak} < \zeta_2 \tag{21}$$
$$\|G_{zwi}(j\omega)\|_{peak} < \zeta_3,\ i = 1, \dots, N_s \tag{22}$$
$$\|G_{rwi}(j\omega)\|_{peak} < \zeta_4,\ i = 1, \dots, N_s \tag{23}$$
$$\sigma_{max}(G_n(j\omega)) < \gamma_1, \omega = 0 \tag{24}$$
$$\sigma_{min}(G_{rf_i}(j\omega)) > \beta_1, \omega = 0,\ i = 1, \dots, N_s \tag{25}$$

III. Simultaneous design of fault tolerant controller and fault detector

A. Usefull lemmas

The following lemmas are essential for later developments.

Lemma 1: (Generalized KYP Lemma [9]) Given system matrices (A, B, C, D), and a symmetric matrix Π, the following statements are equivalent:

i) The finite frequency inequality

$$\begin{bmatrix} G(j\omega) & I \end{bmatrix} \Pi \begin{bmatrix} G(j\omega)^T \\ I \end{bmatrix} < 0, \text{ for all } |\omega| \le \varpi \tag{26}$$

where $G(j\omega) = C(j\omega I - A)^{-1}B + D$.

ii) There exist matrices P, Q satisfying $Q > 0$, and

$$\begin{bmatrix} A & I \\ C & 0 \end{bmatrix} \begin{bmatrix} -Q & P \\ P & \varpi^2 Q \end{bmatrix} \begin{bmatrix} A & I \\ C & 0 \end{bmatrix}^T + \begin{bmatrix} B & 0 \\ D & I \end{bmatrix} \Pi \begin{bmatrix} B & 0 \\ D & I \end{bmatrix}^T < 0 \tag{27}$$

Lemma 2: (*Projection Lemma* [3]) Let Γ, Λ, Θ be given. There exists a matrix F satisfying $\Gamma F \Lambda + (\Gamma F \Lambda)^T + \Theta < 0$ if and only if the following two conditions hold

$$\Gamma^{\perp} \Theta \Gamma^{\perp^T} < 0, \quad \Lambda^{T^{\perp}} \Theta \Lambda^{T^{\perp^T}} < 0$$

The following lemma provides an alternative condition to (27) by introducing a multiplier R through the projection lemma, which is similar to that of [9]. Firstly, define $J \in \mathbb{R}^{(2n+n_z)}, \bar{H} \in \mathbb{R}^{(2n+n_z)\times(n_w+n_z)}$, and $\bar{L} \in \mathbb{R}^{(2n+n_z)\times n}$ as

$$J := \begin{bmatrix} I & 0 \\ 0 & I \\ 0 & 0 \end{bmatrix}, \bar{H} := \begin{bmatrix} 0 & 0 \\ B & 0 \\ 0 & I \end{bmatrix}, \bar{L} := \begin{bmatrix} -I \\ A \\ C \end{bmatrix}$$

Lemma 3: Let symmetric matrices $P, Q \in \mathbb{R}^{n\times n}$ and $Q > 0, R \in \mathbb{R}^{n\times(2n+n_z)}$. Let N_R be the null space of R. The following statements are equivalent:

i) The condition in (27) holds and

$$N_R^T (J \begin{bmatrix} -Q & P \\ P & \varpi^2 Q \end{bmatrix} J^T + \bar{H}\Pi\bar{H}^T) N_R < 0 \tag{28}$$

ii) There exists a variable matrix $X \in \mathbb{R}^{n\times n}$ such that

$$J \begin{bmatrix} -Q & P \\ P & \varpi^2 Q \end{bmatrix} J^T + \bar{H}\Pi\bar{H}^T < \text{He}(\bar{L}XR) \tag{29}$$

Proof: Notice that the null space of $\bar{L}$ is $\begin{bmatrix} A & I & 0 \\ C & 0 & I \end{bmatrix}$, and using Lemma 2, we have that ii) is equivalent to i). ■

B. Conditions for normal case

a) Conditions for performance index (24)

Firstly, consider the normal system model, combining (5) and (11), we have

$$\begin{aligned} \dot{\xi}(t) &= \bar{A}_n \xi(t) + \bar{B}w(t) + \bar{B}_n \\ z(t) &= \bar{C}_n \xi(t) \\ r(t) &= C_v \xi(t) \end{aligned} \tag{30}$$

The following Lemma 4 is essential for Theorem 1.

Lemma 4: Given the same matrices $\bar{A}_n, \bar{B}_n, C_v$ as stated in (30), the following statements are equivalent:

i) There exist matrix variables $P_1, Q_1,\ X = \begin{bmatrix} X_{11} & X_{12} \\ \star & X_{22} \end{bmatrix}$ and positive scalar γ_1 such that

$$\begin{bmatrix} -Q_1 & P_1 & 0 \\ \star & \bar{B}_n\bar{B}_n^T & 0 \\ \star & \star & -\gamma_1^2 \end{bmatrix} < \text{He}(\begin{bmatrix} -I \\ \bar{A}_n \\ C_v \end{bmatrix} X \begin{bmatrix} 0 \\ -I \\ 0 \end{bmatrix}) \tag{31}$$

holds, where $\bar{A}_n = \begin{bmatrix} A & B_2C_k \\ B_kC_2 & A_k \end{bmatrix}$.

ii) There exist matrix variables $P_{a1}, Q_{a1},\ X_a = \begin{bmatrix} Y & -N \\ -N & N \end{bmatrix}$ and positive scalar γ_1 such that

$$\begin{bmatrix} -Q_{a1} & P_{a1} & 0 \\ \star & \bar{B}_n\bar{B}_n^T & 0 \\ \star & \star & -\gamma_1^2 \end{bmatrix} < \text{He}(\begin{bmatrix} -I \\ \bar{A}_{an} \\ C_v \end{bmatrix} X_a \begin{bmatrix} 0 \\ -I \\ 0 \end{bmatrix}^T) \tag{32}$$

holds, where $\bar{A}_{an} = \begin{bmatrix} A & B_2C_{ke} \\ B_{ke}C_2 & A_{ke} \end{bmatrix}$, and $A_{ke} = X_{12}X_{22}^{-1}A_kX_{22}X_{12}^{-1},\ B_{ke} = -X_{12}X_{22}^{-1}B_k,\ C_{ke} = -C_kX_{22}X_{12}^{-1}$.

Theorem 1: Consider the normal system model (30), let real matrices $\bar{A}_n \in \mathbb{R}^{2n\times 2n}$, $\bar{B}_n \in \mathbb{R}^{2n\times 1}$, $C_v \in \mathbb{R}^{1\times 2n}$, a symmetric matrix $\Pi_1 = \begin{bmatrix} 1 & 0 \\ 0 & -\gamma_1^2 \end{bmatrix}$ be given. Then, the inequality condition

$$\sigma_{max}(G_n(j\omega)) < \gamma_1, \text{ for } \omega = 0 \tag{33}$$

holds if there exist matrix variables $Y, N, \mathcal{A}, \mathcal{C}, \mathcal{V}$, symmtric matrices $P_1 = \begin{bmatrix} P_{11} & P_{12} \\ \star & P_{22} \end{bmatrix}$, $Q_1 = \begin{bmatrix} Q_{11} & Q_{12} \\ \star & Q_{22} \end{bmatrix}$ satisfying $Q_1 > 0$, and

$$\begin{bmatrix} -Q_{11} & -Q_{12} & P_{11} - Y & P_{12} + N & 0 \\ \star & -Q_{22} & P_{12}^T + N & P_{22} - N & 0 \\ \star & \star & \Phi_1 & \Phi_2 & -\mathcal{V}^* \\ \star & \star & \star & \Phi_3 & \mathcal{V}^* \\ \star & \star & \star & \star & -\gamma_1^2 \end{bmatrix} < 0 \tag{34}$$

where

$$\begin{aligned} &\Phi_1 = (B_rr_0)(B_rr_0)^T + AY - B_2\mathcal{C} + (AY - B_2\mathcal{C})^T, \\ &\Phi_2 = -AN + B_2\mathcal{C} + (B_kC_2Y - \mathcal{A})^T, \\ &\Phi_3 = -B_kC_2N + \mathcal{A} + (-B_kC_2N + \mathcal{A})^T \\ &\mathcal{A} = A_kN, \mathcal{C} = C_kN, \mathcal{V} = VN \end{aligned}$$

Proof: Note that for system (30), given $\Pi_1 = \begin{bmatrix}1 & 0\\ 0 & -\gamma_1^2\end{bmatrix}$ and let $\varpi = 0$, from Lemma 1, condition (26) becomes

$$G_n(j\omega)G_n(j\omega)^T < \gamma_1^2, \text{ for } \omega = 0 \tag{35}$$

which is equivalent to (33).

Then applying Lemma 1 and Lemma 3, it can be concluded that inequality

$$J\begin{bmatrix}-Q_1 & P_1\\ P_1 & 0\end{bmatrix}J^T + \begin{bmatrix}0 & 0\\ \bar{B}_n & 0\\ 0 & I\end{bmatrix}\Pi_1\begin{bmatrix}0 & 0\\ \bar{B}_n & 0\\ 0 & I\end{bmatrix}^T < \mathrm{He}(\begin{bmatrix}-I\\ \bar{A}_n\\ C_v\end{bmatrix}XR) \tag{36}$$

provides a sufficient condition for performance index (33).

Let $R = \begin{bmatrix}0 & -I & 0\end{bmatrix}$, after some matrix manipulation, (36) becomes

$$\begin{bmatrix}-Q_1 & P_1 & 0\\ \star & \bar{B}_n\bar{B}_n^T & 0\\ \star & \star & -\gamma_1^2\end{bmatrix} < \mathrm{He}(\begin{bmatrix}-I\\ \bar{A}_n\\ C_v\end{bmatrix}X\begin{bmatrix}0\\ -I\\ 0\end{bmatrix}^T) \tag{37}$$

From Lemma 4, choose $X = \begin{bmatrix}Y & -N\\ -N & N\end{bmatrix}$ without introducing any conservatism. Define $Q_1 := \begin{bmatrix}Q_{11} & Q_{12}\\ \star & Q_{22}\end{bmatrix}$, $P_1 := \begin{bmatrix}P_{11} & P_{12}\\ \star & P_{22}\end{bmatrix}$, after some matrix manipulation, (37) becomes (34), which completes the proof. ■

C. Conditions for faulty cases

a) Conditions for performance index (25)

Consider the ith faulty case, combine (6) and (11), we have

$$\begin{aligned}\dot{\xi}(t) &= \bar{A}_{f_i}\xi(t) + \bar{B}w(t) + \bar{B}_{f_i}\\ z(t) &= \bar{C}_{f_i}\xi(t)\\ r(t) &= C_v\xi(t)\end{aligned} \tag{38}$$

To satisfy condition (25), we have the following theorem.

Theorem 2: Consider the ith faulty system model (38), let real matrices $\bar{A}_{f_i} \in \mathbb{R}^{2n\times 2n}$, $\bar{B}_{f_i} \in \mathbb{R}^{2n\times 1}$, $C_v \in \mathbb{R}^{1\times 2n}$, a symmetric matrix $\Pi_2 = \begin{bmatrix}-1 & 0\\ 0 & \beta_1^2\end{bmatrix}$ be given. Then, the inequality condition

$$\sigma_{min}(G_{rf_i}(j\omega)) > \beta_1, \text{ for } \omega = 0 \tag{39}$$

holds if there exist matrix variables $Y, N, \mathcal{A}, \mathcal{C}, \mathcal{V}$, symmetric matrices $P_2^i = \begin{bmatrix}P_{11}^i & P_{12}^i\\ \star & P_{22}^i\end{bmatrix}$, $Q_2^i = \begin{bmatrix}Q_{11}^i & Q_{12}^i\\ \star & Q_{22}^i\end{bmatrix}$ satisfying $Q_2^i > 0$, and

$$\begin{bmatrix}-Q_{11}^i & -Q_{12}^i & P_{11}^i - Y & P_{12}^i + N & -Y\ell\\ \star & -Q_{22}^i & P_{12}^{i^T} + N & P_{22}^i - N & 0\\ \star & \star & \Upsilon_1^i & \Upsilon_2^i & \Upsilon_4^i\\ \star & \star & \star & \Upsilon_3 & \Upsilon_5\\ \star & \star & \star & \star & \Upsilon_6\end{bmatrix} < 0 \tag{40}$$

with $\Upsilon_1^i = -\bar{B}_{f_i}\bar{B}_{f_i}^T + AY - B_2F_i\mathcal{C} + (AY - B_2F_i\mathcal{C})^T$, $\Upsilon_2^i = -AN + B_2F_i\mathcal{C} + (B_kC_2Y - \mathcal{A})^T$, $\Upsilon_3 = -B_kC_2N + \mathcal{A} + (-B_kC_2N + \mathcal{A})^T$, $\Upsilon_4^i = AY\ell - B_2F_i\mathcal{C}\ell - \mathcal{V}^T$, $\Upsilon_5 = B_kC_2Y\ell - \mathcal{A}\ell + \mathcal{V}^T$, $\Upsilon_6 = \beta_1^2 - \mathcal{V}\ell - (\mathcal{V}\ell)^T$, where $\mathcal{A} = A_kN$, $\mathcal{C} = C_kN$, $\mathcal{V} = VN$, and $\ell = \begin{bmatrix}\ell_1 & \dots & \ell_n\end{bmatrix}^T \in \mathbb{R}^{n\times 1}$ is a vector that should be determined beforehand.

Proof: Let $R = \begin{bmatrix}0 & 0 & -I & 0 & \ell\\ 0 & 0 & 0 & -I & 0\end{bmatrix}$, following the same lines of that for Theorem 1, it is immediate. ■

Remark 2: Note that vector ℓ in inequality (40) results from the projection lemma which should be determined beforehand, and it can be obtained through heuristic method.

b) Conditions for performance indexes (22) and (23)

In order to regulate the performance output $z(t)$ and attenuate the disturbance effects on the residual output, the peak-to-peak gain bounds of $G_{zwi}(j\omega)$ and $G_{rwi}(j\omega)$ are minimized. Then we have the following two lemmas.

Lemma 5: Consider system (38), the peak-to-peak gain of $G_{zwi}(j\omega)$ is bounded by

$$\|G_{zwi}(j\omega)\|_{peak} < \zeta_3$$

if there exist variables $Y, N, \mathcal{A}, \mathcal{C}, \mathcal{V}$, $\lambda_3 > 0, \mu_3$ and ζ_3 such that the following inequalities hold

$$\begin{bmatrix}\phi_{1_i} & \phi_{2_i} & B_1\\ \star & \phi_3 & B_kD_{21}\\ \star & \star & -\mu_3 I\end{bmatrix} < 0 \tag{41}$$

$$\begin{bmatrix}\lambda_3Y & -\lambda_3N & 0 & YC_1^T - \mathcal{C}^TF_iD_{12}^T\\ \star & \lambda_3N & 0 & -NC_1^T + \mathcal{C}^TF_iD_{12}^T\\ \star & \star & (\frac{\zeta_3}{\varpi} - \mu_3)I & 0\\ \star & \star & \star & \frac{\zeta_3}{\varpi}I\end{bmatrix} > 0 \tag{42}$$

where $\phi_{1_i} = AY - B_2F_i\mathcal{C} + (AY - B_2F_i\mathcal{C})^T + \lambda_3Y$, $\phi_{2_i} = -AN + B_2F_i\mathcal{C} + (B_kC_2Y - \mathcal{A})^T - \lambda_3N$, $\phi_3 = -B_kC_2N + \mathcal{A} + (-B_kC_2N + \mathcal{A})^T + \lambda_3N$, $\mathcal{A} = A_kN$, $\mathcal{C} = C_kN$, $\mathcal{V} = VN$.

Proof: Consider inequality conditions for the peak-to-peak gain of a transfer matrix, let the lyapunov variable matrix X be chosen to be $X = \begin{bmatrix}Y & -N\\ -N & N\end{bmatrix}$, the conclusion is immediate. ■

Lemma 6: Consider system (38), the peak-to-peak gain of $G_{rwi}(j\omega)$ is bounded by

$$\|G_{rwi}(j\omega)\|_{peak} < \zeta_4$$

if there exist variables $Y, N, \mathcal{A}, \mathcal{C}, \mathcal{V}$, $\lambda_4 > 0, \mu_4$ and ζ_4 such that the following inequalities hold

$$\begin{bmatrix}\phi_{4_i} & \phi_{5_i} & B_1\\ \star & \phi_6 & B_kD_{21}\\ \star & \star & -\mu_4 I\end{bmatrix} < 0 \tag{43}$$

$$\begin{bmatrix}\lambda_4Y & -\lambda_4N & 0 & -\mathcal{V}^T\\ \star & \lambda_4N & 0 & \mathcal{V}^T\\ \star & \star & (\frac{\zeta_4}{\varpi} - \mu_4)I & 0\\ \star & \star & \star & \frac{\zeta_4}{\varpi}I\end{bmatrix} > 0 \tag{44}$$

where $\phi_{4_i} = AY - B_2F_i\mathcal{C} + (AY - B_2F_i\mathcal{C})^T + \lambda_4 Y, \phi_{5_i} = -AN + B_2F_i\mathcal{C} + (B_kC_2Y - \mathcal{A})^T - \lambda_4 N, \phi_6 = -B_kC_2N + \mathcal{A} + (-B_kC_2N + \mathcal{A})^T + \lambda_4 N, \mathcal{A} = A_kN, \mathcal{C} = C_kN, \mathcal{V} = VN$.

Proof: By following the same lines of that stated in Lemma 5, it is immediate. ■

Remark 3: Let $F_i = I$ in Lemma 5 and Lemma 6 viz., let $F_i = I$ of LMIs (41)-(42), and (43)-(44), then the LMI conditions in Lemma 5 and Lemma 6 also provide sufficient conditions for performance indexes (20), (21), respectively.

D. Solutions of controller parameters and V

Till now, conditions for performance indexes (20)-(25) have been formulated. Summarily, we have the following theorem.

Theorem 3: Consider system model (1), there exists a dynamic output feedback controller (4) and a weighting matrix V such that the augmented system model (5), (6) with residual output $r(t)$ in (11) satisfying performance indexes (20)-(25) if inequality conditions (34), (40), and (41)-(44) for $i = 0, 1, \ldots, N_s$ hold where $F_0 = I$.

Proof: Combining Lemmas 5-6 and Theorems 1-2, it is immediate. ■

It should be pointed that conditions (34), (40), (41), and (43) are all nonlinear due to the product terms between controller parameter B_k and matrix variables Y, N, to solve this problem, a two step procedure is presented which follows from the idea proposed in [12], it can be summarized as follows:

1) Step 1: Initial solution of B_k: Consider system (1) regardless of the constant input r_0 as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1w(t) + B_2u(t) \\ z(t) &= C_1x(t) + D_{12}u(t) \\ y(t) &= C_2x(t) + D_{21}w(t) \end{aligned} \tag{45}$$

An initial solution of B_k can be obtained by solving the following LMIs (46)-(47)

$$\begin{bmatrix} \psi & QB_1 + ZD_{21} \\ \star & -\mu_1 I \end{bmatrix} < 0 \tag{46}$$

$$\begin{bmatrix} \lambda_1 Q & 0 & C_1^T \\ \star & (\frac{\zeta_1}{\bar{w}} - \mu_1)I & 0 \\ \star & \star & \frac{\zeta_1}{\bar{w}} I \end{bmatrix} > 0 \tag{47}$$

where $\psi = QA + ZC_2 + (QA + ZC_2)^T + \lambda_1 Q$, variables $\lambda_1, \mu_1, \zeta_1$ are scalars, and matrix variables $Q > 0, Z = QB_k$.

2) Step 2: Design of controller parameters and the weighting matrix: After B_k has been determined in Step 1, inequalities (34), (40), (41), and (43) for $i = 0, 1, \ldots, N_s$ become LMIs. Then, the output feedback controller parameters and the weighting matrix V can be obtained by solving LMI conditions (34), (40), (41), and (43) for $i = 0, 1, \ldots, N_s$ simultaneously which are

$$A_k = \mathcal{A}N^{-1}, C_k = \mathcal{C}N^{-1}, V = \mathcal{V}N^{-1}$$

Lemma 7: Conditions (46)-(47) are necessary conditions of (41)-(42) with $F_i = I$ for the fault free case, respectively.

Proof: The proof is omitted due to space limit. ■

Remark 4: As conditions (41)-(42) with $F_i = I$ are satisfied in Step 2, we have that it is feasible to obtain the initial solution of B_k by satisfying conditions (46)-(47)(necessary conditions of (41)-(42)).

IV. NUMERICAL EXAMPLE

Consider the following decoupled linearized longitudinal dynamical equation of motion of the F-18 aircraft model given in [1]

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1w(t) + B_2u(t) \\ y(t) &= C_2x(t) + D_{21}w(t) \end{aligned} \tag{48}$$

where $x = \begin{bmatrix} \alpha \\ q \end{bmatrix}$ denotes the angle of attack and the pitch rate, $u = \begin{bmatrix} \delta_E \\ \delta_{PTV} \end{bmatrix}$ denotes the symmetrical elevator position and the symmetric pitch thrust velocity nozzle position, $w(t)$ denotes the external disturbance input, satisfying $\|w(t)\| \le 0.5$, where $\|w(t)\| = \sqrt{w(t)^T w(t)}$. And system matrices

$$A = \begin{bmatrix} -1.175 & 0.9871 \\ -8.458 & -0.8776 \end{bmatrix}, B_2 = \begin{bmatrix} -0.194 & -0.03593 \\ -19.29 & -3.803 \end{bmatrix}$$

which are borrowed from [1], and the other matrices are assumed to be

$$B_1 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 2 \end{bmatrix}, D_{21} = 0.2$$

Let the output $y(t)$ track the reference input $r_0 = 0.5$, we get the tracking error $e(t) = r_0 - y(t)$. Define $\xi(t) = \begin{bmatrix} (\int_0^t e(\tau)d\tau)^T & x^T(t) \end{bmatrix}^T$. Using the new state variable $\xi(t)$, we have the following augmented system model

$$\begin{aligned} \dot{\xi}(t) &= \bar{A}\xi(t) + \bar{B}_1w(t) + \bar{B}_2u(t) + \bar{B}_r r_0 \\ z(t) &= C_1\xi(t) + D_{12}u(t) \\ y_f(t) &= \bar{C}_2\xi(t) + \bar{D}_{21}w(t) \\ \bar{A} &= \begin{bmatrix} 0 & -C_2 \\ 0 & A \end{bmatrix}, \bar{B}_1 = \begin{bmatrix} -D_{21} \\ B_1 \end{bmatrix}, \bar{B}_2 = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}, \\ \bar{B}_r &= \begin{bmatrix} I \\ 0 \end{bmatrix}, \bar{C}_2 = \begin{bmatrix} I & 0 \\ 0 & C_2 \end{bmatrix}, \bar{D}_{21} = \begin{bmatrix} 0 \\ D_{21} \end{bmatrix} \end{aligned}$$

and $y_f(t) = \begin{bmatrix} \int_0^t e(\tau)d\tau \\ y(t) \end{bmatrix}$ is to be used as the input of the dynamic output feedback controller. Firstly, the controller parameter B_k obtained using the method given in Step 1 of Section III is $B_k = \begin{bmatrix} -1.3118 & 1.0000 \\ 0.3024 & -0.5000 \\ -0.4613 & -1.0000 \end{bmatrix}$. Then, let $\ell = \begin{bmatrix} -2 & -2 & 1 \end{bmatrix}^T$ be chosen beforehand, the other controller parameters and the weighting matrix V obtained through Step 2 are

$$A_k = \begin{bmatrix} -1.4526 & -0.8808 & 0.5261 \\ 0.7739 & -1.4345 & -0.4016 \\ 5.6281 & -1.9517 & -7.5015 \end{bmatrix},$$

$$C_k = \begin{bmatrix} 0.2916 & 0.3039 & -0.2522 \\ 1.4770 & 1.5382 & -1.2775 \end{bmatrix},$$

$$V = \begin{bmatrix} -2.4185 & -1.1569 & 1.7330 \end{bmatrix}$$

then we get the residual output as

$$r(t) = \begin{bmatrix}-2.4185 & -1.1569 & 1.7330\end{bmatrix} x_k(t) \qquad (49)$$

the performance indexes are obtained as $\beta_1 = 0.5661$, $\gamma_1 = 0.3$, $\zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = 0.5$.

To illustrate the simulation results, assume that the disturbance $w(t) = 0.5sin(t)$. When actuator 1 is of outage at $t = 50s$, the residual output is shown in Fig. 1(a), and for the other case, when actuator 2 is of outage, the residual output is shown in Fig. 1(b).

The threshold for this example is chosen as $r_{th} = \sigma_{max}(G_n(j0)) + \|G_{rw}(j\omega)\|_{peak}\bar{w} = 0.3895$ which is denoted by the dashed line in Fig. 1(a,b). Then from Fig. 1(a-b), it can easily be formulated that by comparing the steady-state value of the residual output with the threshold, either actuator 1 or actuator 2 is of outage, it can be detected.

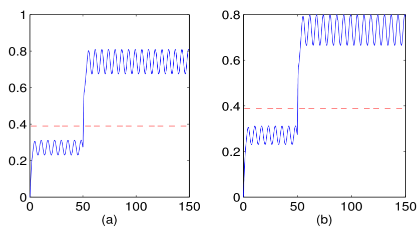

Fig. 1. Residual outputs $|r(t)|$ for different cases.

V. Conclusions

In this paper, the problem of simultaneous design of fault tolerant controller and fault detector for output feedback control systems with bounded disturbances and nonzero constant inputs has been investigated. The considered system models are modeled via multiple modes, namely, nominal case and faulty cases. The actuator outage faults are considered, which is nontrivil in the literature. The numerical example has illustrated the effectiveness of the proposed approach.

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