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Control System Design for Retinal Imaging Adaptive Optics Systems

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Abstract- **In this paper, the control problem in retinal imaging adaptive optics systems is generalized to that of shape control for a flexible membrane representing a deformable membrane mirror. Due to the dynamic nature of the aberrations in the eye, the shape control problem addressed is the tracking of an unknown and time-varying shape for a distributed membrane (i.e. desired shape of the mirror). The proposed controller design approach relies on constructing a *Q*-parameterized set of stabilizing controllers for the system under consideration and the online tuning of the *Q*-parameter so that the controller converges to the controller needed to achieve regulation. Partial decoupling of the multi-input multi-output closed loop system dynamics is introduced to allow the tuning to be performed based on decentralized adaptation algorithms.**

I. INTRODUCTION

In recent years, adaptive optics (AO) systems have been proposed as a means of providing early detection of eye diseases via retinal imaging. Originally developed to correct for optical atmospheric distortions in ground-based telescopes, AO systems are being used to obtain clear images of the cellular structures in the retina tissue in the back of the eye [1][2]. In a normal eye, the presence of aberrations, which are optical defects in the optical path of light inside the eye, leads to distorted and blurred images of the retina tissue. AO systems continuously measure aberrations introduced by the medium through which the light waves travel, and correct for those distortions automatically. The basic set-up of a typical retinal imaging AO system is shown in Figure 1. The main components of the system include a wavefront sensor (WS) for measuring the aberrations, a wavefront corrector (usually a deformable membrane mirror (DMM)), and a controller. A laser light source directs a plane wave[1] onto the retina of the eye. The wave passes through the eye and exits as an aberrated wavefront. The laser light reflects off the DMM and is detected by the WS.

Compensation is achieved through a closed-loop feedback controller, which receives signals from the WS in the form of wavefront measurements and outputs control commands to the DMM in the form of actuator signals. The actuators apply a spatially distributed force to the DMM which causes it to deform to the desired shape required to compensate for the aberrations present in the wavefront. The DMM compensates for the aberrations in the laser light, resulting in a planar wavefront. Sharp images of the retina can then be taken using a camera.

The main control system design problem in an AO system is that of shape control for a distributed membrane. The objective of shape control in an AO system is to obtain, for the membrane mirror under consideration, a shape that is as close as possible to the desired shape needed to cancel the wavefront aberrations present in the incoming wavefront. In order to increase the image quality that can be achieved by an AO system, the DMM must be able to track and compensate for unknown and time-varying changes in the aberrations of the human eye in real-time. The controller design is further complicated by the fact that a typical DMM is a coupled system. In such a system, each individual actuator input influences all of the outputs, the latter being the displacements of points on the mirror surface above the actuators.

Within the literature, a number of control design techniques have been proposed for the control of AO systems. The control strategies suggested in the literature include proportional-integral-derivative (PID), optimal and adaptive controllers. Traditionally, decentralized PID type controllers have been used in AO systems [3][4]. These controllers use the measured wavefront error above each actuator to control the displacement of the mirror above the actuators, with the objective of driving the wavefront error to zero. Typically, a SISO PID controller is first developed for a single actuator and then duplicated for each actuator in the system. Optimal centralized LQG, H_∞, and H_2 controllers [5][6] have been used to minimize the error between the actual shape and the desired shape of the mirror by choosing control inputs that minimize a specified performance criterion. A number of adaptive control algorithms based on different gradient decent type of algorithms have been developed for AO systems [7][8]. Most of these controllers use a centralized control architecture, which uses information from all the measured signals, making them computationally expensive and difficult to implement in real time systems.

In a typical retinal imaging AO system, the difficulty lies in attempting to track an unknown time-varying reference signal (i.e., desired shape of the mirror) for a coupled membrane. Since the desired shape of the mirror is unknown, it is proposed in this paper to tune the controller online to compensate for the lack of information on the dynamics of the time-varying aberrations in the eye. This is done iteratively, by taking advantage of the Q-parameterization of stabilizing controllers, so that the tuned

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[1] A plane wave is a flat or unaberrated wavefront.

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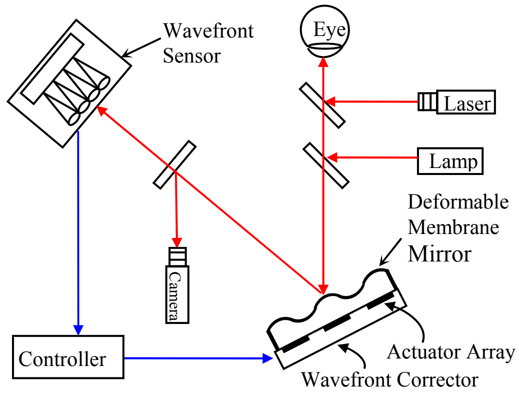

Fig. 1. Schematic diagram of a typical retinal imaging AO system.

controller converges to the ideal controller. The proposed controller design approach relies on two main steps. The first step involves the construction of a parameterized set of stabilizing controllers. The main feature of the constructed set of stabilizing controllers is that appropriate decoupling is introduced in the dynamics of the resulting closed loop system. The proposed decoupling only concerns the part of the closed loop system dynamics that are involved in the expression of the performance variable as a function of the parameter estimation error. The second step in the controller design approach is to use adaptation algorithms to tune the Q-parameter in the expression for the parameterized stabilizing controller. The Q parameter adaptation allows the controller to converge to the controller needed to achieve regulation for the case of unknown and time-varying aberrations. The decoupling approach introduced in the construction of the parameterized set of stabilizing controllers allows the use of a decentralized adaptation approach, hence practically simplifying the controller implementation process and allowing adaptive regulation to be achieved.

The rest of the paper is organized as follows. The next section presents the model of the DMM used in this work. This is followed by the development of an offline controller designed using the Q-parameterization of stabilizing controllers. Next, adaptive regulation for the wavefront corrector is presented. The performance of the adaptive system is then illustrated using simulations results followed by the conclusion in the final section.

In the rest of the paper, $\Sigma:\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$ is used to refer to a state space realization of a system Σ, RH_∞ is used to denote the set of proper real-rational stable transfer matrices, $diag(.)$ refers to a diagonal transfer matrix. Proofs of some results are not included due to space limitation.

II. Modeling of the Wavefront Corrector

In this section, a model of a distributed parameter system representing the DMM will be presented.

1) *Membrane Model*

Consider a circular membrane extending over a domain D defined by $0<r<a$. The boundary of the domain is the circle S given by the equation $r=a$. The membrane is modeled as a two-dimensional system which, in the equilibrium position, lies in a plane. The inputs used to adjust the shape of the membrane are represented by n concentrated forces $F_1(t),\ldots,F_n(t)$ generated by n actuators acting on the membrane at locations $(r_1,\theta_1),\ldots,(r_n,\theta_n)$ expressed in polar coordinates. The outputs of interest are the displacements of points on the membrane surface at the locations $(r_1,\theta_1),\ldots,(r_n,\theta_n)$. The response of the membrane can be expressed in the polar coordinates r and θ as follows [9],

$$w(r,\theta,t)=\sum_{\tilde{n}=1}^{\infty}W_{0\tilde{n}}(r,\theta)\eta_{0\tilde{n}}(t)+\sum_{\tilde{m}=1}^{\infty}\sum_{\tilde{n}=1}^{\infty}W_{\tilde{m}\tilde{n}c}(r,\theta)\eta_{\tilde{m}\tilde{n}c}(t) \qquad (1)$$
$$+\sum_{\tilde{m}=1}^{\infty}\sum_{\tilde{n}=1}^{\infty}W_{\tilde{m}\tilde{n}s}(r,\theta)\eta_{\tilde{m}\tilde{n}s}(t),$$

where $W_{0\tilde{n}}(r,\theta)$, $W_{\tilde{m}\tilde{n}c}(r,\theta)$, and $W_{\tilde{m}\tilde{n}s}(r,\theta)$ are the mode shapes of the uniform membrane clamped at $r=a$ and $\eta_{0\tilde{n}}(t)$, $\eta_{\tilde{m}\tilde{n}c}(t)$, $\eta_{\tilde{m}\tilde{n}s}(t)$ are time dependent generalized coordinates associated with each normal mode. In general, a discrete time model of the membrane can be given in state-space form as follows:

$$\begin{aligned} x(k+1)&=Ax(k)+B_2u(k),\\ y(k)&=C_2x(k)+D_{22}u(k), \end{aligned} \qquad (2)$$

where $x\in\mathbb{R}^{n_x}$ is the state vector, $u\in\mathbb{R}^{n}$ is the vector of control inputs given by $u(k)=\left[F_1(k)\ldots F_n(k)\right]^T$, $y\in\mathbb{R}^{n}$ is the vector of membrane displacements and $D_{22}=0_{n\times n}$.

2) *Reference Shape Model*

The reference shape for the DMM can be viewed as a spatially distributed set of signals that are dependent on spatial variables as well as time. The DMM desired shape is represented by a vector where each entry represents the desired displacement at one of the n spatially distributed points with coordinates $(r_1,\theta_1),\ldots,(r_n,\theta_n)$ on the surface of the membrane. Each of the n points is located at the same position as one of the actuators. A key assumption in this paper is that each of the desired displacements representing the unknown desired shape of the membrane is given by

$$w_{r_i}(k)=A_{i0}+\sum_{j=1}^{\bar{n}}A_{ij}\sin(\omega_jk+\varphi_{ij})=\sum_{j=0}^{\bar{n}}\bar{w}_{ij}(k)\,,\, i=1,\ldots,n\,, \qquad (3)$$

with unknown amplitudes A_{ij}, frequencies ω_j, and phases φ_{ij}, $1\le i\le n$, $1\le j\le\bar{n}$. The resulting plant, Σ, can then be described using the following state space representation:

$$\Sigma:\begin{cases} x(k+1)=Ax(k)+B_1w(k)+B_2u(k)\\ e(k)=C_1x(k)+D_{11}w(k)+D_{12}u(k)\\ y(k)=C_2x(k)+D_{21}w(k)+D_{22}u(k)\end{cases} \qquad (4)$$

where $B_1 = 0$, $D_{11} = -I_{n\times n}$, D_{12}=$D_{22} = 0_{n\times n}$. In a typical AO system, the measurement $y(k)$ provided by the wavefront sensor is the same as the performance variable $e(k)$. In such a case, $D_{11} = D_{21} = -I_{n\times n}$. For the system described above, it is desired to design a controller that yields internal stability and regulation against the signal w_r with unknown amplitudes, frequencies and phases.

III. OFFLINE CONTROLLER DESIGN

In this section the development and analysis of a multivariable controller is discussed for the case where the desired shape of the mirror given by (3) is assumed known. The controller is designed using the Q-parameterization of stabilizing controllers. This ensures the design of a controller that yields internal stability of the closed-loop system while tracking the reference signals.

1) Parameterization of a Set of Stabilizing Controllers

The basic configuration of a Q-parameterized feedback system is shown in figure 2, where Σ is the plant, J a fixed block, and Q is the controller free parameter [10]. The control problem in this setup is to design the controller such that the closed-loop system is internally stable and the output e of the plant Σ is driven to zero asymptotically.

The two blocks Σ and J can be lumped together into a single block to form the transfer matrix $T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & 0 \end{bmatrix}$ such that,

$$\begin{bmatrix} e \\ \bar{r} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & 0 \end{bmatrix} \begin{bmatrix} w \\ s \end{bmatrix};\ T_{12} : \left[\begin{array}{c|c} A + B_2K & B_2 \\ \hline C_1 & 0 \end{array}\right] \quad (5)$$

where K is such that $A + B_2K$ is a stability matrix. Let $E(z)$ denote the Z transform of the performance variable e. The closed-loop system transfer function is given by

$$E(z) = \left[T_{11}(z) + T_{12}(z)Q(z)T_{21}(z)\right]W(z). \quad (6)$$

With $W(z)$ and $E(z)$ being n-dimensional vectors, where n is the number of actuators acting on the membrane, the transfer matrices $T_{11}(z)$, $T_{12}(z)$, and $T_{21}(z)$ are each of dimension $n \times n$.

2) Decoupling of the $T_{12}(z)Q(z)$ Transfer Matrix

To simplify the development and analysis of the adaptive regulation system to be presented later in the paper, it is proposed to design the systems $T_{12}(z)$ and $Q(z)$ in such a way that the resulting system given by $T_{12}(z)Q(z)$ is a diagonal transfer matrix. The main idea is to select the state feedback gain K that appears in the state space realization of T_{12} and the parameter Q such that decoupling takes place in $T_{12}(z)Q(z)$. Consider the subsystem Σ_{22} given by the following state space representation:

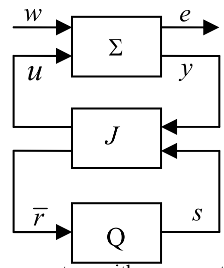

Fig 2. Closed-Loop system with a parameterized controller.

$$\Sigma_{22} : \begin{cases} x(k+1) = Ax(k) + B_2\bar{u}(k) \\ \bar{y}(k) = C_1x(k). \end{cases} \quad (7)$$

Define a state feedback law of the form:

$$\bar{u}(k) = Kx(k) + F\delta(k) \quad (8)$$

where $\delta(k)$ is an external signal. Substituting (8) into (7) yields a system with the following state space representation:

$$\Sigma_{22}^{cl} : \begin{cases} x(k+1) = (A + B_2K)x(k) + B_2F\delta(k), \\ \bar{y} = C_1x(k), \end{cases} \quad (9)$$

and a corresponding transfer function $C_1\left(zI - (A + B_2K)\right)^{-1} B_2F$. It is desired to find gains K and F such that the system (9) is decoupled, i.e. the transfer function $C_1\left(zI - (A + B_2K)\right)^{-1} B_2F$ is a diagonal transfer matrix. Let

$$C_1 = \begin{bmatrix} c_1^T \\ c_2^T \\ \vdots \\ c_n^T \end{bmatrix}, \quad (10)$$

and define the integers σ_i, $i = 1, 2, \ldots, n$, by

$$\sigma_i = \begin{cases} \min\left(j \mid c_i^T A^{j-1} B_2 \neq 0^T, j = 1, 2, \ldots, n-1\right) \\ n-1;\ \text{if } c_i^T A^{j-1} B_2 = 0^T, j = 1, 2, \ldots, n. \end{cases} \quad (11)$$

Based on the integers σ_i, $i = 1, 2, \ldots, n$, defined above, introduce the following two matrices:

$$B^* = \begin{bmatrix} c_1^T A^{\sigma_1 - 1} B_2 \\ c_2^T A^{\sigma_2 - 1} B_2 \\ \vdots \\ c_n^T A^{\sigma_n - 1} B_2 \end{bmatrix}, \quad C^* = \begin{bmatrix} c_1^T A^{\sigma_1} \\ c_2^T A^{\sigma_2} \\ \vdots \\ c_n^T A^{\sigma_n} \end{bmatrix} \quad (12)$$

Then we have the following result.

Theorem 2 [11]: There exists gains K and F that diagonally decouple the system (9) if and only if the matrix B^* in (12) is non-singular. If this is the case, then by choosing

$$F = \left(B^*\right)^{-1}, \qquad K = -\left(B^*\right)^{-1} C^*, \quad (13)$$

the resulting feedback system in (9) has a diagonal transfer function matrix given by

$$C_1\left(zI-\left(A+B_2K\right)\right)^{-1}B_2F = diag\left(z^{-\sigma_1}, z^{-\sigma_2}, \ldots, z^{-\sigma_n}\right). \quad \square$$

Lemma 1: Consider the system T_{12} in (5) and assume the conditions of Theorem 2 are satisfied. Let K and F be as in (13) and assume that $\left(A+B_2K\right)$ is a stability matrix. Let $Q(z) \in RH_\infty$ be a transfer matrix of the form:

$$Q(z) = F \times diag\left(Q_1(z), Q_2(z), \ldots, Q_n(z)\right).$$

It follows that the system $T_{12}(z)Q(z)$ is decoupled and we have that

$$T_{12}(z)Q(z) = diag\left(\frac{1}{z^{-\sigma_1}}Q_1(z), \frac{1}{z^{-\sigma_2}}Q_2(z), \ldots, \frac{1}{z^{-\sigma_n}}Q_n(z)\right).$$

□

The proof of Lemma 1 follows immediately from the results of Theorem 2.

3) Interpolation Condition for Regulation

Assume in the following that the conditions of Lemma 1 are satisfied and that $Q(z)$ is chosen as:

$$Q(z) = F\begin{bmatrix} Q_1(z) & & 0 \\ & \ddots & \\ 0 & & Q_n(z) \end{bmatrix}. \quad (14)$$

Then equation (6) can be rewritten as,

$$\begin{bmatrix} E_1(z) \\ \vdots \\ E_n(z) \end{bmatrix} = \underbrace{\begin{bmatrix} \sum_{j=1}^{n}\left[T_{11}^{1j}(z) + z^{-\sigma_1}Q_1(z)T_{21}^{1j}(z)\right]\frac{N_j(z)}{D_o(z)} \\ \vdots \\ \sum_{j=1}^{n}\left[T_{11}^{nj}(z) + z^{-\sigma_n}Q_n(z)T_{21}^{nj}(z)\right]\frac{N_j(z)}{D_o(z)} \end{bmatrix}}_{T(z)} \frac{D_o(z)}{D(z)}, \quad (15)$$

where $D_o(z)$ is a polynomial of the same order as $D(z)$ and with roots inside the unit circle, which yields a proper stable transfer function in (15). Let p_k, $k=1,\ldots,2\overline{n}$, denote the complex conjugate poles of $W_{r_i}(z)$, $i=1,\ldots,n$, and let $p_{n_p}=1$,where $n_p = 2\overline{n}+1$. All the poles of $W_{r_i}(z)$ are simple and located on the unit circle.

Lemma 2: Consider the closed loop system transfer function $T(z)$ in (15). Assume the conditions of Lemma 1 are satisfied, the gains K and F are chosen as in (13), $Q(z) \in RH_\infty$ is chosen as in (14), and $\left(A+B_2K\right)$ is a stability matrix. Define the following interpolation conditions

$$T(z)\big|_{z=p_k} = 0, \; k=1,\ldots,n_p. \quad (16)$$

Then regulation is achieved if and only if the interpolation conditions (16) are satisfied. □

It is impractical to search over the whole space for a function Q that satisfies the interpolation conditions (16). Instead, a special form of Q will be defined, based on a Ritz-type parameterization [12] that restricts the domain of the search. Each of the Q_i parameters in (14) is then expressed as

$$Q_i(z) = \sum_{j=1}^{n_q}\theta_{ij}\psi_{ij}(z), \; i=1,\ldots,n, \quad (17)$$

where $\psi_{ij}(z) = z^{1-j}$; $i=1,\ldots,n$, $j=1\ldots n_q$. As $n_q \to \infty$, (17) can be used to represent any transfer function in RH_∞. For each Q_i, define the following parameter vector:

$$\theta_i = \left[\theta_{i1}, \ldots, \theta_{in_q}\right]^T, \; i=1,\ldots,n. \quad (18)$$

Lemma 3: The interpolation conditions are equivalent to the following constraint on the parameter vectors θ_i, $i=1,\ldots,n$:

$$A_{\theta_i}\theta_i + B_{\theta_i} = 0, \quad (19)$$

where A_{θ_i} and B_{θ_i} are real matrices and A_{θ_i} is $\left(2\overline{n}+1\right)\times n_q$ and B_{θ_i} is $\left(2\overline{n}+1\right)\times 1$. □

Thus, solving equation (19) yields an offline-designed controller that achieves regulation based on the known desired shape for the DMM.

IV. Adaptive Regulation of the Wavefront Corrector

In this section, recursive algorithms are used in adaptive regulation to track reference signals of the form (3) with unknown and time-varying properties. In the adaptation process, θ is tuned online with no prior knowledge of the desired shape of the DMM.

In the following, it is assumed that the conditions of lemma 1 are satisfied. Consequently, there exists gains K and F such that with Q chosen as in (14), the system $T_{12}Q$ is diagonally decoupled. During adaptation, the subsystem represented by Q is time-varying. Therefore, systems will be considered operators on signals. Let q^{-1} denote the one time step delay operator. The performance variable is given by:

$$e(k) = T_{11}(q^{-1})w_r(k) + T_{12}(q^{-1})Q(q^{-1})T_{21}(q^{-1})w_r(k). \quad (20)$$

Let $Q^o(z) = F \times diag\left(Q_1^o(z), Q_2^o(z), \ldots, Q_n^o(z)\right)$ be a desired Q that yields regulation, where $Q_i^o(z) = \sum_{j=1}^{n_q}\theta_{ij}^o\psi_{ij}(z)$, $i=1,\ldots,n$. To simplify the analysis of the above system, modified error signals are defined as follows

$$\begin{aligned} \tilde{e}_i(k) &= \left[\sum_{j=1}^{n_q}\theta_{ij}(k)\psi_{ij}\left(q^{-1}\right) - \sum_{j=1}^{n_q}\theta_{ij}^o\psi_{ij}\left(q^{-1}\right)\right]q^{-\sigma_i}\overline{r}_i(k) + e_i^o(k) \\ &= \phi_i^T(k)\tilde{\theta}_i(k) + e_i^o(k), \; i=1,\ldots,n, \end{aligned} \quad (21)$$

where ,

$$\phi_i(k)=\begin{bmatrix}\psi_{i1}\left(q^{-1}\right)q^{-\sigma_i}\bar{r}_i(k)\\ \vdots \\ \psi_{in_q}\left(q^{-1}\right)q^{-\sigma_i}\bar{r}_i(k)\end{bmatrix},\ i=1,\ldots,n, \tag{22}$$

$$\bar{r}(k)=\left[\bar{r}_1(k),\ldots,\bar{r}_n(k)\right]^T=T_{21}\left(q^{-1}\right)w_r(k)$$

$$\tilde{\theta}_i(k)=\left[\left(\theta_{i1}(k)-\theta_{i1}^o\right)\ldots\left(\theta_{in_q}(k)-\theta_{in_q}^o\right)\right]^T,\ i=1,\ldots,n. \tag{23}$$

It should be noted that the signal $\bar{r}(k)$ represents the second output of the J block, and is therefore accessible. In the following, a decentralized recursive least squares (RLS) algorithm with dead zone is considered to tune the parameters of the systems Q_i. It is assumed that an upper bound, $\bar{n}^o$, on the number of sinusoids in the expression of the reference signals, w_{r_i}, $i=1,\ldots,n$, is known. In the design of the adaptation algorithm, the number of sinusoids in the reference signals w_{r_i}, $i=1,\ldots,n$, can be assumed to be arbitrarily large as long as it is greater than $\bar{n}^o$. To use the RLS algorithm with dead zone, it is necessary to define an upper bound on $\left|e_i^o(k)\right|$, $i=1,\ldots,n$. Let $\alpha_i>0$ and $0<\beta_i<1$, $i=1,\ldots,n$, be such that

$$\left|e_i^o(k)\right|<\alpha_i\beta_i^k,\ i=1,\ldots,n. \tag{24}$$

The scalars β_i, $i=1,\ldots,n$, can be found by examining the poles of $T_{11}(z)+T_{12}(z)Q(z)T_{21}(z)$. The quantities $\alpha_i>0$, $i=1,\ldots,n$, are assumed known a priori and can be determined based on the closed loop dynamics and the reference signal properties.

For $i=1,\ldots,n$, the adaptation algorithm is given by

$$\begin{aligned}\hat{\theta}_i(k+1)&=\hat{\theta}_i(k)+\lambda_i(k)\frac{P_i(k)\phi_i(k+1)}{1+\phi_i^T(k+1)P_i(k)\phi_i(k+1)}\tilde{e}_i(k+1),\\ P_i(k)&=P_i(k-1)-\frac{\lambda_i(k)P_i(k-1)\phi_i(k)\phi_i^T(k)P_i(k-1)}{1+\lambda_i(k)\phi_i^T(k)P_i(k-1)\phi_i(k)},\end{aligned} \tag{25}$$

with $P_i(0)>0$ and where

$$\lambda_i(k)=\begin{cases}1 & if\quad \left|\dfrac{\tilde{e}_i(k)}{1+\phi^T(k)P(k-1)\phi(k)}\right|>\left|\alpha_i\beta_i^k\right| \\ 0 & otherwise\end{cases}. \tag{26}$$

Based on the convergence results for a similar algorithm in[12], we have the following theorem.

Theorem 3: Assume the conditions of lemma 1 are satisfied such that K and F can be chosen as in (13) so that $\left(A+B_2K\right)$ is a stability matrix and with Q as in (14), the system $T_{12}Q$ is decoupled. Moreover, assume α_i and β_i, $i=1,\ldots,n$, and an upper bound, $\bar{n}^o$, on the number of sinusoids in the reference signals are all known a priori . Then the algorithm given by (25) and (26) yields:

$$\lim_{k\to\infty}\tilde{\theta}_i(k)=0;\ i=1,\ldots,n, \tag{27}$$

and the performance variables $e_i(k)$, $i=1,\ldots,n$, asymptotically converge to zero. □

V. SIMULATIONS

This section presents simulation results for a circular DMM in a retinal imaging AO system. The DMM has an aperture of 1 *cm* and is represented using the membrane model developed above by considering twelve modes, i.e. $\tilde{m}=\tilde{n}=3$ in (3). The surface shape of the mirror is adjusted using 35 microactuators, shown in figure 3. Wavefront measurements of the surface shape error are taken at points on the membrane surface corresponding to the locations of the actuators.

The simulations were carried out by using reference signals w_{r_i}, $i=1,\ldots,35$, as in (3), where each signal is represented as the sum of 2 sinusoids (i.e. $\bar{n}=2$) with varying amplitudes and phases, and with zero offset. The frequencies of the individual sinusoids range from 15 to 25 rads/s, the amplitudes of the sinusoids range from 1 to 14 μm and the phases range from 0 to 3.7 radians. The actuators are positioned on the DMM such that the C_2 and B_2 matrices in (2) are full rank and that $A+B_2K$ is a stability matrix.

1) Closed Loop System with the Offline Designed Controller

Assuming the properties of the reference signals are known a priori, and that $n_q=2\bar{n}$, the offline-designed controller is obtained by finding the unique Q that yields regulation. This Q-parameter is then used to drive the performance variable to zero. Simulations results for the closed loop system with the offline-designed controller are shown in Figure 4. It can be seen that the DMM is capable of tracking the desired shape required to compensate for the aberrations in the eye.

2) Adaptive System using the RLS algorithm with Dead Zone

In this case, the reference signals shown have unknown frequencies ω_j, amplitudes A_{ij}, and phases φ_{ij}, $i=1,\ldots,35; j=1,\ldots,2$. It is also assumed that $\bar{n}$ is unknown. An upper bound on the assumed number of sinusoids of $\bar{n}^o=10$ was chosen. In order to use the RLS algorithm with dead zone in (25) and (26) it is necessary to determine β_i and α_i, $i=1,\ldots,n$ in (24). Conservative values of $\beta_i=0.9$ and $\alpha_i=1$ were used during the simulations. Each of the Q_i parameters in (14) is such that $n_q=2\bar{n}^o=20$. The initial conditions of the algorithm were $\hat{\theta}_i(0)=0_{20\times1}$ and $P_i(0)=10^5\times I_{20\times20}$. The performance of the resulting adaptive closed loop system is shown in figure 5. The closed loop system was able to quickly track the reference inputs. Hence, as shown in the simulation results, regulation takes place even when the number of parameters

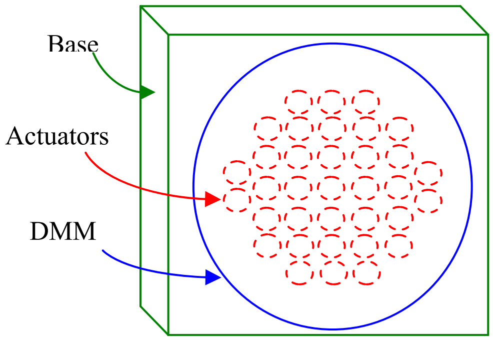

Fig. 3. Deformable membrane mirror and actuator arrangement underneath the mirror.

in Q_i, $i = 1,\ldots,n$, is larger than the minimal number of parameters needed to achieve regulation, in this case 4 parameters in each Q_i.

VI. Conclusion

The mirror control problem in AO systems is generalized as a shape control problem. The objective of shape control is to obtain, for the membrane under consideration, a shape that is as close as possible to the desired shape. The desired shape is the mirror shape that is needed in order to cancel the wavefront aberrations present in the incoming wavefront. Due to the dynamic nature of the wavefront aberrations in the eye and the desired shape of the mirror, the main control problem addressed is the tracking of an unknown shape for a distributed membrane. In order to effectively compensate for the unknown dynamic aberrations in the eye, the controller in an AO system must be tuned online in order to realize the controller needed to achieve regulation. A two-step controller design approach is proposed. The first step is to construct a parameterized set of stabilizing controllers for the system under consideration and to derive conditions on the *Q*-parameter in the controller expression to achieve regulation. Partial decoupling is introduced in the closed loop system dynamics to facilitate the design of the adaptive regulator. The second step is to use online tuning algorithms for the *Q*-parameter in the expression for the parameterized stabilizing controller. The *Q* parameter adaptation allows the controller to converge to the controller needed to achieve regulation. The proposed decentralized adaptive controller tuning algorithms are shown to effectively compensate for the unknown time-varying aberrations. This will allow for retinal images to be taken with improved resolution, hence facilitating the early detection of some eye diseases via retinal imaging.

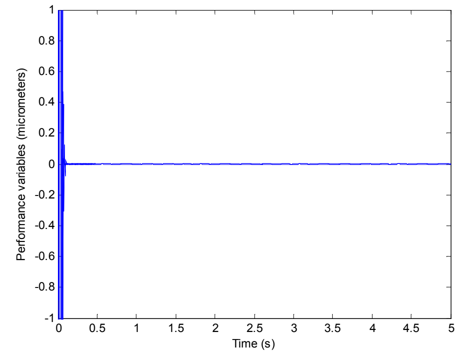

Fig. 4. The closed loop system with the offline-designed controller.

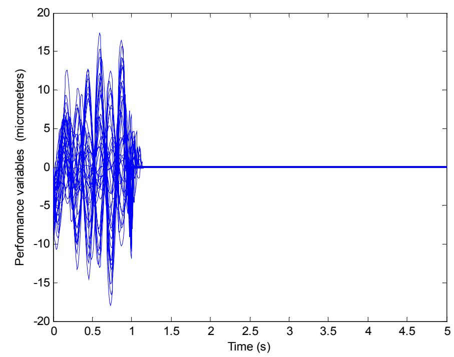

Fig. 5. Response of the adaptive closed loop system using the RLS algorithm with dead zone.

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