2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

Fault Tolerant Control for Unknown Nonlinear Systems with Actuator Failures: An Adaptive Fuzzy Approach

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Abstract— **In this paper, an adaptive fuzzy approach is proposed to deal with output regulation of unknown nonlinear systems with actuator failures. The actuator failures under consideration can be lock-in-place or/and loss of effectiveness. Based on fuzzy logic systems (FLS) equipped with adaptive algorithms to approximate the nonlinear system functions and the occurred failures together, a fault tolerant control law is developed to achieve asymptotic output regulation and closed-loop stability, despite the system and actuator faults are all unknown. A numerical simulation study illustrates the effectiveness of the proposed control approach.**

I. INTRODUCTION

The design of fault tolerant control (FTC) is very important for safety and reliability of modern engineering systems. As actuator faults may cause undesired system behavior and sometimes lead to instability or even catastrophic accidents, there is necessary to develop approaches that would accommodate actuator faults during operation. Adaptive control has been widely used in fault tolerant control field to deal with actuator faults since it can learn some unknown information of the controlled system and made appropriate adjustment adaptively. In the last decade, lots of adaptive approaches were developed for linear systems with actuator faults. In [1], a stable scheme was proposed by Boskovic for automatic control reconfiguration in the presence of actuator failures, which can achieve efficient energy consumption when there is no failure and maintain the stability and asymptotic tracking of the given reference outputs at the presence of failures. [2] developed multiple model based adaptive flight control to accommodate both total and partial loss of effectiveness. [3] extended the results of [2] to flight control with second-order actuator dynamics. In [4], Tao *et al* considered actuator failure in redundant actuation structure based on matching conditions using model-reference adaptive approach. The cases of state feedback design for state tracking, state feedback design for output tracking, output feedback design for output tracking and pole placement design are included in [4]. Yang and Ye combined robust control with adaptive control to dealt with loss of actuator effectiveness in the framework of linear matrix inequality in [5]-[7].

However, most real-life systems are nonlinear in nature. Therefore, much attention has been attracted to concentrate on adaptive nonlinear fault tolerant control in recent years. Also in [4], adaptive control of nonlinear systems with actuator failures was presented. [8] proposed virtual grouping based adaptive actuator failure compensation for multiple-input-multiple-output systems. But only feedback linearizable or parametric-strick-feedback nonlinear systems are considered in them. A multiple model-based fault tolerant control was developed in [9] for decentralized nonlinear system with second-order actuator dynamics. [10] applied robust adaptive predictive fault-tolerant control on a nonlinear chemical process, and [11] proposed fault accommodation approach for Lipschitz nonlinear systems. Since it was proved that adaptive fuzzy systems are universal approximators [12], and stable adaptive fuzzy control design was showed in [13], fuzzy logic and neural network (NN) have been used to nonlinear systems, and also FTC systems. In [14], a general framework for constructing automated fault diagnosis and accommodation architectures was presented using on-line approximators and adaptive schemes, [15]-[19] provided several FTC methods based on fuzzy logic systems (FLSs) or/and NNs. [20] and [21] applied FTC to practical systems for a turbine engine and aircraft autolanding respectively. Most of the existing works on fuzzy or neural-networks FTC is to detection and diagnosis/isolation faults with FLS or NN. Thus, good fault detection and diagnosis (FDD) is very important since if there are false or omitted alarms of the faults, the overall system may even become unstable. Inspired by the work of [4], we want to develop a adaptive FTC approach without resorting to FDD mechanism to accommodate both total and partial loss of effectiveness of actuators in unknown affine nonlinear systems.

This paper studies fault tolerant control for unknown nonlinear systems against actuator faults. The main properties compared with the existing results are that: first, adaptive fuzzy systems are introduced to tolerate actuator faults of unknown nonlinear systems without need of FDD mechanism, this is achieved by approximating the system functions and the effects caused by actuator faults together, so the unexpected system behavior caused by false or omitted

This work is supported in part by Program for New Century Excellent Talents in University (NCET-04-0283), the Funds for Creative Research Groups of China (No. 60521003), Program for Changjiang Scholars and Innovative Research Team in University (No. IRT0421), the State Key Program of National Natural Science of China (Grant No. 60534010), the Funds of National Science of China (Grant No. 60674021) and the Funds of PhD program of MOE, China (Grant No. 20060145019), the 111 Project (B08015).

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alarms can be avoided and the control structure of [4] can be applied to unknown affine nonlinear systems; second, we consider the actuator lock-in-place and loss of effectiveness together when design the fault tolerant control scheme, furthermore, any combination of these failures can be dealt with if only the system is still controllable, thus the fault set which can be accommodated is widened. We also considered the system with external disturbance in another paper.

This paper is organized as follows. Section II formulates the problem. Section III introduces the proposed fault tolerant control scheme. In Section IV, a numerical simulation example illustrates the effectiveness of the control method. Finally, Section V concludes the paper.

II. Problem Formulation:

Consider the following affine nonlinear plant

$$\begin{aligned} \dot{x} &= f(x) + \sum_{i=1}^{m} g_i(x)u_i \\ y &= h(x) \end{aligned} \tag{1}$$

where $x \in U \subseteq R^n$ is the state vector, U is a compact set in R^n. $y \in R$ is the output, and $u_i \in R$, $i \in \{1,2,\cdots,m\}$ is the control input which may fail during operation, $f(x) \in R^n$ and $g_i(x) \in R^n$ are function vectors whose components are unknown nonlinear smooth functions. So we cannot get the model of the controlled plant. $h(x) \in R$ is known and smooth. The failure model under consideration for fault tolerant control of system (1) is

$$u_i^F = \rho_i u_i, \quad \rho_i \in [\underline{\rho}_i, \bar{\rho}_i], \quad i \in \{1,2,\cdots,m\} \tag{2}$$

and

$$u_j^F = \bar{u}_j, \quad j \in \{1,2,\cdots,m\} \tag{3}$$

In this failure model, (2) describes the loss of effectiveness, where ρ_i denotes the percentage of the remaining effective part of the corresponding actuator, $0 < \underline{\rho}_i \leq 1$, $0 < \bar{\rho}_i \leq 1$, are the lower and upper bounds of ρ_i respectively, $\underline{\rho}_i \leq \bar{\rho}_i$. If $\underline{\rho}_i = \bar{\rho}_i = 1$, there is no failure occurred, i.e. the actuator is normal. For (2), $\rho_i = 0$ which means the complete loss of effectiveness is not considered since it is included in (3) for $\bar{u}_j = 0$. (3) describes the lock-in-place (stuck at an unknown value) failure. If (3) has occurred, the actuator must lose the effectiveness completely, thus the control input has no impact on the controlled system, but the actuator failure bring some disturbance if $\bar{u}_j \neq 0$. Actuators can be failed as (2) or (3), also, both (2) and (3) may occur during operation.

The objective for actuator fault tolerant control is to use feedback control design for the plant (1) with the actuator failures (2) or/and (3), to guarantee that all signals in the closed-loop system are bounded and asymptotic regulation of the plant output $y(t)$. In order to accomplish this task, the following basic assumption for the actuator fault tolerant control problem is needed.

Assumption 1: The plant (1) is so constructed that for any p_1 actuators fail as (2), $0 \leq p_1 \leq m$, and any p_2 actuators fail as (3), $0 \leq p_2 \leq m-1$, the remaining effective part of the actuators can still achieve the desired control objective.

III. Fault Tolerant Control Scheme

In this section, the design of the fault tolerant control scheme for plant (1) with actuator failures (2) or/and (3) is presented. Inspired by [4], a specific structure is considered so as to obtain the closed-loop stability and the asymptotic output regulation when the plant model and the actuator failures are all uncertain, here we respect to the nonlinear functions of the plant ($f(x)$ and $g(x)$), the failure style (which actuator has failed and as which failure model), the failure value (ρ_i and $\bar{u}_j$), and the failure time (at which the failure occurred) are all unknown. So Assumption 2 is set.

Assumption 2: $g_i(x) \in span\{g_0(x)\}$, for $i = 1,2,\cdots,m$, $g_0(x) \in R^n$, and the nominal system

$$\begin{aligned} \dot{x} &= f(x) + g_0(x)u_0 \\ y &= h(x) \end{aligned} \tag{4}$$

is feedback linearizable with a relative degree λ. Note that u_0 is a scalar in (4). Since (4) is feedback linearizable with an index λ, there exits a diffeomorphism $[\xi,\eta]^T = T(x) = [T_c(x), T_z(x)]^T$, $\xi \in R^\lambda$ and $\eta \in R^\gamma$, $\lambda + \gamma = n$, that

$$\xi = T_c(x) = \begin{pmatrix} h(x) \\ L_{f(x)}h(x) \\ \vdots \\ L_{f(x)}^{\lambda-1}h(x) \end{pmatrix}, \quad \eta = T_z(x) = \begin{pmatrix} T_{\lambda+1}(x) \\ T_{\lambda+2}(x) \\ \vdots \\ T_n(x) \end{pmatrix} \tag{5}$$

where $L_{f(x)}h(x)$ is the Lie derivative of $h(x)$ along $f(x)$, to transform the system (4) into the following form.

$$\begin{aligned} \dot{\xi} &= A\xi + B(\varphi(\xi,\eta) + \beta_0(\xi,\eta)u_0), \\ \dot{\eta} &= \psi(\xi,\eta), \\ y &= C\xi \end{aligned} \tag{6}$$

where

$$A = \begin{bmatrix} 0\,1\,0 \cdots 0\,0 \\ 0\,0\,1 \cdots 0\,0 \\ \cdots\cdots \\ 0\,0 \cdots 0\,0\,1 \\ 0\,0 \cdots 0\,0\,0 \end{bmatrix} \in R^{\lambda\times\lambda}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in R^{\lambda\times 1}$$
$$C = [1\;0\;\cdots\;\cdots 0\;0] \in R^{1\times\lambda} \tag{7}$$

$$\begin{aligned} \varphi(\xi,\eta) &= L_{f(x)}^{\lambda}h(x) \\ \beta_0(\xi,\eta) &= L_{g_0(x)}L_{f(x)}^{\lambda-1}h(x) \neq 0 \end{aligned} \tag{8}$$

$\dot{\eta} = \psi(\xi,\eta)$, is the zero dynamics of the nominal system, and η is the state vector of zero dynamic subsystem.

Remark 1: Assumption 2 indicates that the functions $g_i(x)$, $i = 1,\ 2,\ \cdots,\ m$, are similar in constitution. Take a linear system $\dot{x} = A_0x + b_1u_1 + b_2u_2$ for example, Assumption 2 means b_1 and b_2 are parallel to each other.

It follows from Assumption 2 that with the diffeomorphism (5), the plant (1) can be transformed into

$$\begin{aligned} \dot{\xi} &= A\xi + B(\varphi(\xi,\eta) + \beta^T(\xi,\eta)u), \\ \dot{\eta} &= \psi(\xi,\eta), \\ y &= C\xi \end{aligned} \tag{9}$$

where

$$\beta(\xi,\eta)=[\beta_1(\xi,\eta),\ \cdots,\ \beta_m(\xi,\eta)]^T = \left[L_{g_1(x)}L_{f(x)}^{\lambda-1}h(x),\ \cdots,\ L_{g_m(x)}L_{f(x)}^{\lambda-1}h(x)\right]^T \tag{10}$$

During the actuator fault tolerant control design, the states η in the zero dynamics will become unobservable in the output. So, for the stability of the closed-loop system, the following input-to-state stable assumption is needed.

Assumption 3: The system (4) is minimum-phase, i.e., the zero dynamics $\dot{\eta}=\psi(\xi,\eta)$ are input-to-state stable (ISS).

Consider the actuator failures described by (2) and (3), the actual control input vector $u=[u_1,u_2,\cdots,u_m]^T$ can be expressed as

$$u=\rho\upsilon(t)+\sigma(\bar{u}-\rho\upsilon(t)) \tag{11}$$

where $\upsilon(t)=[\upsilon_1(t),\upsilon_2(t),\cdots,\upsilon_m(t)]$ is the applied control to be designed, and

$$\begin{aligned}&\rho=diag\{\rho_1,\ \rho_2,\ \cdots,\ \rho_m\},\\ &\sigma=diag\{\sigma_1,\ \sigma_2,\ \cdots,\ \sigma_m\},\\ &\sigma_j=\begin{cases}1 & if\ the\ jth\ actuator\ fails\ as\ (3)\\ 0 & otherwise\end{cases}\\ &\bar{u}=[\bar{u}_1,\ \bar{u}_2,\ \cdots,\ \bar{u}_m]^T\end{aligned} \tag{12}$$

Note that $\rho\upsilon(t)$ includes the case where the actuator has no failure when the corresponding ρ_i, $i\in\{1,\cdots,m\}$, equals to 1.

With the actual input (11) in the presence of actuator failures (2) or/and (3), the plant (9) can be rewritten as

$$\begin{aligned}&\dot{\xi}=A\xi+B(\varphi(\xi,\eta)+\beta^T(\xi,\eta)\sigma\bar{u}+\beta^T(\xi,\eta)\rho(I-\sigma)\upsilon),\\ &\dot{\eta}=\psi(\xi,\eta),\\ &y=C\xi\end{aligned} \tag{13}$$

Here a specific proportional actuation structure is used,

$$\upsilon_i(t)=b_i\upsilon_0(t)\quad i=1,\cdots,m \tag{14}$$

where b_i is a nonzero constant, $\upsilon_0(t)$ is the control signal needs to be designed.

Remark 2: Such an actuation structure together with the condition in Assumption 2 implies that plant (1) is an actuator redundancy system. This is reasonable because for some practical systems, the control surfaces can be divided into several individually actuated segments. For example, the aileron segments of an aircraft provide some redundancy needed for failure compensation.

With the proportional actuation structure(14), (13) is equivalent to the following form

$$\begin{aligned}\dot{\xi}=&A\xi+B(\varphi(\xi,\eta)+\sum_{j=j_1,\cdots,j_{p_2}}\beta_j(\xi,\eta)\bar{u}_j\\ &+\sum_{j\neq j_1,\cdots,j_{p_2}}\beta_j(\xi,\eta)\rho_jb_j\upsilon_0),\\ \dot{\eta}=&\psi(\xi,\eta),\\ y=&C\xi\end{aligned} \tag{15}$$

where p_2 is the total number of the actuators which fail as (3), $\sum_{j\neq j_1,\cdots,j_{p_2}}\beta_j(\xi,\eta)\rho_jb_j\neq 0$ from Assumption 1 to accomplish the control task. For this condition to be always true, Assumption 4 is set

Assumption 4: $sign(\beta_j(\xi,\eta))$ is known.

Without loss of generality, the sign of b_j is made the same as the sign of $\beta_j(\xi,\eta)$, so $\sum_{j\neq j_1,\cdots,j_{p_2}}\beta_j(\xi,\eta)\rho_jb_j>0$ holds.

Let

$$\begin{aligned}&\bar{f}(x)=\varphi(\xi,\eta)+\sum_{j=j_1\cdots j_{p_2}}\beta_j(\xi,\eta)\bar{u}_j\\ &\bar{g}(x)=\sum_{j\neq j_1\cdots j_{p_2}}\beta_j(\xi,\eta)\rho_jb_j\end{aligned} \tag{16}$$

Then the formation (15) can be described as

$$\begin{aligned}&\dot{\xi}=A\xi+B(\bar{f}(x)+\bar{g}(x)\upsilon_0),\\ &\dot{\eta}=\psi(\xi,\eta),\\ &y=C\xi\end{aligned} \tag{17}$$

Assumption 5: Assume ξ is available.

Then, if $\bar{f}(x)$ and $\bar{g}(x)$ are known, the control signal

$$\upsilon_0=\frac{1}{\bar{g}(x)}(-\bar{f}(x)-k^T\xi) \tag{18}$$

makes the system (17) meet that

$$\dot{\xi}=A\xi-Bk^T\xi=A_c\xi \tag{19}$$

where $A_c=\begin{bmatrix}0 & 1 & 0\cdots 0 & 0\\ 0 & 0 & 1\cdots 0 & 0\\ & \cdots\cdots & & \\ 0 & 0\cdots 0 & 0 & 1\\ -k_n & -k_{n-1} & \cdots & -k_2 & -k_1\end{bmatrix}\in R^{\lambda\times\lambda}$, the constants $k_1,\cdots,k_n$ is designed such that $s^n+k_1s^{(n-1)}+\cdots+k_{(n-1)}s+k_n$ is a Hurwitz polynomial, and $k=(k_n\ \cdots\ k_1)^T$. The matrix A_c is defined stable, so there exit symmetric positive matrices P and Q such that

$$A_c^TP+PA_c\leq -Q \tag{20}$$

is satisfied. Then from the Lyapnov stability theory, one can get $\lim_{t\to\infty}\xi=0$, thus from definition (5) $\lim_{t\to\infty}y=0$. Because the zero dynamic subsystem $\dot{\eta}=\psi(\xi,\eta)$ is ISS, η is bounded. So, the conclusion can be drawn that if the nonlinear functions $\bar{f}(x)$ and $\bar{g}(x)$ are known, the control law (18) will achieve the control objective.

But the problem is that the nonlinear functions $\bar{f}(x)$ and $\bar{g}(x)$ are unknown, thus the ideal control law (18) can not be applied directly. Since FLS are universal approximators, they can be used to approximate the unknown functions.

Lemma 1: For any given real continuous function $F(x)$, on a compact set $U\in R^n$, there exits an FLS of the following form that can uniformly approximate $F(x)$ over U to arbitrary accuracy.

$$y(x)=\theta^T\zeta(x)$$

where $\theta=(\theta_1,\theta_2,\cdots,\theta_M)^T$ is the estimate parameter vector, and $\zeta(x)=(\zeta_1(x),\zeta_2(x),\cdots,\zeta_M(x))^T$ is the vector of fuzzy basis functions, M is the number of fuzzy rules. One can refer to [12] and [13] for more details.

Since x is in a compact set $U \in R^n$, we define

$$\begin{array}{l}\hat{\bar{f}}(x) = \theta_f^T \zeta(x) \\ \hat{\bar{g}}(x) = \theta_g^T \zeta(x)\end{array}$$

where estimate parameter vectors θ_f and θ_g can be adjusted by the corresponding adaptive laws respectively. Define the optimal parameters as

$$\begin{array}{l}\theta_f^* = \arg\min\left[\sup\left|\hat{\bar{f}}(x) - \bar{f}(x)\right|\right] \\ \theta_g^* = \arg\min\left[\sup\left|\hat{\bar{g}}(x) - \bar{g}(x)\right|\right]\end{array}$$

one can get the parameter errors $\tilde{\theta}_f = \theta_f - \theta_f^*$ and $\tilde{\theta}_g = \theta_g - \theta_g^*$. The applied control law is obtained as

$$\upsilon_0 = \frac{1}{\hat{\bar{g}}(x)}(-\hat{\bar{f}}(x) - k^T\xi) = \frac{1}{\theta_g^T\zeta(x)}(-\theta_f^T\zeta(x) - k^T\xi) \tag{21}$$

We design the adaptive laws as follows:

$$\dot{\theta}_f = \gamma_1 \xi^T PB\zeta(x)$$
$$\dot{\theta}_{gi} = \begin{cases} \gamma_2\xi^T PB\zeta_i(x)\upsilon_0 & if\ \theta_{gi} > \delta\ or \\ & (\theta_{gi} = \delta\ and\ \xi^T PB\zeta_i(x)\upsilon_0 > 0) \\ 0 & if\ \theta_{gi} = \delta\ and\ \xi^T PB\zeta_i(x)\upsilon_0 < 0 \end{cases} \tag{22}$$

where γ_1 and γ_2 are adaptive gain, and δ is a chosen small positive constant. The project algorithm of the adaptive law is used to keep $\hat{\bar{g}}(x)$ off an neighborhood of zero, so that the proposed control law will be nonsingular. And the adaptive fuzzy control scheme has the following properties.

Theorem 1: With the ISS assumption of the zero dynamics, the controller (21) with the adaptive laws (22) guarantees that all closed-loop signals are bounded, and the asymptotic output regulation of the plant (1) with actuator failures as (2) or/and (3).

Proof: Suppose that one or more than one actuators fail at time instant t_j, $j = 1, 2, \cdots, q$, $1 \le q \le m-1$ if all faults are of the form (3), $1 \le q \le m$ otherwise. And at time $t \in (t_{j-1}, t_j)$, there are $p_1 (0 \le p_1 \le m)$ actuators fail as (2), and $p_2 (0 \le p_2 \le m-1)$ actuators fail as (3). Define Lyapunov function on the interval (t_{j-1}, t_j) where the actuator failure pattern is unchanged, as

$$V_{j-1} = \frac{1}{2}\xi^T P\xi + \frac{1}{2\gamma_1}\tilde{\theta}_f^T\tilde{\theta}_f + \frac{1}{2\gamma_2}\tilde{\theta}_g^T\tilde{\theta}_g \tag{23}$$

It's derivative along (17) with the control signal (21) is

$$\begin{aligned}\dot{V}_{j-1} &= \tfrac{1}{2}\left(\dot{\xi}^T P\xi + \xi^T P\dot{\xi}\right) + \tfrac{1}{\gamma_1}\tilde{\theta}_f^T\dot{\tilde{\theta}}_f + \tfrac{1}{\gamma_2}\tilde{\theta}_g^T\dot{\tilde{\theta}}_g \\ &= \tfrac{1}{2}[\xi^T A^T P\xi + (\bar{f}(x) - \hat{\bar{f}}(x) + \bar{g}(x)\upsilon_0 - \hat{\bar{g}}(x)\upsilon_0 - k^T\xi) \\ &\quad B^T P\xi\ + \xi^T PA\xi + \xi^T PB(\bar{f}(x) - \hat{\bar{f}}(x) + \bar{g}(x)\upsilon_0 - \\ &\quad \hat{\bar{g}}(x)\upsilon_0 - k^T\xi)] + \tfrac{1}{\gamma_1}\tilde{\theta}_f^T\dot{\tilde{\theta}}_f + \tfrac{1}{\gamma_2}\tilde{\theta}_g^T\dot{\tilde{\theta}}_g \\ &= \tfrac{1}{2}[\xi^T(A - Bk^T)^T P\xi + \xi^T P(A - Bk^T)\xi] - \xi^T PB[(\theta_f \\ &\quad -\theta_f^*)^T\zeta(x) + (\theta_g - \theta_g^*)^T\zeta(x)\upsilon_0] - \xi^T PB[(\theta_f^{*T}\zeta(x) \\ &\quad -\bar{f}(x)) + (\theta_g^{*T}\zeta(x) - \bar{g}(x))\upsilon_0] + \tfrac{1}{\gamma_1}\tilde{\theta}_f^T\dot{\tilde{\theta}}_f + \tfrac{1}{\gamma_2}\tilde{\theta}_g^T\dot{\tilde{\theta}}_g\end{aligned} \tag{24}$$

Let

$$\begin{aligned}\omega &= (\hat{\bar{f}}(x|\theta_f^*) - \bar{f}(x)) + (\hat{\bar{g}}(x|\theta_g^*) - \bar{g}(x))\upsilon_0 \\ &= (\theta_f^{*T}\zeta(x) - \bar{f}(x)) + (\theta_g^{*T}\zeta(x) - \bar{g}(x))\upsilon_0\end{aligned}$$

Then (24) can be rewritten as

$$\begin{aligned}\dot{V}_{j-1} &= \tfrac{1}{2}\xi^T\left(A_c^T P + PA_c\right)\xi - \xi^T PB\tilde{\theta}_f^T\zeta(x) - \xi^T PB\tilde{\theta}_g^T\zeta(x)\upsilon_0 \\ &\quad + \tfrac{1}{\gamma_1}\tilde{\theta}_f^T\dot{\tilde{\theta}}_f + \tfrac{1}{\gamma_2}\tilde{\theta}_g^T\dot{\tilde{\theta}}_g - \xi^T PB\omega\end{aligned} \tag{25}$$

By Lemma 1 and the definition of ω, one can believe that ω is an arbitrarily small value. This can be true if the number of the rules is large enough, thus the last term of (25) can be ignored. Consider the adaptive laws (22) and note the fact that $\dot{\tilde{\theta}}_f = \dot{\theta}_f$, $\dot{\tilde{\theta}}_g = \dot{\theta}_g$, (25) is equivalent to

$$\dot{V}_{j-1} \le -\frac{1}{2}\xi^T Q\xi - \varepsilon \tag{26}$$

where $\varepsilon = 0$ if for all i, $\theta_{gi} > \delta$ or $\theta_{gi} = \delta$ *and* $\xi^T PB\zeta_i(x)\upsilon_0 > 0$ is true; otherwise, $\varepsilon = \sum_i \tilde{\theta}_{gi}\xi^T PB\zeta_i(x)\upsilon_0$, wherein i denotes i-th component of parameter vector θ_g which meets $\theta_{gi} = \delta$ *and* $\xi^T PB\zeta_i(x)\upsilon_0 < 0$. It can be proved easily that $\varepsilon \ge 0$, then the following inequality is obtained

$$\dot{V}_{j-1} \le -\frac{1}{2}\xi^T Q\xi \le 0 \tag{27}$$

At time $t = t_j$, $j = 1, 2, \cdots, q$, when actuator failures occur, there may be a finite jumping in the optimal parameters θ_f^* and θ_g^*, so $\tilde{\theta}_f$ and $\tilde{\theta}_g$ will change their values to form the new Lyapunov function V_j on the next time interval (t_j, t_{j+1}). From (27), it follows that

$$V_{j-1}(t_j^-) \le V_{j-1}(t_{j-1}^+) \tag{28}$$

which indicates that ξ, $\tilde{\theta}_f$, and $\tilde{\theta}_g$ are all bounded on the interval (t_{j-1}, t_j) if $V_{j-1}(t_{j-1}^+)$ is finite. Because the changes of $\tilde{\theta}_f$ and $\tilde{\theta}_g$ are finite, it in turn implies that $V_j(t_j^+)$, the initial value of the function $V_j(t)$ is finite. Due to the chosen value of $V_0(t_0)$ is finite , it turns out that the initial value of the last function V_q defined on the interval (t_q, ∞) is finite, then it can be concluded that $\tilde{\theta}_f$ and $\tilde{\theta}_g$ are bounded, which means the estimate parameters θ_f and θ_g are bounded; ξ is bounded and square integrable(see (27)), which indicates ξ will go to zero when t goes to infinity. With the ISS zero dynamics, the closed-loop stability and the asymptotic output regulation can be established. Thus the proof has been completed.

IV. Simulation Example

In this section, the presented adaptive fuzzy fault tolerant controller is applied to a nonlinear system with actuator faults described as (2) and (3).

Example: We consider that after certain transformation, the nonlinear system can be written as the following form which has redundancy actuation structure.

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{5\sin x_1 - 0.02x_2^2 \cos x_1 \sin x_1}{3-0.2\cos^2 x_1} + \frac{\cos^2 x_1}{3-0.2\cos^2 x_1} u_1 + \frac{2\cos^2 x_1}{3-0.2\cos^2 x_1} u_2 \tag{29}$$

where the actuator of u_1 is stuck at 2 at $t = 6s$, i.e. $u_1^F = 2$ when $t \geq 6$, while the actuator of u_2 loses 70% effectiveness at $t = 20s$ that is $u_2^F = 0.3u_2$ when $t \geq 20$ in simulation. The situation of the failure is already very severe. In order to control system (29) to get closed-loop stability and asymptotic output regulation $\lim_{t\to\infty} y = 0$, control law (21) is applied with the following simulation settings. Choose $k_1 = 2$, $k_2 = 1$, and $Q = diag(10, 10)$. Then by solving equation (20) one can obtain

$$P = \begin{bmatrix} 15 & 5 \\ 5 & 5 \end{bmatrix}$$

$\gamma_1 = 5$, and $\gamma_2 = 1$ for adaptive adjusting. We define five fuzzy sets over each axis, which label as $F_i^1, \cdots, F_i^5$, $i = 1, 2$, respectively. The fuzzy membership functions are

$$\mu_{F_i^1}(x_i) = \frac{1}{1+\exp[5\pi/3 \times (x_i + \pi/3)]}$$

$$\mu_{F_i^2}(x_i) = \exp[-\frac{(x_i+\pi/6)}{(\pi/10)^2}]$$

$$\mu_{F_i^3}(x_i) = \exp[-\frac{x_i}{(\pi/10)^2}]$$

$$\mu_{F_i^4}(x_i) = \exp[-\frac{(x_i-\pi/6)}{(\pi/10)^2}]$$

$$\mu_{F_i^5}(x_i) = \frac{1}{1+\exp[-5\pi/3 \times (x_i - \pi/3)]}$$

So we have $M = 5 \times 5 = 25$ rules for each fuzzy logic system. The initial values are selected as $\theta_f(0) = \theta_g(0) = 2I_{25\times 1}$, and $x(0) = (-\pi/9, 0.05)^T$. The simulation results for $0 \leq t \leq 40$ are shown in Fig.1 and Fig.2

We can see from the results that even though both of the actuators have failures, the output of the controlled system can still be regulated that $\lim_{t\to\infty} y = 0$ as long as Assumption 1 is still satisfied.

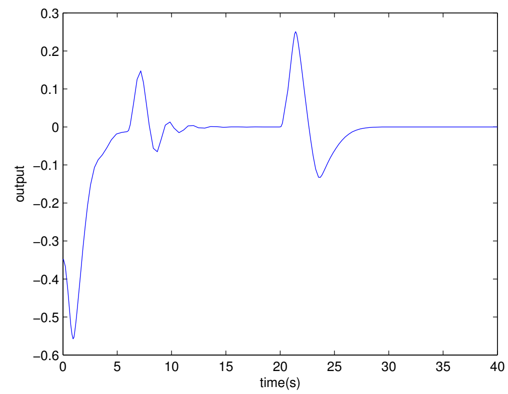

Fig. 1. The output of the controlled system

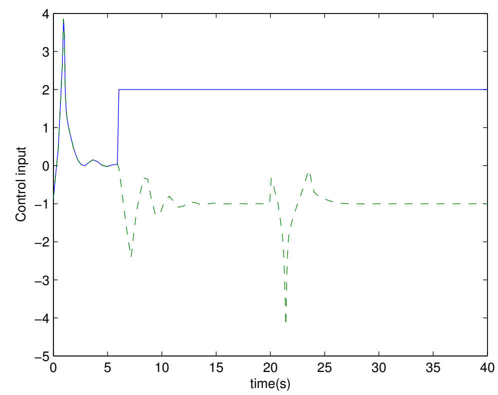

Fig. 2. The control signal of the controlled system with actuator faults

V. Conclusion

This paper deals with fault tolerant control problem for unknown nonlinear systems with actuator faults which can be lock-in-place, loss of effectiveness or both of them. Fuzzy logic systems with free variable parameters are used to approximate the unknown nonlinear functions and the failures together, with which the control law can be developed to estimate an ideal one. On the basis of Lyapunov stability theory, adaptive laws are derived to adjust the estimate parameters of the fuzzy logic systems so that the control objective can be achieved. Note that the proposed adaptive fuzzy fault tolerant control approach need not resort to the fault detection and diagnosis mechanism by continuously adaptive the control law. And the types of faults that can be tolerant have been broadened even in unknown nonlinear systems as long as the systems are still controllable. A simple output regulation problem is studied, and the simulation results show the effectiveness of the proposed control scheme even though the faults occurred are severe.

References

[1] J. D. Boskovic, S. H. Yu, and R. K. Mehra, "A stable scheme for automatic control reconfiguration in the presence of actuator failures" *American Control Conference*, Jun. 1998 vol. 4, pp. 2455 - 2459.

[2] J. D. Boskovic and R. K. Mehra, "Stable multiple model adaptive flight control for accommodation of a large class of control effector failures", *American Control Conference*, Jun. 1999, vol 3, pp. 1920 - 1924.

[3] J. D. Boskovic, S. E. Bergstrom, and R. K. Mehra, "Adaptive accommodation of failures in second-order flight control actuators with measurable rates" *American Control Conference*, Jun. 2005 vol. 2, pp. 1033 - 1038.

[4] G. Tao, S. H. Chen, X. D. Tang, and S. M. Joshi, *Adaptive Control of Systems with Actuator Failures.*, New York: Springer, Mar. 2004.

[5] G. H. Yang and D. Ye, "Adaptive fault-tolerant h_∞ control via state feedback for linear systems against actuator faults", *Conference on Decision and Control*, San Diego, CA, USA, Dec. 2006, pp. 3530 - 3535.

[6] G. H. Yang and D. Ye, "Adaptive fault-tolerant h_∞ control via dynamic output feedback for linear systems against actuator faults", *Conference on Decision and Control*, San Diego, CA, USA, Dec. 2006, pp. 3524 - 3529.

[7] D. Ye, and G. H. Yang, "Adaptive fault-tolerant tracking control against actuator faults with application to flight control", *IEEE Transactions on Control Systems Technology*, Nov. 2006, vol. 14, iss. 6, pp. 1088 - 1096.

[8] X. D. Tang, G. Tao, S. M. Joshi, "Virtual grouping based adaptive actuator failure compensation for MIMO nonlinear systems", *IEEE Transactions on Automatic Control*, vol 50, iss. 11, Nov. 2005, pp. 1775 - 1780.

[9] J. D. Boskovic and R. K. Mehra, "A multiple model-based decentralized system for accommodation of failures in second-order flight control actuators" *American Control Conference*, Jun. 2006, pp. 4436-4440.

[10] D. Zumoffen, M. Basualdo, M. Jordan, A. Ceccatto, "Robust adaptive predictive fault-tolerant control linked with fault diagnosis system applied on a nonlinear chemical process", *IEEE Conference on Decision and Control*, San Diego, CA, USA, Dec. 2006, pp. 3512 - 3517.

[11] B. Jiang, M. Staroswiecki, V. Cocquempot, "Fault accommodation for nonlinear dynamic systems", *IEEE Transactions on Automatic Control*, Sep. 2006, vol. 51, no. 9, pp. 1578 - 1583.

[12] L. X. Wang, J. M. Mendel, "Fuzzy basis functions, universal approximation, and orthogonal least-squares learning", *IEEE Transactions on Neural Networks*, Sep. 1992, vol 3, iss 5, pp. 807 - 814.

[13] L. X. Wang, *Adaptive Fuzzy Systems and Control: Design and Stability Analysis.*, Upper Saddle River, NJ: Prenctice-Hall, 1994.

[14] M. M. Polycarpou and A. J. Helmicki, "Automated fault detection and accommodation: a learning systems approach", *IEEE Transactions on Systems, Man, and Cybernetics*, Nov. 1995, vol. 25, no. 11, pp. 1447 - 1458.

[15] M. M. Polycarpou, X. D. Zhang, R. Xu, Y. L. Yang, C. Kwan, "A neural network based approach to adaptive fault tolerant flight control", *Proceedings of IEEE Intemational Symposium on Intelligent Control*, Taipei, Taiwan, Sep. 2004, pp. 61 - 66.

[16] X. D. Zhang, T. Parisini, M. M. Polycarpou, "Adaptive fault-tolerant control of nonlinear uncertain systems: an information-based diagnostic approach", *IEEE Transactions on Automatic Control*, Aug. 2004, vol. 49, no. 8, pp. 1259 - 1274.

[17] X. D. Zhang, Y. Liu, R. Rysdyk, C. Kwan, R. Xu, "An intelligent hierarchical approach to actuator fault diagnosis and accommodation", *IEEE Aerospace Conference*, Mar. 2006, pp. 1 - 14.

[18] Z. H. Mao, B. Jiang, F. Chowdhury, "Fault accommodation for a class of nonlinear flilght control systems", *International Symposium on Systems and Control in Aerospace and Astronautics*, Jan. 2006, pp. 758 - 763.

[19] H. Xue and J.G. Jiang, "Fault detection and accommodation for nonlinear systems using fuzzy neural networks", *IEEE 5th International Power Electronics and Motion Control Conference*, Aug. 2006, vol. 3, pp. 1 - 5.

[20] Y. X. Diao, K. M. Passino, "Stable fault-tolerant adaptive fuzzy/neural control for a turbine engine", *IEEE Transactions on Control Systems Technology*, vol. 9, iss. 3, May 2001, pp. 494 - 509.

[21] H. J. Rong, G. B. Huang, N. Sundararajan, P. Saratchandran, "Fuzzy fault tolerant controller for actuator failures during aircraft autolanding", *IEEE International Conference on fuzzy systems*, Jul. 2006, pp. 1200 - 1204.