# Repetitive Control of an Electro-Hydraulic Engine Valve Actuation System

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Abstract-Fully flexible engine valve systems serve as powerful rapid prototyping tools in research laboratories. With the ability to quickly design innovative valve strategies, researchers can explore the possibilities of improving fuel efficiency, power output and emissions through appropriately varying the valve lift, phasing and timing. One means of achieving variable valve motion is through an electro-hydraulic valve system (EHVS). However, with an EHVS, it is difficult to achieve the high acceleration necessary for tracking cam profiles while maintaining the same level of accurate position control that a mechanical cam provides. In particular, the response time delay and the nonlinear dynamics of the hydraulic system can lead to error in position control. The paper first describes an identification method for obtaining a mathematical model of the EHVS. Based on the model, a linear feedback controller is developed. Finally, a repetitive feed-forward controller is added to augment the feedback controller, improving rootmean-square tracking performance to below forty micrometers.

## I. INTRODUCTION

Flexible valve strategies are widely used in the automotive industry to enable advanced engine control strategies. By appropriately varying the valve lift, timing and phasing, one can improve the fuel efficiency, increase power output and reduce emissions (Stein *et al.*,1995 [1]), (Leone *et al.*, 1996 [2]). There are various systems that manage to achieve variable valve profiles. For example, an electro-mechanical variable valve system (Peterson *et al.*, 2003 [3]) can change its valve timing independently, however, its lift is fixed at a constant value. In a research environment, it is particularly desirable to have fully flexible systems so that new valve strategies can be quickly prototyped and validated, as long as the valve position control is accurate. The particular fully flexible valve system we discuss in this paper is an electrohydraulic variable valve system (EHVS).

While there are significant gains from replacing mechanical cams with EHVS, it is difficult to control such a system to the level of accuracy inherent in a mechanical cam. This is largely due to the response time delay that results from the compressibility of the hydraulic fluid and the nonlinearity of the system. In particular, the time delay decreases the phase and gain margins of the system and limits the bandwidth of the system. As a result, tracking performance of EHVS deteriorates as engine speed increases, since shorter rise time is required at higher engine speeds.

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A/D+CPU+D/A A/D+C

Fig. 1. Schematics of the EHVS

This is particularly undesirable in a research environment where the inconsistency of EHVS performance might bias the subsequent combustion results.

Various researchers have developed controllers for EHVS. Anderson *et al.* present an adaptive controller for EHVS in [4]. However, this work only addresses maximum lift control and not valve profile tracking. In [5], Sun *et al.* show that repetitive control (Tomizuka *et al.*, 1989 [6]) can be very effective for EHVS tracking problem, since valve motion is largely repetitive for steady state engine operation. However, their approach assumes a fundamental period for the desired valve profile which only allows the EHVS to operate at several specific engine speeds.

In this paper, we first develop a linear quadratic controller to achieve the baseline tracking performance. A repetitive controller that does not assume a fixed fundamental period of the desired valve profile is implemented. This enables fully flexible valve profile tracking to micrometer level.

The presentation of this work is divided into five sections. The next section, Two, briefly describes the EHVS system and the identification method we use to model the system. Section Three presents a linear feedback controllers and explains the difficulties with using feedback for EHVS. Section Four shows a repetitive feed-forward controller that augments the feedback controller. The performance of the combined controller is shown in Section Five and the achieved root mean square (RMS) tracking error is under 40 ( $\mu$ m). Section Six presents a brief stability statement on the repetitive controller. The controller development framework can be and has been applied to other EHVS hardware.

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Fig. 2. Valve and Actuator Illustration

## **II. SYSTEM IDENTIFICATION**

Fig. 1 shows the schematic for a single actuator. Analog voltage generated by a computer drives the voice coil in the system. The voice coil position determines the area of opening of the hydraulic path and generates a pressure difference across the piston as shown in Fig. 1. The pressure difference creates a force that moves the valve actuator. A linear variable differential transformer (LVDT) is attached to the valve actuator to measure its position.

The dynamics of the EHVS can be represented by a five state nonlinear model as shown by Richman [7]. Hathout *et al.* show in [8] that it can be further simplified and controlled using a three state linear model. We follow the same approach in this work. We use a frequency domain identification technique and assume a third-order linear model. It is important that the identification input is rich in frequency content to sufficiently excite the system dynamics. However, the valve actuators are not rigidly linked to the engine valves as shown in Fig. 2. Thus, we need to carefully choose the identification input so that the valve contacts the actuator at all times. Detachment of the two elements might alias the identification results and, more importantly, could damage the valve-actuator contact surface due to large impact forces.

To ensure continuous contact, the two elements must experience the same acceleration and the reaction force between the two elements must be positive. The first condition gives us the following equation:

$$\ddot{x_1} = \frac{F+R}{m_1} = \frac{-d_2\dot{x_2} - k_2(x_2 - x_{20}) - R}{m_2} = \ddot{x_2} \qquad (1)$$

where

- $m_1$  is the actuator mass
- $m_2$  is the valve mass
- $k_2$  is the valve spring constant
- $d_2$  is the valve damping coefficient
- $x_1$  is the actuator position
- $x_2$  is the valve position
- $x_{20}$  is the valve spring relaxed position
- F is the actuator force
- R is the reaction force between the two elements

Solving for R and restricting it to be positive, we have the



Fig. 3. ETFE and Model Fit

following condition:

$$\frac{m_1k_2(x_{20} - x_2) - m_1d_2\dot{x_2} - m_2F}{m_1 + m_2} > 0 \tag{2}$$

While the valve damping coefficient is difficult to measure, the valve spring force is very large compared to the force generated by the damping term. The valve spring is also heavily preloaded from its relaxed position of  $x_{20}$ . We can thus bound the actuator force conservatively to be negative and, equivalently, bound the input voltage to be negative, since negative input voltage represents negative force. With this bound applied to our input, we can ensure that (2) is satisfied and the actuator and valves remain in contact.

Conveniently, a pseudo random binary sequence (PRBS) fits into this conservative force bound and possesses rich frequency content. Since it is a binary signal, we either feed some fixed negative voltage or zero (Volt) to the system input. In addition, a dither at 800 (Hz) is added to overcome the static friction present in the system. Without the dither, the system response delay is increased. An empirical transfer function estimate (ETFE), as shown in Fig. 3, is obtained using this setup. One important characteristic of the system is the presence of a 1 (ms) response delay. This can be seen from the ETFE phase plot. In general, the system behaves much like a third order system with input delay. Therefore, we fit an eighth order discrete model with a sampling rate of 5000 (Hz) to the ETFE. Five of the eight states in this discrete model are used to represent the 1 (ms) response delay. The Bode plot of this discrete model is shown in red in Fig. 3. Also plotted in green in the phase plot of Fig. 3 is a model that has the same dynamics except for the response delay, highlighting the phase loss that the delay introduces. In general, we have a good fit to the ETFE using this model structure.

We notice some nonlinearity of the system when varying the amplitude of the PRBS. As shown in Fig. 4, the ETFE generated by a 1.3 (Volt) PRBS has a higher resonance peak than that of the ETFE generated by a 0.9 (Volt). We pick the ETFE that has the highest resonance peak to fit our model, since we do not want the valve to overshoot and, as a result, do not want to underestimate the resonance peak.



Fig. 4. Nonlinearity

## III. FEEDBACK CONTROLLER

With a model to approximate the dynamics of the system, we design a feedback controller for the EHVS. A typical valve profile that we wish to track is shown in green in Fig. 5. It has a maximum lift of 5 (mm). The rising and falling edges of the profile are 70 crank angle degrees (CAD) and the dwelling length at maximum lift is 60 (CAD). Unlike a generic reference tracking problem in which the feedback controller has no prior information about the reference to be tracked, we wish to utilize the fact that the desired valve profile is known ahead of time. Since the future desired trajectory has the desired velocity and acceleration information implicit in it, we should be able to control the EHVS better if we give the controller the information of where it should be in the future. We set up a finite horizon quadratic cost function (3) based on tracking error and input.

$$J = \sum_{t}^{t+N} [y_{des}(t) - y(t)]^T \cdot Q \cdot [y_{des}(t) - y(t)] + u(t)^T \cdot R \cdot u(t)$$
(3)

where

 $y_{des} \in R$  is the desired value position

- $y \in R$  is the valve position
- $u \in R$  is the voltage input to the voice coil
- Q, R are positive scalars that weigh tracking error and control effort

N is a positive constant that defines the time horizon For each time step, we wish to minimize the cost function described above. The optimal input u(t) for the future trajectory  $Y_{des}(t) = [y_{des}(t) \ y_{des}(t+1)...y_{des}(t+N)]^T$ given the dynamics of the system can be solved recursively by using dynamic programming. The optimal input reduces to a time-invariant linear function of the future trajectory and current states. This is given as:

$$u(t) = L \cdot Y_{des}(t) + K \cdot x(t) \tag{4}$$

where

$$\begin{aligned} Y_{des}(t) &= [y_{des}(t) \quad y_{des}(t+1)...y_{des}(t+N)]^T \\ Y_{des} & \text{is the desired future trajectory} \\ x & \text{is the current state} \end{aligned}$$



Fig. 5. Tracking Performance of the Linear Quadratic Tracking Controller

#### L, K are gains found by solving the cost function

Thus, for each time step we update the future trajectory and the current states then multiply them by the static gains L and K respectively to determine the input for the system. This feedback controller is implemented on our EHVS and experiments are conducted at an engine speed of 1800 (rpm). The tracking performance is shown in blue in Fig. 5. As can be seen, the valve trajectory tracks the desired profile well at the rising and falling edges, but shows some ringing at the maximum valve lift.

The 1 (ms) delay in the loop makes it very difficult to damp out the resonance mode using feedback. To put the control problem into perspective, at the resonance frequency of 350 (Hz) of the system, the 1 (ms) delay costs 126 (deg) of phase. Therefore, gain and phase margins are seriously hurt by this response delay. Furthermore, at an engine speed of 1800 (rpm) and with a typical valve opening event that is 70 crank angle degree (CAD), the response delay is roughly twenty percent of the rise time. This shows the difficulty in achieving cam-like tracking performance which motivates the work of the next section.

## IV. REPETITIVE CONTROLLER

In order to eliminate any variability that might be brought into the subsequent combustion research by using the EHVS, it is highly desirable to achieve cam-like precision position control. As shown in the previous section, using only the feedback controller is not sufficient to achieve the accurate position control offered by a mechanical cam. To further improve the tracking performance of our feedback controller, we utilize the fact that the tracking error given by the feedback controller is highly repeatable from engine cycle to engine cycle. As can be seen in Fig. 6, the tracking error from ten engine cycles are over-plotted and the error traces almost lie on top of each other. This motivates us to set up a repetitive controller that adaptively generates an input to compensate for the repetitive tracking error using feedforward.

The algorithm of the repetitive controller is described as follows. First we partition the tracking error evenly into several pieces, as shown in the top plot of Fig. 7. We refer



Fig. 6. Repetitive Tracking Error

to each of these partitions as a repetitive state. For example, if we choose the state width to be 20 CAD, a valve opening and closing event that lasts 140 CAD consists of 7 repetitive states. We then compute the mean tracking error of each state as shown in the bottom plot of Fig. 7. The calculation of the mean tracking error can be represented by a linear function and we give a specific example as follows:

$$E^{(k)} = P \cdot (\tilde{y}_{des}^{(k)} - \tilde{y}^{(k)})$$
 (5)

where

$$P = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

 $P \in \mathbb{R}^{m \times n}$ 

 $E \in \mathbb{R}^m$  is the mean error of each state

 $\tilde{y}_{des} \in \mathbb{R}^n$  is the desired valve trajectory

 $\tilde{y} \in \mathbb{R}^n$  is the actual valve trajectory

k represents the  $k^{th}$  engine cycle

- m is the number of repetitive states
- n is the number of samples that the repetitive controller covers

For this particular example, the structure of P implies that there are three samples within one repetitive state and the mean tracking error is simply the average of the tracking error of the three samples.

After computing the mean tracking error, our goal is to find a input that drives the mean error of each state to zero. To determine the repetitive control input for each state, we update the control input according to the following equation:

$$U_{rep}^{(k)} = U_{rep}^{(k-1)} + k_I \cdot M \cdot E^{(k-1)}$$
(6)

$$M = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$



Fig. 7. Repetitive States and Mean Error

 $\begin{array}{ll} M & \in R^{m \times m} \\ U_{rep} & \in R^m \text{ is the repetitive control input for each state} \\ k_I & \text{ is the integral gain} \end{array}$ 

Notice that in (6),  $U_{rep}$  is a vector with each of the indices representing the repetitive input to the corresponding state, and the vector is updated with respect to engine cycles. Thus,  $U_{rep}$  has the same length as the number of repetitive states. In this specific example, the structure of the M matrix implies that the repetitive control input for each state is a function of the error from the current state and the adjacent states. This dependency on adjacent states helps the stability of the repetitive controller because the controller determines the input for each state with the knowledge of its neighbors' mean error as well as its own. So far, we only describe the repetitive control input as a vector in  $\mathbb{R}^m$  with each of its indices correspond to a state. In reality, we need to convert  $U_{rep}$  into a vector of length  $\mathbb{R}^n$ , the domain of the system input. As an example, we again look at the case where we have three samples per state:

$$u_{rep}^{(k)} = S \cdot U_{rep}^{(k)}$$

(7)

 $S^{T} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 1 & 1 & 1 \end{bmatrix}$  $S \qquad \in R^{n \times m}$  $u_{rep} \qquad \in R^{n} \text{ is the repetitive control input}$ 

The structure of S suggests that we use the same repetitive control input for the samples that exist in the same state. We now arrive at a vector of length n that corresponds to all the samples where the repetitive states are defined. As a final step, we need to feed-forward this trajectory  $u_{rep}$  in time by d time steps to account for the fact we have a response delay in the system. The number of feed-forward time steps, d, is empirically determined to be 12 for this particular system.

#### V. RESULTS

The algorithm is implemented and the same desired profile is tested on our EHVS. Fig. 8 shows the tracking perfor-

where



Fig. 8. Tracking Performance with the Repetitive Controller

mance achieved by the combination of the linear quadratic tracking controller and the repetitive controller. As can be seen, the tracking error with this set-up is excellent. The maximum tracking error in this case is within  $\pm 0.08$  (mm). To see how the repetitive controller copes with drastic valve profile changes, the following experiments are carried out.

## A. Step Lift Change

The controller is validated in the case of switching between different valve lifts. The two profiles are shown in Fig. 9. We switch between these two profiles every five seconds. At 1800 (rpm), this is equivalent to 75 engine cycles. Fig. 10 shows the RMS tracking error for 300 engine cycles. As can be seen, the RMS tracking error jumps up every time we switch to a different desired profile and decays as time progresses. The higher RMS tracking error immediately after the profile change is expected, though its value of 0.12 (mm) is still acceptable. The repetitive controller manages to squash the RMS error to below 0.04 (mm) as it learns the new input.

## B. Step Opening Duration Change

One immediate problem arises if we want to drastically change the valve opening duration. That is, we need to have different number of repetitive states to accommodate the different opening duration. One example of such a change is shown in Fig. 11. We fix the valve opening timing at 100 (CAD) and varies the closing timing between 300 (CAD) and 240 (CAD). Since the difference between the two profiles is that one has a longer duration at maximum valve lift, we throw away the extra states at maximum lift for the short duration profile, and simply recall the states that we took out before if we are to switch back to the long duration profile. The results of this test is shown in Fig. 12. The engine speed is set at 1800 (rpm) and the desired valve profile is switched every 5 (s). As can be seen, the RMS tracking error is below 0.04 (mm) for all times.

#### C. Engine Speed Transient

In this test, we ramp up the engine speed from 1500 (rpm) to 2500 (rpm) in 3 (s). The engine speed variation is shown in the top plot of Fig. 13. The corresponding RMS tracking



Fig. 9. Low and High Lift Profile



Fig. 10. RMS Tracking Error for Variable Lift

error is shown in the bottom plot of Fig. 14. As can be seen from the plot, the RMS error jumps up when we start increasing the engine speed. After the engine speed settles, the repetitive controller learns a new input and the tracking error converges to a steady state value under 0.04 (mm).

#### VI. CONVERGENCE

Assuming the system is linear and time invariant, the closed loop system response can be represented as a linear function of the reference input and the system initial conditions. During steady state engine operation, the reference input to be tracked and the initial conditions of each engine cycle remain unchanged. Therefore, with the addition of the repetitive controller, the mean tracking at any engine cycle can by related to the mean tracking error at the previous engine cycle through the following equation.

where

$$T = \begin{bmatrix} CA^{d-1}B & CA^{d-2}B & \dots & 0\\ CA^{d}B & CA^{d-1}B & \dots & 0\\ CA^{d+1}B & CA^{d}B & \dots & 0\\ \vdots & \vdots & \ddots & \vdots\\ CA^{d+n-2}B & CA^{d+n-3}B & \dots & CA^{d-1}B \end{bmatrix}$$

 $E^{(k)} = (I - k_I \cdot P \cdot T \cdot S \cdot M) \cdot E^{(k-1)}$ 

(8)

A, B and C are matrices that describe the closed loop dynamics of the system and I is the identity matrix with



Fig. 11. Long and Short Duration Profile



Fig. 12. RMS Tracking Error for Variable Opening Duration

the appropriate dimension. We can further define a matrix  $A_E$  to be :

$$A_E = I - k_I \cdot P \cdot T \cdot S \cdot M \tag{9}$$

Essentially,  $A_E$  describes the error dynamics of the system. Therefore, the mean tracking error approaches zero asymptotically if the eigenvalues of  $A_E$  satisfy the following condition:

$$|\lambda_i(A_E)| < 1 \quad \forall i = 1, 2, \dots, m \tag{10}$$

If all the eigenvalues of  $A_E$  lies within the unit circle on the complex plane then the mean tracking error converges to zero asymptotically.

#### VII. CONCLUSION

In this paper, we present a framework that covers the system identification, feedback controller design and feedforward repetitive controller design that enables accurate tracking performance that has a RMS tracking error under 0.04 (mm). The nonlinear dynamics of the EHVS are approximated by a third order linear model with response delay. A feedback controller is designed and it is found that the tracking performance with using only feedback is insufficient. With the addition of the repetitive controller, however, the tracking error is improved. The repetitive controller performs well during a step valve lift change, a step opening duration change and engine speed transients. It is



Fig. 13. Effect of Engine Speed Transients

furthermore amenable to a simple analysis of stability and convergence. The work presented is also generalizable to other EHVS that operate under similar physical principles.

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