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Uncertainty Measurement Based on General Relation

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***Abstract*— In incomplete information system, new information entropy and conditional entropy based on general relation are proposed. The results that the information entropy is extended from the general relation to equivalent relation and tolerance relation are found. Then the conclusion that the conditional entropy based on general relation decreases monotonously as the neighbor operators become finer is obtained. This paper presents some useful exploration about the incomplete information system from information views.**

***Index Terms*— Rough set; Information entropy; Conditional entropy; General relation; Incomplete information system**

I. INTRODUCTION

Rough set theory is a mathematical tool to deal with vagueness and uncertainty of imprecise data. The theory introduced by Pawlak in 1982 has been developed and found applications in the field of decision analysis, data analysis, pattern recognition, machine learning, and knowledge discovery in databases. While the equivalence relation is too harsh to meet and is extended to tolerance relation and similarity relation. For example, equivalence relation can't be established based on the null value of attribute. In incomplete information systems, which relations are established can be the base of further study for rough computation, knowledge reduction and rule extraction.

For the incomplete information systems, Kryszkietxricz [1] proposed tolerance relation (reflexivity and symmetry), J Stefamowski and A Tsoukeas [2] studied non-symmetric similarity relation (reflexivity and transitivity), Wang [3] analyzed limited tolerance relation (reflexivity and symmetry). Yao [4] introduced neighborhood operators and discussed interrelationship of rough models in general relation.

Information entropy can be used as an uncertainty measurement. In the recent years, many researchers have discussed the relationship between roughness of knowledge, roughness of rough sets and information entropy [5-9], which can be regarded as an interpretation about roughness of knowledge and roughness of rough sets from the information views. Huang et al. discussed rough entropy based on generalized covering [10]. He proposed a tolerance fuzzy-rough sets based on fuzzy tolerance relation. Tolerance fuzzy-rough attribute reduction and the method for measuring fuzziness are also given. Incomplete fuzzy information system is studied by Yang et al.. Furthermore, fuzzy covering on universe is proposed, three different operations on the coverings are formed and then some significant results are gained. Besides, with the two new definitions of fuzzy rough entropies, uncertain factors can be effectively measured, some important relationships between the varieties of uncertain factors and the strength of those entropies are discussed carefully [12]. Liang analyzed information entropy, rough entropy and knowledge granulation in complete information system systemically, and also discussed in incomplete information systems [13]. However, the incomplete information systems is pointed as tolerance relation, few research has be shown about the information entropy in general relation.

General relation exists widely in social relation, for instance, mutual spread of virus, citing behavior of references, acquaintance relation of people etc. So it is necessary to discuss the measurements of general relation. If the information entropies of equivalence relation and tolerance relation are regarded as that of general relation, which only considered the neighborhood size and ignored number of element. Since the number of element also contains information quantity, considering this a new information entropy of general relation is introduced. We can demonstrate the new information entropy of general relation is the extensions of information entropies based on equivalence relation and tolerance relation respectively. The conclusion that the new information entropy of general relation decreases monotonously as the neighborhood becomes finer is obtained.

The paper is organized as follows. In section II, basic notions related to information entropies are introduced. And new information entropy based on general relation is proposed. Furthermore, the interrelationships of entropies based on equivalence relation, tolerance relation, and general relation are discussed. In section III, conditional entropy based on general relation is proposed. Section IV concludes the whole paper.

II. AN NEW INFORMATION ENTROPY BASED ON GENERAL RELATION

Information entropy based on equivalence relation is defined by domain partition. If equivalence relation is extended to tolerance relation or similarity relation, the tolerance class or similarity class can not treated as domain partition,

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but covering. It is that all subclasses may overlap in the tolerance class or the similarity class. In the following, a new information entropy based on general relation is introduced.

Definition 1. (See[14]) Let U be universe, $\{X_1, X_2, \cdots, X_n\}$ is a partition, $p_i = P(X_i)$ is the probability distribution of $\{X_1, X_2, \cdots, X_n\}$. Then $H(X) = -\sum_{i=1}^{n} p_i log p_i$ is information entropy of information source X, where the base of logarithm is 2, and if $p_i = 0$, then $0 \times log 0 = 0$.

Definition 2. (See[15]) Let $S = (U, A)$ be a complete information systems, $U/A = \{X_1, X_2, \cdots, X_m\}$. Information entropy is defined by

$$E_{ind}(A) = \sum_{i=1}^{m} \frac{|X_i|}{|U|}\frac{|X_i^c|}{|U|} = \sum_{i=1}^{m} \frac{|X_i|}{|U|}(1 - \frac{|X_i|}{|U|})$$

where, X_i^c denotes the complementary set of X_i, namely, $X_i^c = U - X_i$. $\frac{|X_i|}{|U|}$ is the probability of X_i in U and $\frac{|X_i^c|}{|U|}$ is the probability of complementary set of X_i in U. Complete information system implies it is equivalence relation.

Definition 3. (See[13]) Let $S = (U, A)$ be an incomplete information systems, $U/SIM(A) = S_A(x_1), S_A(x_2), \cdots, S_A(x_{|U|})$. Then information entropy of A is defined by

$$E_{sim}(A) = \sum_{i=1}^{|U|} \frac{1}{|U|}(1 - \frac{|S_A(u_i)|}{|U|})$$

Generally, the relation we usually defined in incomplete systems is the following: $SIM(A) = \{(x,y) \in U \times U, a \in A, a(x) = a(y)$ or $a(x) = *$ or $a(y) = *\}$. Obviously, $SIM(A) = \bigcap_{a \in A} SIM(a)$ is the similarity relation.

Definition 4. Let $S = (U, A)$, $U = \{x_1, x_2, \cdots, x_{|U|}\}$ and $|x_i|_A = |\{n_A(x_j) | x_i \in n_A(x_j), 1 \le j \le |U|\}|$. Then information entropy of A in general relation is defined by

$$E_n(A) = \sum_{i=1}^{|U|} \left[\frac{1}{|U|} \sqrt{(1 - \frac{|n_A(x_i)|}{|U|}) \times (1 - \frac{|x_i|_A}{|U|})} \right]$$

Where $|x_i|_A$ is number of x_i appearance in each neighborhood x_j $(1 \le j \le |U|)$. n_A denotes the neighborhood operator in general relation. For reflexivity of the general relation, $x_i \in n_A(x_i), i = 1, 2, \cdots, |U|$.

Lemma 1. Let $S = (U, A)$, $U = \{x_1, x_2, \cdots, x_{|U|}\}$. The information entropy of A in general relation can obtain its maximum $E(A) = 1 - 1/|U|$ if and only if $n_A(x_i) = (x_i)$, and it can also obtain its minimum 0 if and only if $n_A(x_i) = U$.

Proof: If information entropy of A in general relation obtains maximum $E(A) = 1 - 1/|U|$, then $|x_i|_A$ and $|n_A(x_i)|$ need to be gotten the minimum values at the same time. Since A is the general relation, each x_i satisfies reflexivity $x_i \in n_A(x_i)$, thus $|x_i|_{min} = 1$ and $|n_p(x_i)|_{min} = 1$, namely, $n_A(x_i) = \{x_i\}$. On the contrary, if $n_A(x_i) = \{x_i\}$, $|x_i|_A$ and $|n_A(x_i)|$ can all obtain minimum values 1, which make information entropy of A obtaining maximum value $E(A) = 1 - 1/|U|$.

With the same method, the information entropy of A in general relation can obtain its minimum value 0 if and only if $n_A(x_i) = U$.

Lemma 2. Let $S = (U, A)$ be a complete information system, $U/A = \{n_A(x_1), n_A(x_2), \cdots, n_A(x_{|U|})\}$, $U/IND(A) = \{X_1, X_2, \cdots, X_m\}$, then we have $E_n(A) = E_{ind}(A)$.

Proof: Let $U/IND(A) = \{X_1, X_2, \cdots, X_m\}$, $X_i = \{x_{i1}, x_{i2}, \cdots, x_{iS_i}\}$, where $IND(A)$ be an equivalence relation, $|X_i| = S_i$, and $\sum_{i=1}^{m} |S_i| = |U|$. Then we have the following equations,

$$X_i = n_A(x_{i1}) = n_A(x_{i2}) = \cdots = n_A(x_{iS_i}).$$

It is that

$$|X_i| = |n_A(x_{i1})| = |n_A(x_{i2})| = \cdots = |n_A(x_{iS_i})|.$$

Where number of appearance $x_{ij} (1 \le j \le S_i)$ in X_i is denoted by $|x_{ij}|_A$, so we get

$$|x_{i1}|_A = |x_{i2}|_A == |x_{iS_i}|_A = |X_i| = S_i.$$

Therefore, the following equality hold

$$\begin{aligned}
&\frac{|X_i|}{|U|}(1 - \frac{|X_i|}{|U|}) \\
=&\frac{1}{|U|}\left[(1 - \frac{|X_i|}{|U|}) + (1 - \frac{|X_i|}{|U|}) + \cdots + (1 - \frac{|X_i|}{|U|})\right] \\
=&\frac{1}{|U|}\left[\sqrt{(1 - \frac{|n_A(x_{i1})|}{|U|})(1 - \frac{|x_{i1}|}{|U|})} + \cdots \right.\\
&\left. + \sqrt{(1 - \frac{|n_A(x_{iS_i})|}{|U|})(1 - \frac{|x_{iS_i}|}{|U|})}\right]
\end{aligned}$$

Hence,

$$\begin{aligned}
&E_{ind}(A) \\
=&\sum_{i=1}^{m} \frac{|X_i|}{|U|}(1 - \frac{|X_i|}{|U|}) \\
=&\sum_{i=1}^{m} \left[\frac{1}{|U|}(1 - \frac{|X_i|}{|U|}) + \cdots + (1 - \frac{|X_i|}{|U|})\right] \\
=&\sum_{i=1}^{m} \frac{1}{|U|}\left[\sqrt{(1 - \frac{|n_A(x_{i1})|}{|U|})(1 - \frac{|x_{i1}|_A}{|U|})} + \cdots \right.\\
&\left. + \sqrt{(1 - \frac{|n_A(x_{iS_i})|}{|U|})(1 - \frac{|x_{iS_i}|_A}{|U|})}\right] \\
=&\sum_{i=1}^{|U|} \frac{1}{|U|}\left[\sqrt{(1 - \frac{|n_A(x_i)|}{|U|})(1 - \frac{|x_i|_A}{|U|})} + \cdots \right.\\
&\left. + \sqrt{(1 - \frac{|n_A(x_{|U|})|}{|U|})(1 - \frac{|x_{|U|}|_A}{|U|})}\right] \\
=&E_n(A)
\end{aligned}$$

From the Lemma 2, we have the information entropy of general relation is the extension of that of equivalence relation.

Lemma 3. Let $S = (U, A)$ be an incomplete information system, $U/A = \{n_A(x_1), n_A(x_2), \cdots, n_A(x_{|U|})\}$, $U/SIM(A) = \{S_A(x_1), S_A(x_2), \cdots, S_A(x_{|U|})\}$. $SIM(A)$ denotes a tolerance relation. Then we have the following equality $E_n(A) = E_{sim}(A)$.

Proof: $SIM(A)$ denotes a tolerance relation , obviously. For any $S_A(x_i) = n_A(x_i) = \{x_{i1}, x_{i2}, \cdots, x_{iT_i}\}$, by reflexivity,

$x_i \in S_A(x_i)$; by symmetry, $x_i \in S_A(x_{i1}), x_i \in S_A(x_{i2}), \cdots, x_i \in S_A(x_{iT_i})$, and $\forall x_j \in \{U - S_A(x_i)\}$, $x_i \notin S_A(x_j)$. It is that x_i belongs to the neighborhood of elements which are in $S_A(x_i)$, and will not appear in other neighborhoods. Hence, the number of x_i appearance is $|x_i|_A = |S_A(x_i)|$. Obviously,

$$\frac{1}{|U|}(1 - \frac{|S_A(x_i)|}{|U|}) = \frac{1}{|U|}\sqrt{(1 - \frac{|n_A(x_i)|}{|U|}) \times (1 - \frac{|x_i|_A}{|U|})}.$$

Therefore,

$$\begin{aligned} E_{sim}(A) =& \sum_{i=1}^{|U|} \frac{1}{|U|}(1 - \frac{|S_A(x_i)|}{|U|}) \\ =& \sum_{i=1}^{|U|} \left[\frac{1}{|U|}\sqrt{(1 - \frac{|n_A(x_i)|}{|U|}) \times (1 - \frac{|x_i|_A}{|U|})} \right] \\ =& E_n(A) \end{aligned}$$

From the Lemma 3, we have the information entropy of general relation is the extension of that of tolerance relation.

Lemma 4. Let $S = (U, A)$ be an information system of general relation, $P, Q \subseteq A$. If $P \subset Q$, then $E_n(Q) < E_n(P)$.

Proof. Let $P \subset Q$, for $\forall i \in \{1, 2, \cdots, |U|\}$, we have $n_P(x_i) \subseteq n_Q(x_i)$. Clearly, $|x_i|_P \le |x_i|_Q$. There exists $i \in \{1, 2, \cdots, |U|\}$ such that $n_P(x_i) \subset n_Q(x_i)$ or $|x_i|_P < |x_i|_Q$. Therefore

$$\begin{aligned} E_n(Q) =& \sum_{i=1}^{|U|} \frac{1}{|U|}\sqrt{(1 - \frac{|n_Q(x_i)|}{|U|})(1 - \frac{|x_i|_Q}{|U|})} \\ <& \sum_{i=1}^{|U|} \frac{1}{|U|}\sqrt{(1 - \frac{|n_P(x_i)|}{|U|})(1 - \frac{|x_i|_P}{|U|})} \\ =& E_n(P). \end{aligned}$$

III. Conditional entropy based on general relation

Definition 5. Let $S = (U, A)$ be an information systems of general relation, $P, Q \subseteq A$. The conditional entropy is defined as follows

$$\begin{aligned} & E_n(Q|P) \\ =& \frac{1}{|U|}\sum_{i=1}^{|U|}\sum_{j=1}^{|U|} \left[\sqrt{\frac{|n_Q(x_i) \cap n_P(x_j)|}{|U|}\frac{|x_{Qi}x_{Pj}|}{|U|}} \right. \\ & \left. \times \sqrt{\frac{|n_P(x_j) - n_Q(x_i)|}{|U|}\frac{|x_{Pj}\bar{x}_{Qi}|}{|U|}} \right]. \end{aligned}$$

Where $|x_{Qi}x_{Pj}|$ denotes number of appearance of x_i and x_j not only in the relation P, but also in the relation Q; $x_{Pj}\bar{x}_{Qi}$ denotes number of appearance of x_i and x_j in the relation P, but not in the relation Q.

Lemma 5. Let $S = (U, A)$ be an information systems of general relation, $P, Q \subseteq A$. If $P \subset Q$, then $E_n(D|P) < E_n(D|Q)$.

Proof: Let $P \subset Q$, for $\forall i \in \{1, 2, \cdots, |U|\}$, $n_P(x_i) \subseteq n_Q(x_i)$, we have $|n_P(x_i)| < |n_Q(x_i)|$, $|n_D(x_i) \cap n_P(x_j)||x_{Di}x_{Pj}| < |n_D(x_i) \cap n_Q(x_j)||x_{Di}x_{Qj}|$, $|n_P(x_j) - n_D(x_i)||x_{Pj}\bar{x}_{Di}| < |n_Q(x_j) - n_D(x_i)||x_{Qj}\bar{x}_{Di}|$. Therefore

$$\begin{aligned} E_n(D|P) =& \frac{1}{|U|}\sum_{i=1}^{|U|}\sum_{j=1}^{|U|} \sqrt{\frac{|n_D(x_i) \cap n_P(x_j)|}{|U|}\frac{|x_{Di}x_{Pj}|}{|U|}} \\ & \times \sqrt{\frac{|n_P(x_j) - n_D(x_i)|\frac{|x_{Pj}\bar{x}_{Di}|}{|U|}}{|U|}} \\ <& \frac{1}{|U|}\sum_{i=1}^{|U|}\sum_{j=1}^{|U|} \sqrt{\frac{|n_D(x_i) \cap n_Q(x_j)|}{|U|}\frac{|x_{Di}x_{Qj}|}{|U|}} \\ & \times \sqrt{\frac{|n_Q(x_j) - n_D(x_i)|\frac{|x_{Qj}\bar{x}_{Di}|}{|U|}}{|U|}} \\ =& E_n(D|Q) \end{aligned}$$

namely, $E_n(D|P) < E_n(D|Q)$.

Lemma 6. Let $S = (U, A)$ be a complete information system, $U/P = \{n_P(x_1), n_P(x_2), \cdots, n_P(x_{|U|})\}$, $U/Q = \{n_Q(x_1), n_Q(x_2), \cdots, n_Q(x_{|U|})\}$; $U/IND(P) = \{X_1, X_2, \cdots, X_m\}$, $|X_i| = S_i$, $U/IND(Q) = \{Y_1, Y_2, \cdots, Y_n\}$, $|Y_i| = V_i$. Then the relationship of conditional entropy between in general relation and in equivalence is as follows

$$E_n(Q|P) = \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{V_j S_i}{|U|}\frac{|X_i \cap Y_j|}{|U|}\frac{|X_i^c - Y_j^c|}{|U|}.$$

Proof: Let $U/IND(P) = \{X_1, X_2, \cdots, X_m\}$, $X_i = \{x_{i1}, x_{i2}, \cdots, x_{iS_i}\}$, where $|X_i| = S_i$ and $\sum_{i=1}^{m}|S_i| = |U|$; $U/IND(Q) = \{Y_1, Y_2, \cdots, Y_n\}$, $Y_i = \{x_{i1}, x_{i2}, \cdots, x_{iV_i}\}$, where $|Y_i| = V_i$ and $\sum_{i=1}^{n}|Y_i| = |U|$. Then the elements in U/P and these in $U/IND(P)$ have the following relationship

$$X_i = n_P(x_{i1}) = n_P(x_{i2}) = \cdots = n_P(x_{iS_i}),$$

namely,

$$|X_i| = |n_P(x_{i1})| = |n_P(x_{i2})| = \cdots = |n_P(x_{iS_i})|.$$

The elements in U/Q and these in $U/IND(Q)$ have the following relationship

$$|Y_i| = |n_Q(x_{i1})| = |n_Q(x_{i2})| = \cdots = |n_Q(x_{iS_i})|.$$

We can obtain $|n_Q(x_i) \cap n_P(x_j)| = |x_{Qi}x_{Pj}| = |X_i \cap Y_j|$, $|n_P(x_j) - n_Q(x_i)| = |x_{Pj}\bar{x}_{Qi}| = |X_i^c - Y_j^c|$, so

$$\begin{aligned} & E_n(Q|P) \\ =& \frac{1}{|U|}\sum_{i=1}^{|U|}\sum_{j=1}^{|U|} \left[\sqrt{\frac{|n_Q(x_i) \cap n_P(x_j)|}{|U|}\frac{|x_{Qi}x_{Pj}|}{|U|}} \right. \\ & \left. \sqrt{\frac{|n_P(x_j) - n_Q(x_i)|}{|U|}\frac{|x_{Pj}\bar{x}_{Qi}|}{|U|}} \right] \\ =& \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{V_j S_i}{|U|}\frac{|X_i \cap Y_j|}{|U|}\frac{|X_i^c - Y_j^c|}{|U|}. \end{aligned}$$

From the lemma 6, we can see weighed sum of components which is of conditional entropy in equivalence relation is the conditional entropy of general relation, where weight coefficients relate to the sizes of two domain.

Lemma 7. Let $S=(U,A)$ be a complete system, $U/P=\{n_P(x_1),n_P(x_2),\cdots,n_P(x_{|U|})\}$, $U/Q=\{n_Q(x_1),n_Q(x_2),\cdots,n_Q(x_{|U|})\}$; $U/IND(P)=\{X_1,X_2,\cdots,X_m\}$, $U/IND(Q)=\{Y_1,Y_2,\cdots,Y_n\}$. If $n_P(x_i)=\{x_i\}$ and $n_Q(x_i)=\{x_i\}$, $E_n(Q|P)=\frac{1}{|U|}E_{ind}(Q|P)$; if $n_P(x_i)=U$ and $n_Q(x_i)=U$, $E_n(Q|P)=0$.

Proof: From the proof procedure of Lemma 6, we can know

$$E_n(Q|P)=\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{V_jS_i}{|U|}\frac{|X_i\cap Y_j|}{|U|}\frac{|X_i^c-Y_j^c|}{|U|}.$$

If $n_P(x_i)=\{x_i\}$ and $n_Q(x_i)=\{x_i\}$, then $V_j=S_i=1$. Therefore, $E_n(Q|P)=\frac{1}{|U|}E_{ind}(Q|P)$. If $n_P(x_i)=U$ and $n_Q(x_i)=U$, according to the definition of conditional entropy, $E_n(Q|P)=0$.

Example 1. Let $\{x_1,x_2,\cdots,x_{10}\}$, $U/P=\{\{x_1,x_5\},\{x_2,x_3,x_4,x_6,x_7\},\{x_8,x_9,x_{10}\}\}$, $U/Q=\{\{x_1,x_3,x_4\},\{x_2,x_5,x_6\},\{x_7,x_8,x_9,x_{10}\}\}$, $U/D=\{\{x_1,x_3,x_5,x_8,x_9\},\{x_2,x_4,x_6,x_7,x_{10}\}\}$. If it is general relation, then $E_n(D|P)=0.26$, $E_n(D|Q)=0.33$. Obviously, $E_n(D|P)<E_n(D|Q)$, while $P\subset Q$ does not hold.

Lemma 8. Let $S_1(U,P)$ and $S_2(U,Q)$ be two information systems of general relations, $P\subset Q$ if and only if $E(Q|P)=0$.

Proof: Suppose $P\subset Q$, for any $n_P(x_j)$ and $n_Q(x_j)$, we have $n_P(x_j)\cap n_Q(x_j)=\emptyset$. Therefore for any $n_P(x_j)$ and $n_Q(x_j)$, we obtain $\sqrt{|n_Q(x_i)\cap n_P(x_j)|\times|x_{Qi}x_{Pj}|}=0$, $\sqrt{|n_P(x_j)-n_Q(x_i)|\times|x_{Pj}\bar{x}_{Qi}|}=0$. Thus

$$\begin{aligned}&E_n(Q|P)\\ =&\frac{1}{|U|}\sum_{i=1}^{|U|}\sum_{j=1}^{|U|}\left[\sqrt{\frac{|n_Q(x_i)\cap n_P(x_j)|}{|U|}\frac{|x_{Qi}x_{Pj}|}{|U|}}\right.\\ &\left.\sqrt{\frac{|n_P(x_j)-n_Q(x_i)|}{|U|}\frac{|x_{Pj}\bar{x}_{Qi}|}{|U|}}\right]\\ =&0.\end{aligned}$$

Suppose $E_n(Q|P)=0$, if $P\subset Q$ is not hold, there at least exist $n_P(x_j)$ and $n_Q(x_j)$ such that $n_P(x_j)\supset n_Q(x_j)$, then $\sqrt{|n_Q(x_i)\cap n_P(x_j)|\times|x_{Qi}x_{Pj}|}\neq 0$, $\sqrt{|n_P(x_j)-n_Q(x_i)|\times|x_{Pj}\bar{x}_{Qi}|}\neq 0$. Therefore, $E_n(Q|P)>0$. If $n_P(x_j)\cap n_Q(x_j)\neq\emptyset$, in a similar way, we can get $E_n(Q|P)>0$, which is contrary to the assumption. Thus $P\subset Q$.

IV. CONCLUSIONS

New information entropy of general relation are proposed. After analyzing the information entropies of the equivalence relation and tolerance relation, we conclude the information entropy of general relation is the extension of two information entropies in two above relations and corresponding proof are given. Furthermore, the property of the conditional entropy is studied. Then the conclusion that the conditional entropy based on general relation decreases monotonously as the neighbor operators become finer is obtained.

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