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An Adaptive Sliding Mode Observer for Induction Machines

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***Abstract*— Control of induction machines in the so-called sensorless paradigm forms an interesting academic and practical problem. In this paper, an observer-controller system is proposed that can be used to control an induction machine with unknown parameters and partial state variable information. The observer is used to obtain the rotor flux linkage vector, needed for the sliding mode control based torque and flux control laws, while estimating speed and the unknown rotor resistance at the same time. In a nut shell, this system provides a unified sensorless control technique.**

I. INTRODUCTION

Discontinuous control offered by sliding mode control theory makes it a natural choice for controlling inverter driven induction machines. Apart from this advantage, this theory provides simplicity in control design and robustness against unknown or time-varying plant parameters. However, irrespective of the control algorithm, information on motor speed, flux linkages etc. are required. Recent advanced developments in data acquisition and processing allow the implementation of the sensorless control idea, this paper presumes the availability of such devices and offers a sliding mode based observer-controller system.

Control of an induction machine has been a subject of much study over the past several decades. Sliding mode control has also been presented abundantly in technical literature, both from a theoretical and implementation perspective. Hence, in this paper, background details are kept to the minimum and the interested or novice control engineer is directed to [1], [2], and [3] for fundamentals of sliding mode control theory and induction machines.

Design of torque and flux control in an induction machine without the use of speed or flux sensors is the main focus of this paper. In terms of stator currents and rotor flux variables, an induction machine is described by a fifth-order non-linear system with stator voltages as inputs and stator currents as output; a detailed derivation of the equations used in this paper is provided in [4]. The unknown quantities are the speed, rotor fluxes, and finally, the rotor resistance, which can vary significantly, albeit slowly, with temperature. The problem statements can be described as follows: first, estimate the unknown variables and second, use these estimates to design control. An observer is designed to solve the first problem and a controller based on the Field-Oriented Control (FOC) technique [2], [4], is used for the second; sliding mode control theory is applied in solving both problems. For more ideas on observers: [5] is an exhaustive survey on sliding mode based observer designs for AC machines; a more basic version of the observer proposed in this paper can be found in [6], [7].

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II. MATHEMATICAL MODELS

A. Induction machine

From [4], [6],

$$\begin{aligned}
\dot{i}_\alpha &= \beta\eta\lambda_\alpha + \beta\omega\lambda_\beta - \gamma i_\alpha + \frac{1}{\sigma L_S}u_\alpha \\
\dot{i}_\beta &= \beta\eta\lambda_\beta - \beta\omega\lambda_\alpha - \gamma i_\beta + \frac{1}{\sigma L_S}u_\beta \\
\dot{\lambda}_\alpha &= -\eta\lambda_\alpha - \omega\lambda_\beta + \eta L_M i_\alpha \\
\dot{\lambda}_\beta &= -\eta\lambda_\beta + \omega\lambda_\alpha + \eta L_M i_\beta \\
\dot{\omega} &= \frac{PL_M}{JL_R}(\lambda_\alpha i_\beta - \lambda_\beta i_\alpha) - \frac{T_L}{J}
\end{aligned} \tag{1}$$

are the induction machine equations in the standard $(\alpha,\ \beta)$ frame. The known entities in (1) are: the stator currents $[i_\alpha\ i_\beta]^T$, the input stator voltages $[u_\alpha\ u_\beta]^T$; also known are the positive constants: $\sigma = 1 - \frac{L_M^2}{L_S L_R}$, $\beta = \frac{L_M}{\sigma L_S L_R}$, and $\gamma = \frac{R_S + (L_M^2/L_R^2)R_R}{\sigma L_S}$. The unknowns, which need to be estimated, are: the rotor fluxes $[\lambda_\alpha\ \lambda_\beta]^T$, the electrical rotor angle velocity ω, and the positive constant $\eta = \frac{R_R}{L_R}$. L_S, L_R are the stator and rotor inductances, R_S, R_R are the stator and rotor resistances, and L_M is the mutual inductance; on the mechanical side, J is the known rotor inertia and T_L is a constant and known load torque. Although γ depends on time-varying R_R, it is assumed to be constant with a chosen nominal R_R.

B. Adaptive Observer

In keeping with the problem statement mentioned before, the unknowns have to be estimated. Consider the first 4 equations of (1) - the electrical equations. The observer model is selected as

$$\begin{aligned}
\dot{\hat{i}}_\alpha &= \beta\hat{\eta}\hat{\lambda}_\alpha + \beta\hat{\omega}\hat{\lambda}_\beta - \gamma i_\alpha + \frac{1}{\sigma L_S}u_\alpha - \beta\hat{\lambda}_\alpha\mu \\
\dot{\hat{i}}_\beta &= \beta\hat{\eta}\hat{\lambda}_\beta - \beta\hat{\omega}\hat{\lambda}_\alpha - \gamma i_\beta + \frac{1}{\sigma L_S}u_\beta - \beta\hat{\lambda}_\beta\mu \\
\dot{\hat{\lambda}}_\alpha &= -\hat{\eta}\hat{\lambda}_\alpha - \hat{\omega}\hat{\lambda}_\beta + \hat{\eta}L_M i_\alpha + C\hat{\lambda}_\beta\mu \\
\dot{\hat{\lambda}}_\beta &= -\hat{\eta}\hat{\lambda}_\beta + \hat{\omega}\hat{\lambda}_\alpha + \hat{\eta}L_M i_\beta - C\hat{\lambda}_\alpha\mu
\end{aligned} \tag{2}$$

where $\hat{i}_{\alpha,\beta}$ and $\hat{\lambda}_{\alpha,\beta}$ are current and flux estimates; $\hat{\eta}$ and $\hat{\omega}$ are rotor resistance and speed estimates; μ and C form the observer input which is to be selected. In order to justify the choice of the observer model, the problem should be described in some detail.

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III. PROBLEM STATEMENT

The conventional control problem concerning induction machines is that of position or speed control. In this paper, only speed control will be considered with the induction machine acting as a motor; the position control problem can be similarly tackled, but with the use of a position sensor. From the mechanical equation in (1)

$$\dot{\omega} = \frac{PL_M}{JL_R}T_u - \frac{T_L}{J} \qquad (3)$$
$$T_u = (\lambda_\alpha i_\beta - \lambda_\beta i_\alpha)$$

the speed ω can be controlled using the motor output torque T_u as control. The desired control torque can be provided by assigning appropriate values to the stator currents and rotor fluxes using the input stator voltages, which are, in fact, the real control inputs.

It is clear from (3), that $\lambda_{\alpha,\beta}$ should be known to achieve desired T_u. The observer model (2) can provide this information accurately only if the estimates $\hat{\eta}$ and $\hat{\omega}$ are correct. Hence, the observer should be equipped to estimate parameters as well. The following sections are devoted to showing the convergence of the estimates to the true values.

IV. OBSERVER ANALYSIS

The estimation of currents in (2) might appear to be a redundant feature. Since the fluxes depend on the known currents, they may be calculated using a combination of integration techniques and parameter update laws to estimate ω and η. But, integration without the knowledge of initial fluxes leads to errors in the form of dc drift. Moreover, open-loop techniques which need some knowledge of parameters suffer from lack of robustness in case of parameter variations. Hence, a full-order observer with discontinuous inputs designed from principles of sliding mode control theory is used.

A. Current Estimate Errors

Define the estimate errors: $\tilde{i}_{\alpha,\beta} = \hat{i}_{\alpha,\beta} - i_{\alpha,\beta},\ \tilde{\lambda}_{\alpha,\beta} = \hat{\lambda}_{\alpha,\beta} - \lambda_{\alpha,\beta},\ \tilde{\eta} = \hat{\eta} - \eta,\ \tilde{\omega} = \hat{\omega} - \omega$. From (1) and (2), the error dynamics are

$$\begin{aligned}\dot{\tilde{i}}_\alpha &= \beta\tilde{\eta}\hat{\lambda}_\alpha + \beta\eta\tilde{\lambda}_\alpha + \beta\hat{\omega}\hat{\lambda}_\beta - \beta\omega\lambda_\beta - \beta\hat{\lambda}_\alpha\mu \\ \dot{\tilde{i}}_\beta &= \beta\tilde{\eta}\hat{\lambda}_\beta + \beta\eta\tilde{\lambda}_\beta - \beta\hat{\omega}\hat{\lambda}_\alpha + \beta\omega\lambda_\alpha - \beta\hat{\lambda}_\beta\mu \end{aligned} \qquad (4)$$

$$\begin{aligned}\dot{\tilde{\lambda}}_\alpha &= -\tilde{\eta}\hat{\lambda}_\alpha - \eta\tilde{\lambda}_\alpha - \hat{\omega}\hat{\lambda}_\beta + \omega\lambda_\beta + \tilde{\eta}L_m i_\alpha + C\hat{\lambda}_\beta\mu \\ \dot{\tilde{\lambda}}_\beta &= -\tilde{\eta}\hat{\lambda}_\beta - \eta\tilde{\lambda}_\beta + \hat{\omega}\hat{\lambda}_\alpha - \omega\lambda_\alpha + \tilde{\eta}L_m i_\beta - C\hat{\lambda}_\alpha\mu \end{aligned} \qquad (5)$$

The principles of sliding mode control theory will be implemented to drive the estimate errors to zero. This requires the definition of a switching surface and a control that undergoes discontinuities on this surface [1]. So, define the surfaces

$$\begin{aligned} s_1 &= \tilde{i}_\beta\hat{\lambda}_\alpha - \tilde{i}_\alpha\hat{\lambda}_\beta \\ s_2 &= \tilde{i}_\alpha\hat{\lambda}_\alpha + \tilde{i}_\beta\hat{\lambda}_\beta \end{aligned} \qquad (6)$$

and from [1], the derivative of $s_{1,2}$ needs to be calculated to choose discontinuous control. From (2), (4)

$$\begin{aligned} \dot{s}_1 &= \beta\eta(\hat{\lambda}_\alpha\tilde{\lambda}_\beta - \tilde{\lambda}_\alpha\hat{\lambda}_\beta) - \beta(\hat{\omega}(\hat{\lambda}_\alpha^2 + \hat{\lambda}_\beta^2)) \\ &\quad + \beta\omega(\hat{\lambda}_\alpha\lambda_\alpha + \hat{\lambda}_\beta\lambda_\beta) - \hat{\eta}(s_1 + L_M(i_\beta\tilde{i}_\alpha - i_\alpha\tilde{i}_\beta)) \\ &\quad + (C\mu - \hat{\omega})s_2 \\ \dot{s}_2 &= \beta(\hat{\lambda}_\alpha^2 + \hat{\lambda}_\beta^2)(\tilde{\eta} - \mu) + \beta\eta(\hat{\lambda}_\alpha\tilde{\lambda}_\alpha + \hat{\lambda}_\beta\tilde{\lambda}_\beta) \\ &\quad + \beta\omega(\lambda_\alpha\hat{\lambda}_\beta - \hat{\lambda}_\alpha\lambda_\beta) - \hat{\eta}(s_2 + L_M(i_\alpha\tilde{i}_\alpha + i_\beta\tilde{i}_\beta)) \\ &\quad + (\hat{\omega} - C\mu)s_1 \end{aligned} \qquad (7)$$

By choosing $\hat{\omega} = \omega_0 sign(s_1)$ and $\mu = \mu_0 sign(s_2)$, sliding mode can be enforced on the intersection of the surfaces $s_{1,2}$, so

$$\begin{aligned} s_{1,2} &\to 0 \\ \dot{s}_{1,2} &\to 0 \end{aligned} \qquad (8)$$

Proof of existence of sliding mode and method of selection of control magnitudes ω_0 and μ_0 can be found in [1]. With the occurrence of sliding mode

$$\tilde{i}_{\alpha,\beta} \to 0 \qquad (9)$$

B. Flux Estimate Errors

The flux error equations (5) become the sliding mode equations after substitution of the so-called equivalent control [1]. This reduction in system order (the error dynamics (4), (5) form a 4^{th} order system) - equal to the order of the switching surface vector - which sliding mode theory offers, makes feedback analysis relatively easier. Thus, the stability and convergence properties of (5) alone should be analysed. Hence, equivalent control - the solution to $\dot{s}_{1,2} = 0$ - should be found. From (7), (8), (9)

$$\begin{aligned} \hat{\omega}_{eq} &= \frac{\omega(\hat{\lambda}_\alpha\lambda_\alpha + \hat{\lambda}_\beta\lambda_\beta)}{\hat{\lambda}^2} + \frac{\eta(\hat{\lambda}_\alpha\tilde{\lambda}_\beta - \tilde{\lambda}_\alpha\hat{\lambda}_\beta)}{\hat{\lambda}^2} \\ \mu_{eq} &= \tilde{\eta} + \frac{\omega(\lambda_\alpha\hat{\lambda}_\beta - \hat{\lambda}_\alpha\lambda_\beta)}{\hat{\lambda}^2} + \frac{\eta(\hat{\lambda}_\alpha\tilde{\lambda}_\alpha + \hat{\lambda}_\beta\tilde{\lambda}_\beta)}{\hat{\lambda}^2} \end{aligned} \qquad (10)$$

where $\hat{\lambda}^2 = \hat{\lambda}_\alpha^2 + \hat{\lambda}_\beta^2$. From [1], the continuous equivalent control can be found as the output of a low pass filter whose input is the actual discontinuous control, thus, $\hat{\omega}_{eq}$ and μ_{eq} are available. The filtering process needs to be performed as the right-hand sides of (10) contain unknown entities such as $\eta,\ \lambda_{\alpha,\beta}$ and so $\hat{\omega}_{eq},\ \mu_{eq}$ cannot be directly calculated.

To simplify matters, define new error variables

$$\begin{aligned} e_1 &= \hat{\lambda}_\alpha\tilde{\lambda}_\alpha + \hat{\lambda}_\beta\tilde{\lambda}_\beta \\ e_2 &= \hat{\lambda}_\alpha\tilde{\lambda}_\beta - \tilde{\lambda}_\alpha\hat{\lambda}_\beta \end{aligned} \qquad (11)$$

and find their derivatives

$$\begin{aligned} \dot{e_1} &= -\tilde{\eta}\hat{\lambda}^2 - 2\eta e_1 - \tilde{\eta}e_1 + (\hat{\omega}_{eq} - C\mu_{eq} - \omega)e_2 \\ &\quad + L_M(\tilde{\eta}(i_\alpha\hat{\lambda}_\alpha + i_\beta\hat{\lambda}_\beta) + \hat{\eta}(i_\alpha\tilde{\lambda}_\alpha + i_\beta\tilde{\lambda}_\beta)) \\ \dot{e_2} &= -2\eta e_2 - \tilde{\eta}e_2 - (\hat{\omega}_{eq} - C\mu_{eq} - \omega)(e_1 - \hat{\lambda}^2) \\ &\quad + L_M(\tilde{\eta}(i_\beta\hat{\lambda}_\alpha - i_\alpha\hat{\lambda}_\beta) + \hat{\eta}(i_\alpha\tilde{\lambda}_\beta - i_\beta\tilde{\lambda}_\alpha)) \end{aligned} \qquad (12)$$

using $(\hat{\lambda}_\alpha \lambda_\alpha + \lambda_\beta \hat{\lambda}_\beta) = (\hat{\lambda}^2 - e_1)$ and $(\hat{\lambda}_\alpha \lambda_\beta - \lambda_\alpha \hat{\lambda}_\beta) = -e_2$.

Introduce the transforms

$$
\begin{aligned}
i_d &= (\hat{\lambda}_\alpha i_\alpha + \hat{\lambda}_\beta i_\beta)/\hat{\lambda} \\
i_q &= (\hat{\lambda}_\alpha i_\beta - \hat{\lambda}_\beta i_\alpha)/\hat{\lambda} \qquad \hat{\lambda} = (\hat{\lambda}_\alpha^2 + \hat{\lambda}_\beta^2)^{0.5}
\end{aligned} \tag{13}
$$

which are actually similar to the current vectors in the $(d,\ q)$ frame [4]. By solving the pair of simultaneous equations (11) and (13), for $i_{\alpha,\beta}$ and $\tilde{\lambda}_{\alpha,\beta}$, (12) becomes

$$
\begin{aligned}
\dot{e_1} &= -\tilde{\eta}\hat{\lambda}^2 - 2\eta e_1 - \tilde{\eta} e_1 + (\hat{\omega}_{eq} - C\mu_{eq} - \omega) e_2 \\
&\quad + L_M \tilde{\eta}\hat{\lambda} i_d + \frac{L_M \hat{\eta}}{\hat{\lambda}}(i_d e_1 + i_q e_2) \\
\dot{e_2} &= -2\eta e_2 - \tilde{\eta} e_2 - (\hat{\omega}_{eq} - C\mu_{eq} - \omega)(e_1 - \hat{\lambda}^2) \\
&\quad + L_M \tilde{\eta}\hat{\lambda} i_q + \frac{L_M \hat{\eta}}{\hat{\lambda}}(i_d e_2 - i_q e_1)
\end{aligned} \tag{14}
$$

Before continuing the analysis of (14), which requires controller design for the assignment of i_q and i_d, some discussion on the adaptive capacity of the observer is in order.

C. Rotor Resistance Estimation

For the moment, assume that the errors $e_{1,2} \to 0$, then from (10), the equivalent controls $\hat{\omega}_{eq} \to \omega$ and $\mu_{eq} \to \tilde{\eta}$. Using this result and that $\eta = \hat{\eta} - \mu_{eq}$, the rotor resistance $R_R = L_R * \hat{\eta}$ can be found. But, this means that the initial estimate of $\hat{\eta}$ should be very close to the true η, which is unknown. Moreover, η varies with the motor temperature, so a static calculation definitely leads to an error in parameter estimation. Therefore, a dynamic law should be used so that the estimate converges to the true value. Proposition: the law

$$
\dot{\hat{\eta}} = -\gamma \mu_{eq} \tag{15}
$$

in tandem with the observer and where the gain $\gamma > 0$ will ensure parameter convergence. The proof is quite simple. Indeed, if η varies very slowly in time, $\dot{\hat{\eta}} = \dot{\tilde{\eta}}$. Hence, from (15)

$$
\dot{\tilde{\eta}} = -\gamma \tilde{\eta} \tag{16}
$$

and for $\gamma > 0$, $\hat{\eta} \to \eta$. However, for (16) to be true, that $e_{1,2} \to 0$ is still to be proved and this is the subject of the next section.

V. CONTROLLER-OBSERVER SYSTEM

A. Controller Design

In (14), some knowledge about the variables $i_q,\ i_d$ is required. This knowledge can be gained from the very idea of the FOC technique, which involves controlling these exact variables. Hence, to prove the convergence of the observer errors, the joint controller-observer system must be studied.

The FOC technique has proved to be a very convenient method to control an induction machine. It involves transforming the machine variables in the stationary $(\alpha,\ \beta)$ frame to the $(d,\ q)$ frame making them rotate with the rotor flux variables $(\lambda_{\alpha,\beta})$. From [4], the motor equations in the $(d,\ q)$ frame are

$$
\begin{aligned}
\dot{i}_q &= -\gamma i_q - \omega i_d - \beta\omega\lambda_d - \eta L_M \frac{i_d i_q}{\lambda_d} + \frac{1}{\sigma L_S} u_q \\
\dot{i}_d &= -\gamma i_d + \omega i_q + \beta\omega\lambda_d + \eta L_M \frac{i_q^2}{\lambda_d} + \frac{1}{\sigma L_S} u_d \\
\dot{\lambda}_d &= -\eta\lambda_d + \eta L_M i_d \\
T_u &= \lambda_d i_q
\end{aligned} \tag{17}
$$

As mentioned in the problem statement, the control problem in the $(d,\ q)$ frame becomes that of making the state variables $i_q,\ \lambda_d$ achieve desired values. In the present scenario, these state variables can be evaluated from the rotor flux estimates $\lambda_{\alpha,\beta}$, hence control can be designed. (For convenience, the 'hat' superscript is dropped, it is assumed that the flux estimates are used). Again, based on sliding mode control theory, define the surfaces

$$
\begin{aligned}
s_q &= i_q^* - i_q \\
s_d &= \lambda_d^* - \lambda_d
\end{aligned} \tag{18}
$$

which are the errors between the desired and current values; for simplicity, let $i_q^*,\ \lambda_d^*$ be constants.

By choosing $u_q = M_q sign(s_q)$ with appropriate M_q, sliding mode is enforced on the surface s_q and $i_q \to i_q^*$. To enforce sliding mode on the surface s_d, the Block-control principle [1] is used. The state i_d is chosen as a fictitious control and its desired value is given by

$$
\begin{aligned}
\dot{s}_d &= -\dot{\lambda}_d = -\eta\lambda_d + \eta L_M i_{deq} = 0 \quad \text{thus} \\
i_{deq} &= \lambda_d^*/L_M
\end{aligned} \tag{19}
$$

To use the true control u_d, define the new surface

$$
s = i_{deq} - i_d \tag{20}
$$

and with $u_d = M sign(s)$, sliding mode can be enforced by choosing M properly. Thus, $i_d \to \lambda_d^*/L_M$ and $\lambda_d \to \lambda_d^*$.

B. Closed-loop Behaviour

With the results of control implementation, the system (14) in the closed-loop is

$$
\begin{aligned}
\dot{e_1} &= -\eta e_1 + (F_1 + F_2) e_2 \\
\dot{e_2} &= -\eta e_2 - (F_1 + F_2) e_1 + F_1 \hat{\lambda}^2 + F_3 \tilde{\eta} \\
\dot{\tilde{\eta}} &= -\gamma\tilde{\eta} - \frac{\gamma\eta}{\hat{\lambda}^2} e_1 - \frac{\gamma\omega}{\hat{\lambda}^2} e_2
\end{aligned} \tag{21}
$$

where $F_1 = (\hat{\omega}_{eq} - C\mu_{eq} - \omega)$, $F_2 = (L_M \hat{\eta} i_q^*)/\hat{\lambda}$, and $F_3 = L_M i_q^*/\hat{\lambda}$.

By selecting a very high $\gamma > 0$, from singular perturbation theory,

$$
\tilde{\eta} = \frac{1}{\hat{\lambda}^2}(-\eta e_1 - \omega e_2)
$$

will be true. As a result of this, and from (10),

$$
F_1 = \frac{1}{\hat{\lambda}^2}(-\omega e_1 + \eta e_2)
$$

Finally, from the above two results, (21) becomes

$$\begin{aligned} \dot{e_1} &= -\eta e_1 + A_1 e_2 \\ \dot{e_2} &= -(A_1 + \delta_1)e_1 - \delta_2 e_2 \quad A_1 = (F_1 + F_2) \end{aligned} \tag{22}$$

where $\delta_1 = \frac{L_M \eta i_q^*}{\hat{\lambda}_d} + \omega$, $\delta_2 = \frac{L_M \eta i_q^* \omega}{\hat{\lambda}_d}$, and $A_1 = f(\eta, \omega)$; the stability of which can be proved by choosing the Lyapunov function

$$V = 0.5(\alpha e_1^2 + \beta e_2^2) \qquad \alpha, \beta > 0$$

and evaluating its derivative

$$\dot{V} = -\alpha \eta e_1^2 + (\alpha A_1 - \beta(A_1 + \delta_1))e_1 e_2 - \beta \delta_2 e_2^2$$

In the above equation, $\eta > 0$ and $\delta_2 = (L_M \eta i_q^* \omega)/\hat{\lambda_d} = (L_M \eta T \omega)/\hat{\lambda_d}^2$, where T is the output torque. $T\omega$ is the expression for power and as the induction machine has been assumed to act as a motor, the power output is always positive for $\omega \neq 0$, which means $\delta_2 > 0$ when the rotor is in motion. Hence, α and β can be selected so that $\dot{V} < 0$; and from (11), (21), $\tilde{\lambda}_{\alpha,\beta},\ \tilde{\eta} \to 0$.

A common and valid objection that can be raised to the above result is that the induction machine need not act as a motor at all times. If the motor is to operate in the braking mode, then sign(i_q^*) = $-$sign(ω) [8] and indeed the closed-loop equations (22) become unstable. A simple solution can be offered: switch off the adaptation process and use the latest value of $\hat{\eta}$ to continue with the estimation of the flux and speed even during braking. This solution is not very restrictive as during braking, the emphasis is more on stopping the motor than on its speed control. The regenerative mode is not considered in this paper, as it is beyond its scope and would require analysis with the motor hardware [9].

To analyse stability in this case, assume that at the moment braking begins, $\tilde{\eta} = 0$. The stability analysis is now similar to the one offered in [7]. Only a brief description of this analysis is provided here. On substituting $\hat{\omega}_{eq}$ and μ_{eq} from (10) and linearizing the dynamics of $e_{1,2}$ about $e_{1,2}^* = 0$, the linear system of equations

$$\begin{aligned} \dot{e_1} &= -\eta e_1 + F_2 e_2 \\ \dot{e_2} &= -(F_2 + \hat{\omega}_{eq} + C\eta)e_1 - C\hat{\omega}_{eq} e_2,\ F_2 = (L_M \hat{\eta} i_q^*)/\hat{\lambda} \end{aligned} \tag{23}$$

result which are stable for a high value of C and such that sign(C) = sign$(\hat{\omega}_{eq})$. The parameter C can be chosen to satisfy this sign constraint as $\hat{\omega}_{eq}$ is available. Thus the observer can be used to estimate the unknown electrical quantities in all operating modes and in the motoring mode, the time-varying rotor resistance can be estimated as well.

VI. SIMULATION RESULTS

A speed control algorithm with the proposed observer was simulated for an induction motor having the following constants [10]: $L_S = L_R = 0.47$H, $L_M = 0.44$H; $R_S = 8\Omega$ and a nominal value of the rotor resistance $R_R = 3.6\Omega$ ($\eta = (R_R/L_R) = 7.7$); $P = 2$; and $J = 0.05$Nms^2. The desired flux and speed values were 1.5Wb and 100rad/s respectively. The constant load torque $T_L = 5$Nm was applied.

To use the FOC technique to control speed and flux, i_q^* and i_d^* - the desired values - should be calculated (the latter is given by 19). To calculate i_q^*: Assuming a constant and known load torque, from (3)

$$i_q^* = \frac{1}{K\lambda_d^*}\left(\frac{T_L}{J} + \alpha(\omega^* - \omega)\right) \tag{24}$$

where $K = (PL_M)/(JL_R)$; λ_d^* and ω^* are reference flux and speed values respectively; and constant $\alpha > 0$ which decides the rate of convergence of $\omega \to \omega^*$. The sliding mode controller design procedure that was described in the previous section is followed to obtain the currents $i_{q,d}^*$, the results of the simulation are shown in Figures 1 to 2. Fig.1 shows the errors between the actual and estimated i_q and λ_d respectively while Fig.2 shows the convergence of the actual (unmeasured) speed to the reference speed profile and the adaptively estimated $\hat{\eta}$ to the actual η.

The equivalent controls μ_{eq} and $\hat{\omega}_{eq}$ were extracted as the output of a filter with a cut-off frequency of 200Hz and 40Hz respectively. As the filtered component $\mu_{eq} \neq 0$, there is no need of an external signal having the persistency of excitation property for adaptation, even at standstill. The estimation yields the exact value for $\hat{\eta}$ at constant speed.

The simulation was performed using the simple Euler scheme with a step-size of 0.1msec. This is equivalent to implementing a switching frequency of 10kHz, which can easily be achieved by today's inverter designs that can also provide a voltage output with minimum ripple. In fact, since the observer is completely implemented in the computer, a much lower step-size can be used to accurately determine its output as near ideal sliding mode can be enforced.

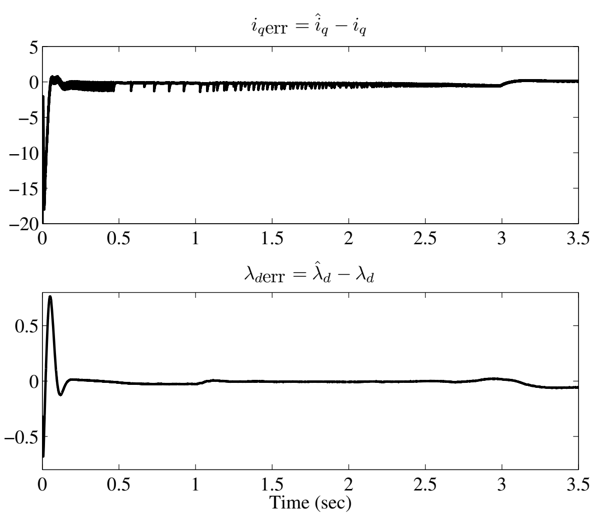

Fig. 1. Flux and Current Estimate Errors

VII. ACKNOWLEDGMENTS

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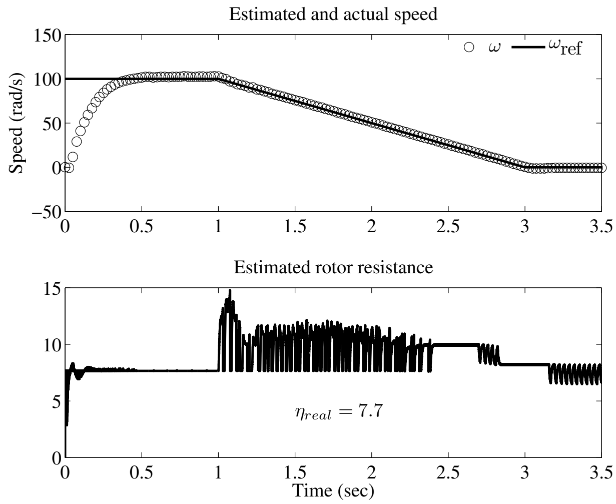

Fig. 2. Actual and Reference Speeds; Estimated Rotor Resistance

which the authors are grateful.

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