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Optimal Control of Continuous Tandem Cold Metal Rolling

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Abstract— **The continuous tandem cold rolling of metal strip is a highly nonlinear multi-variable complex process the control of which presents additional challenges over those encountered in the control of stand-alone tandem cold rolling. These challenges arise during the passage of the weld, which joins the head of the next strip to the tail of the present strip to maintain the continuity of rolling. This paper presents the results of an investigation of the application of a control technique based on the state-dependent Riccati equation (SDRE) method to continuous tandem cold rolling, particularly during passage of the weld. Two methods of control during this regime of operation are evaluated and a preferred method is selected. Using the preferred method it was determined by simulation that the SDRE-based technique has the capability for successfully controlling the mill during weld passage, so that this approach offers a strong potential for improvement in the control of the continuous tandem cold rolling processes.**

I. Introduction

In the tandem cold rolling process (Fig. 1) a metal strip is passed through five pairs of independently driven work rolls, with each work roll supported by a back-up roll of larger diameter. As the strip passes through the individual pairs of work rolls, the thickness is successively reduced by very high compression stress in a small region (i.e. the roll bite) between the work rolls. The necessary compression force is applied by hydraulic rams, or by a screw arrangement driven by an electric motor.

The instrumentation provided to monitor the process includes sensors which produce signals that represent the roll force at each stand, the interstand strip tension forces, the strip thickness at the mill entry and at the exit of the first and last stands, the roll gap actuator (hydraulic ram or screw) positions, and the work roll speeds.

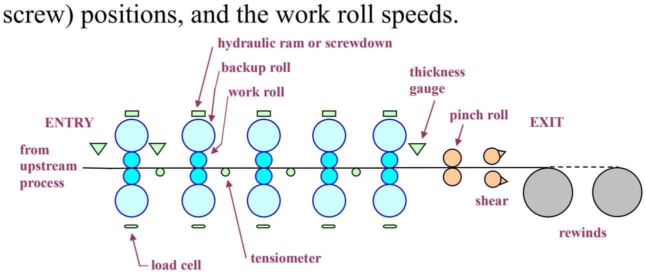

Fig. 1. Typical five stand tandem cold rolling mill

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Unlike the stand-alone mill, in a continuous mill the strip is fed from an upstream process through storage devices so that rolling is not interrupted for a coil change. At the entry of the upstream process, the strip of an incoming coil is welded to the strip of the coil being processed. As the weld exits the upstream process and approaches the tandem cold mill, generally the mill speed is lowered to reduce the likelihood of strip breakage during the weld passage. The passage of the weld through the roll bite results in a smooth but very steep change between the thickness of the present strip and the thickness of the next strip, which puts somewhat severe requirements on the method of control to reduce undesirable excursions in tension, and to reduce undesirable excursions in roll force which could possibly mark the work rolls.

The operation of the continuous mill is similar to the operation of the stand-alone mill except during passage of the weld, so that the improvements realized using the SDRE-based control in the stand-alone case [1, 2] also apply to the continuous case. Thus the efforts described in this paper mostly are concentrated on the investigation of the performance of the continuous mill during passage of the weld. Two control techniques which supplement the SDRE method are evaluated and a preferred technique is selected for further evaluation.

As we will show in this paper, the preferred technique exhibits successful performance as judged on the basis of the following criteria which are important for control during weld passage: (1) reduction in the length of strip in the neighborhood of the weld that has excessive excursions in thickness, (2) reduction in the excursions in tension and roll force as the weld goes through the roll bite, (3) maintaining the mass flow balance in the mill to enhance the stability of rolling, and (4) simplicity of design and implementation.

II. The Process Model

The mathematical model describing the tandem cold rolling process is derived based on a series of algebraic equations developed for control purposes by Bryant [3] and on empirical equations given in Roberts [4]. The expressions as presented in [2] comprise the model as developed for stand-alone tandem cold rolling and also apply for continuous tandem cold rolling. The expressions which comprise the model are put into the form of state-space equations (1) and (2) which describe the system,

$$\dot{x} = A(x)x + Bu, \quad x(0) = x_0, \tag{1}$$

$$y = C(x)x, \tag{2}$$

where $x \in R^n$, $y \in R^p$, $u \in R^m$, are vectors whose elements represent the individual state variables, output variables, and control variables respectively, $A(x) \in R^{nxn}$ and $C(x) \in R^{pxn}$ are state-dependent matrices, and $B \in R^{nxm}$ is a constant matrix.

State variables, control variables, and output variables represented by the elements of the vectors *x, u,* and *y* respectively in (1) and (2) are as shown in Table I where, with *i* representing the stand number, $\sigma_{i,i+1}$ is the interstand tension stress, S_i is the work roll position actuator position, V_i is the work roll linear speed, U_{Si} is the work roll position actuator position reference, U_{Vi} is the work roll drive speed reference, $h_{out,i}$ is the output thickness, and P_i is the specific roll force. The state-space model as described by (1) and (2) was verified by simulation as described in [5, 6].

TABLE I VARIABLES REPRESENTED BY ELEMENTS OF THE STATE, CONTROL AND OUTPUT VECTORS

State Vector		Control Vector		Output Vector	
x_1 (σ_{12})	x_8 (S_4)	u_1 (U_{S1})	u_6 (U_{V1})	y_1 (h_{out1})	y_8 (σ_{34})
x_2 (σ_{23})	x_9 (S_5)	u_2 (U_{S2})	u_7 (U_{V2})	y_2 (h_{out2})	y_9 (σ_{45})
x_3 (σ_{34})	x_{10} (V_1)	u_3 (U_{S3})	u_8 (U_{V3})	y_3 (h_{out3})	y_{10} (P_1)
x_4 (σ_{45})	x_{11} (V_2)	u_4 (U_{S4})	u_9 (U_{V4})	y_4 (h_{out4})	y_{11} (P_2)
x_5 (S_1)	x_{12} (V_3)	u_5 (U_{S5})	u_{10} (U_{V5})	y_5 (h_{out5})	y_{12} (P_3)
x_6 (S_2)	x_{13} (V_4)			y_6 (σ_{12})	y_{13} (P_4)
x_7 (S_3)	x_{14} (V_5)			y_7 (σ_{23})	y_{14} (P_5)

III. THE CONTROLLER

The control strategy for the stand-alone mill is used as a basis for control of the continuous mill. This strategy involves the use of the SDRE technique [7, 8] which is augmented by modifications to enhance performance. As shown in previous work [9] and confirmed by simulation (Section IV), this strategy reduces excursions in the thickness and tension during a speed change, which is especially useful for continuous tandem cold rolling where speed changes are required before and after the weld passage. In addition, the technique for stand-alone cold rolling is enhanced to accommodate the rapid changes in the characteristics of the product in the short time (milliseconds) that the weld is in the roll bite.

The optimal control problem in the SDRE technique is to minimize the performance index

$$J = \tfrac{1}{2}\int_0^\infty (x'Q(x)x + u'R(x)u)\,dt \tag{3}$$

with respect to the control vector *u*, subject to the constraint (1), where $Q(x) \geq 0$, $R(x) > 0$ are state-dependent matrices that are weights for the state and control vectors. Under some mild assumptions [5], (3) implies finding a control law which regulates the system to the origin. The method of solution is to solve the state-dependent algebraic Riccati equation

$$A'(x)K(x) + K(x)A'(x) - K(x)BR^{-1}(x)B'K(x) + Q(x) = 0 \tag{4}$$

pointwise for *K(x)*, which results in the control law

$$u = -R^{-1}(x)B'K(x)x \tag{5}$$

and which is the basis for the control of the mill.

Fig. 2 depicts the controller structure, where the vectors *x*, *u*, and *y* at the operating point are represented as x_{op}, u_{op}, and y_{op}. A coordinate change is performed by the introduction of the vector $z = x - x_{op}$ which shifts the operating point to the origin. The performance index (3) is then modified to be

$$J = \tfrac{1}{2}\int_0^\infty (z'Qz + (u-u_{op})'R(u-u_{op}))\,dt\ , \tag{6}$$

where for simplicity *Q* and *R* are diagonal matrices with tunable constant elements. A change in mill speed is effected by changing the operating point, which is done by changing the variables corresponding to the elements of $x_{op,i}$ $(i=10,\ldots,14)$ and of $u_{op,i}$ $(i=6,\ldots,10)$ (Table I) proportionally to a mill master speed reference.

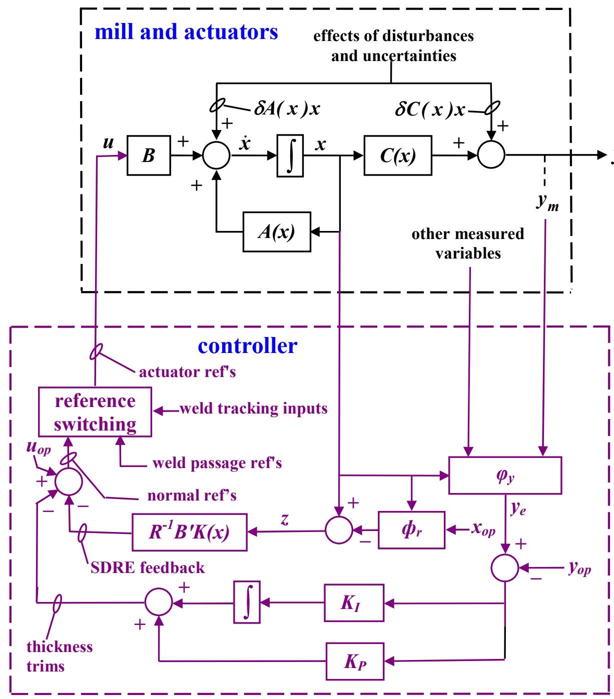

Fig. 2 Model of mill and actuators coupled to controller

The SDRE control of an individual stand estimated output thickness is enhanced by a trim function to achieve zero steady-state error and greatly reduce the effect of the interstand time delay by providing a fast-responding trim loop which does not include the time delay. The strip speeds at the mill entry, at the inputs to stands 2 through 5, and at the output of stand 5 are measured by high accuracy velocimeters. The measured strip speed signals are used for weld tracking, for tracking strip thickness, and by the algorithm φ_y for the estimation of strip thickness at the output of each stand using mass flow techniques. The elements of the diagonal matrices K_I and K_P are the integral and proportional gains for the individual thickness trim functions.

The function ϕ_r which includes a trim for each interstand tension, corrects for slight offsets in the interstand tensions from the operating point values. The final control of the tensions is achieved by the control law (5) that is computed pointwise by the SDRE controller. The measured variables represented by the elements y_m of the output vector y are used by φ_y to estimate the specific roll forces.

Using inputs from the weld tracking system the references to the work roll position actuators and the work roll speed actuators (drives) are switched by the reference switching function (Section III. C) during the weld passage.

A. *Tracking of the Weld*

Tracking of the weld with low error in the weld position is essential for effective control during the weld passage. It is assumed that a system to track the weld is operative and that it updates about every 2 ms or less, which is typical for existing hardware and software platforms. The weld tracking system uses the inputs from the velocimeters to generate signals that indicate the weld position as the weld travels through the mill, and provides a signal to initiate the deceleration to weld passage speed such that the strip is at constant speed just before the weld enters the first stand.

B. *Operating Point Changes*

An operating point for the mill is determined by the product being rolled and the desired mill speed. During the weld passage, the operating point is changed from the operating point for the present strip to the operating point for the next strip. It is desirable that the individual references set in the controller are changed in a manner that reduces the length of strip that has excessive excursions in the thickness and reduces the excursions in tension and roll force. Further, the mass flow balance must be maintained to support the stability of rolling. What follows describes how these references are changed during the transition.

A position actuator movement is initiated as the weld approaches stand 1 to move the actuator from a position corresponding to the thickness of the present strip to a position corresponding to the thickness of the next strip. At some point during the movement, the output thickness and roll force will change very rapidly as the weld enters and passes through the roll bite. To reduce the length of strip that has excessive excursions in thickness, the position actuator is moved at nearly its maximum speed over a path that reduces both the excursions in thickness and the changes in roll force during the transition. Several possible paths were simulated and a preferred path was selected (Section IV).

Actuator positions for stands 2 through 5 are controlled similarly to the control for stand 1, except that the position actuator speed may be less than its maximum to approximately match the thickness profile generated at stand 1. This increases the likelihood that the weld will pass through the roll bite sometime during the movement of the position actuator which reduces the possibility of higher excursions in roll force during the transition.

The likelihood of strip breakage at the weld is reduced by holding the work roll peripheral speed reference at a constant value as the weld is passed through the roll bite of stand *i*, and then switching back to the speed reference with correction from the SDRE feedback after the weld has traveled a preset distance pas the stand. During the switching operations, filtering techniques are applied to reduce excursions in the associated speeds.

The reference for interstand tension between stands *i* and *i+1* is changed slowly from the reference for the present strip to the reference for the next strip. The change is initiated when the weld exits the roll bite of stand *i* and is completed before the weld enters stand *i+1*.

C. *Control of the Position Actuator*

Two techniques (A and B) were evaluated for controlling the position actuator to follow approximately the preferred path. Technique A (Fig. 3) is based on using the process model [2] and the SDRE controller with the thickness trim control loop remaining closed during the transition from the present strip to the next strip. Using the process model, a path for the thickness reference is computed (Appendix) to give the preferred path for the actuator position. The capability of the SDRE controller to control the thickness to follow closely the path for the thickness reference is utilized to assure that the position actuator closely follows its preferred path. It was considered initially that this technique might be simpler than Technique B since switching of the controller from a thickness mode to a position mode would be unnecessary since the thickness loop remains closed during the transition.

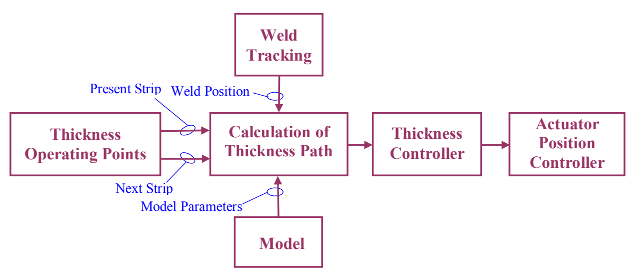

Fig. 3 Configuration for Technique A

Technique B (Fig. 4) changes the mode of the controller for stand 1 from thickness to position by removing the normal reference signal (Fig. 2) from the actuator position controller and providing a position reference signal for weld passage to move the actuator toward a position corresponding to the operating point for the next strip. During the movement of the actuator the thickness reference is ineffective and is set to the operating point for the next strip. While the movement occurs, the estimated strip thickness is monitored, and when the estimated thickness is within a preset amount of the thickness reference for the next strip, the mode is changed from position to thickness by removing the position reference signal for weld passage and reestablishing the normal reference signal. Filtering techniques are used to reduce excursions in the associated variables during the transfers between controller modes.

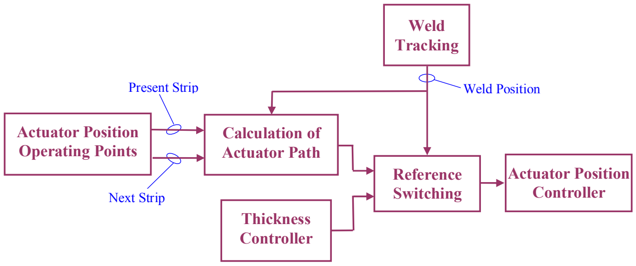

Fig. 4 Configuration for Technique B

Simulations (Section IV) were done using both techniques. With no disturbances or uncertainties, and considering only transitions in the thickness of the incoming strip with no transitions in hardness, the performance of both methods was nearly identical. However, the following are noted:

- In Technique A the inclusion of a model and the computation of a thickness path is required in the controller to determine the position path whereas in Technique B no model is required and the computation of the position path is straightforward. This increases the complexity of Technique A over Technique B.
- The effects of uncertainties in computing the thickness path in Technique A could increase the length of strip with excessive excursions since the position actuator may not be moving near its maximum speed. In Technique B the motion is controlled directly and the uncertainties therefore have less effect.
- During the transition the control of strip thickness is unnecessary and therefore any reasonable thickness deviations using Technique B are acceptable.
- The complexity resulting from the switching of controller modes in Technique B is not excessive compared to Technique A.

Based on the above, Technique B is taken for this initial work as the preferred technique for stand 1 and similarly for stands 2 through 5.

D. *Maintaining Mass Flow Balance*

The mass flow balance in the mill is important to maintaining the stability of rolling. Using the SDRE technique, the steady-state mass flow is balanced inherently by the SDRE controller holding the individual tensions close to their desired values as the weld passes through the mill. From the mathematical model [2], the relationship for the tension stress σ between stands i and $i+1$ is approximately

$$\frac{d\sigma_{i,i+1}}{dt} \equiv \dot{\sigma}_{i,i+1} = \frac{E\left(V_{in,i+1} - V_{out,i}\right)}{L_0}, \quad \sigma_{i,i+1}(0) = \sigma_{0,i,i+1}, \quad (7)$$

where E is Young's modulus, $V_{in(out)}$ are the input (output) strip speeds at a mill stand, and L_0 is the distance between the adjacent mill stands. As can be seen from (7), controlling the tension to be constant between adjacent stands i and $i+1$, with a constant strip thickness and assuming a constant width, implies a mass flow balance between the output of stand i and the input of stand $i+1$, in accordance with the approximation

$$MF = V\ h\ w \quad (8)$$

where MF is the mass flow, V is the strip velocity, h is the strip thickness, and w is the strip width. Since the steady-state mass flow across the roll bite does not change, the mass flow at the output of stand $i+1$ is the mass flow at the output of stand i. The control of mass flow is verified by simulation (Section IV).

IV. SIMULATIONS

Closed loop simulations using MATLAB/Simulink were performed with the controller coupled to the model to confirm the control of thickness and tension during the speed changes before and after the weld passage, and to verify performance during the weld passage including mass flow balance. In addition, initial simulations were done to select a preferred path for the travel of the position actuator. The simulations during weld passage were performed at a speed of about 122 m/min (400 ft/min) which is 10% of run speed.

A. *Speed Changes*

Changes in speed were simulated similarly to simulations performed previously [5] by applying a mill master speed reference (Fig. 5) proportionally to the drive speed controllers of the individual mill stands. The simulations were done without disturbances or uncertainties, with typical mill and strip properties [3], and using the operating point for the Present Strip of Table II [3] for acceleration and deceleration. For both cases the excursions in thickness were negligible (less than 0.001%) and the excursions in tension were less than 0.1%.

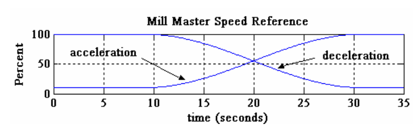

Fig. 5 Mill master speed reference before and after weld passage

B. *Preferred Path for the Actuator Position*

Five different possible paths for moving the position actuator during the transition from the present strip to the next strip were simulated for a weld passing through the roll bite of stand 1, which was taken as typical for stands 2 through 5. The simulations were performed using control techniques A and B, with no disturbances or uncertainties, with the operating points for the two strips as shown in Table II (Next Strip, with a Thickness Transition of 20%), where the variables h_{in1}, σ_{10}, and σ_{50} represent the mill input thickness, the mill input tension, and the mill exit tension respectively, and with the remaining variables representing the thickness and tensions as noted in Table I.

TABLE II Operating Points for the Present Strip and the Next Strip

Variable	Present Strip	Next Strip, with Thickness Transition of 5%	10%	20%
h_{in1}	3.56 mm (0.140 in)	3.73 (0.147)	3.91 (0.154)	4.32 (0.170)
h_{out1}	2.95 (0.116)	3.10 (0.122)	3.23 (0.1275)	3.56 (0.140)
h_{out2}	2.44 (0.096)	2.56 (0.101)	2.67 (0.105)	2.79 (0.110)
h_{out3}	2.01 (0.079)	2.11 (0.083)	2.21 (0.087)	2.29 (0.090)
h_{out4}	1.67 (0.066)	1.75 (0.069)	1.78 (0.070)	1.83 (0.072)
h_{out5}	1.58 (0.062)	1.65 (0.065)	1.68 (0.066)	1.73 (0.068)
σ_{10}	0.024 kN/mm^2 (1.5 long-t/in^2)	0.024 (1.5)	0.024 (1.5)	0.024 (1.5)
σ_{12}	0.086 (5.6)	0.086 (5.6)	0.086 (5.55)	0.085 (5.5)
σ_{23}	0.088 (5.7)	0.088 (5.7)	0.087 (5.65)	0.086 (5.6)
σ_{34}	0.089 (5.8)	0.089 (5.8)	0.089 (5.75)	0.088 (5.7)
σ_{45}	0.092 (6.0)	0.092 (6.0)	0.092 (5.95)	0.089 (5.8)
σ_{50}	0.028 (1.8)	0.028 (1.8)	0.028 (1.8)	0.028 (1.8)

During the simulation the position actuator was moved at its maximum speed (about 1.5 mm/sec, or 0.06 in/sec). In all cases the results using technique A were very similar to the results using technique B. Table III summarizes the results. Path 2 was selected as the preferred path based on reducing the out-of-tolerance length, the peak specific roll force, and the maximum change in the specific roll force.

TABLE III Simulation Results for Several Paths

Path Number	Out-of-Tolerance Length	Max Change in Specific Roll Force	Peak Specific Roll Force
1	0.21 m (8.3 in)	2.1 kN/mm (5.3 long-t/in)	11.6 (29.6)
2	0.22 (8.7)	2.2 (5.6)	10.7 (27.2)
3	0.66 (26.0)	3.1 (7.8)	10.6 (27.1)
4	1.06 (41.7)	4.1 (10.5)	10.6 (27.0)
5	1.63 (64.2)	5.2 (13.3)	10.5 (26.7)

C. Performance

Simulations were performed using control technique B for increases in strip thickness of 5%, 10%, and 20% from the present strip to the next strip (Table II). For each case, the length of strip in the neighborhood of the weld that was out-of-tolerance (i.e. had excessive excursions in the thickness), the maximum change in the specific roll force, the peak of the specific roll force, and the peak excursion in the tension were recorded. The position actuator movement was initiated to start the transition to the next strip such that the weld passed through the roll bite when the actuator was at about one-half of its travel, which allowed for some margin around the half-travel point for an on-going evaluation of the effects of disturbances and uncertainties.

The position actuator for stand 1 was moved at near the typical maximum speed to reduce the out-of-tolerance thickness near the weld. To retain the margin around the half-travel point, the length of the transition was not decreased as the weld passed through the downstream stands, even though the associated position actuators might be moved at less than their maximum speeds. However, if the length of the transition could not be retained with a downstream position actuator near its maximum speed, then the length would be increased as determined by the movement of the actuator at the maximum speed.

Filtering techniques were applied in every case during the switching of controller modes such that the position and speed references would be changed slowly enough that the capabilities of the actuators to control position and speed would not be degraded.

Simulation results are presented in Table IV for a 20% thickness transitions. Results for 5% and 10% thickness transitions are similar.

TABLE IV Results for 20% Thickness Transition

	Stand 1	Stand 2	Stand 3	Stand 4	Stand 5
Out-of-Tolerance Length, m (in)	0.16 (6.3)	0.26 (10.2)	0.33 (13.0)	0.33 (13.0)	0.38 (15.0)
Max Change in Specific Roll Force, kN/mm (long-t/in)	0.94 (2.4)	0.70 (1.8)	0.54 (1.4)	1.9 (4.9)	0.54 (1.4)
Peak Specific Roll Force	9.88 (25.2)	10.3 (26.4)	10.7 (27.3)	11.3 (28.8)	6.59 (16.8)
Max Tension Excursion, percent	−7.7	−8.8	−1.1	−7.2	–

D. Mass Flow Balance

For verification of the mass flow balance, just after the transition has exited the roll bite of stand $i+1$, the magnitude of the percent deviation of the peak of the transient difference between the mass flows at the exits of stand $i+1$ and stand i, with respect to the steady-state mass flow, was determined by simulation to be less than 0.30% for transitions of 5% and 10%, and less than 0.7% for transitions of 20% passing through stands 2,1 and stands 3,2.

In addition, percent deviations of the peaks of the transient differences between the mass flows at other adjacent upstream stands (e.g. at stands i and $i-1$) were determined during passage of the transition through the roll bite of stand $i+1$. For thickness transitions of 5% and 10% the magnitudes of the maximum deviations were less than .1% and .4% respectively, and similarly for a thickness transition of 20%. As indicated by these results the transient differences in the mass flows between adjacent upstream stands are not excessive. Further simulations also were performed which showed similar results for the transient differences in mass flows between non-adjacent upstream stands, and also confirmed that at steady-state there is zero difference between mass flows at the upstream stands. Similar results were obtained for the mass flows at downstream stands (e.g. at the exits of stands *i+2* and *i+3* during passage of the transition through stand *i+1*).

E. Comparison with Existing Applications

Data available from recent installations of continuous tandem cold rolling applications are scant, which makes comparison with existing applications difficult. In general, the data that are available in the literature [e.g. 10, 11] are presented as records from actual operations and require supplementary information for a more thorough understanding. However, comparing the out-of-tolerance lengths of strip in the vicinity of the weld on the basis of an interpretation of the available data and adjusting for the differences in mill speeds, provides an indication that the SDRE technique has the potential for the successful application for control during passage of the weld (Table V). It is expected that more complete data from operating installations, plus the results of on-going work to address the effects of hardness, of disturbances and uncertainties, and of weld passage at higher operating speeds will add to the validity of the comparison.

TABLE V ROUGH COMPARISON WITH MILL DATA A [3] AND MILL DATA B [4]

Controller	Out-of-Tolerance Length Near the Weld, m (in)
SDRE-Based	<1.0 (39)
Mill A	2.0 (79)
Mill B	2.5 (98)

V. CONCLUSION

The inherent capability of the controller to reduce excursions in thickness and tension during speed change, and to accommodate changes in product on the fly make it useful for control of continuous tandem cold rolling. The results of this initial work imply a potential for effective control during passage of the weld, so that the SDRE technique as augmented for control during weld passage has a strong likelihood of offering an improvement in the overall control of continuous tandem cold rolling.

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APPENDIX

Estimation of the Thickness Reference for Technique A

The position of the work roll position actuator at the point where the weld enters the roll bite is computed using the BISRA relation [2] for the output thickness as

$$S_{i,2} = h_{out,i,2} - S_{0,i} - \frac{F_{i,2}}{M_i}, \tag{9}$$

where $h_{out,i,2}$ is the desired thickness corresponding to $S_{i,2}$, $S_{0,i}$ is the intercept of the linear approximation of the mill stretch curve, $F_{i,2}$ is the roll force estimated using the strip model [2] for a thickness of $h_{out,i,2}$, and M_i is the mill stretch parameter taken from the mill model. The desired path of the actuator is

$$S_i = a_1 t + a_2, \tag{10}$$

where a_1 is the desired speed of the actuator, a_2 is a constant computed using $S_{i,2}$, the initial actuator position $S_{i,1}$, the time when the actuator is at $S_{i,1}$, and the estimated time when the actuator is to be at $S_{i,2}$. During each scan of the controller the roll force F_i is estimated [2] in a small neighborhood of the desired output thickness $h_{out,i0}$ by a linear approximation as

$$F_i = c_1 h_{out,i} + c_2, \tag{11}$$

where $h_{out,i}$ is an output thickness in the neighborhood of $h_{out,i0}$, and c_1, c_2 are constants that are computed during each scan. Using (10), (11), and the BISRA relation for output thickness, the expression for the thickness reference is derived as

$$h_{out,i0} = \frac{(M_i)(a_1 t + a_2 + S_{0,i}) + c_2}{(M_i - c_1)}, \tag{12}$$

where c_1 and c_2 are as computed during the previous scan.