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A New Class of Modular Adaptive Controllers, Part II: Neural Network Extension for Non-LP Systems

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Abstract— **The development in this (Part II) paper augments the result developed in Part I by considering uncertain dynamic systems that are not necessarily linear-in-the-parameters (LP), and have additive non-LP bounded disturbances. For non-LP uncertainties, a model-based adaptive feedforward formulation cannot be used. Therefore, in this paper, a multilayer neural network (NN) structure is used as a feedforward element (that learns and compensates for the non-LP dynamics) in conjunction with the Robust Integral of the Sign of the Error (RISE) feedback term. Similar to the result in Part I, a NN-based controller is developed in this paper with modularity in NN weight tuning laws and control law. Specifically, the results in this paper allow the NN weight tuning laws to be determined from a developed generic update law (rather than be restricted to a gradient update law).**

I. Introduction

Numerous adaptive control results have been developed for nonlinear systems with parametric uncertainty that is assumed to satisfy the linear-in-the-parameters (LP) condition. The transient performance of these adaptive controllers is often degraded because of the restriction to use a gradient-based adaptive update law to cancel cross terms in a Lyapunov-based stability analysis. Motivated by the fact that gradient update laws often exhibit slow parameter convergence in comparison to other possible adaptive update laws (e.g., least-squares update law), several efforts have been made to redesign the closed-loop error system and stability analysis to accommodate different adaptive update laws (cf. [1]–[4]). Researchers have also developed a class of modular adaptive controllers (cf. [5]–[9]) where nonlinear damping (ND) [6], [10] is used to stabilize the error dynamics provided certain conditions are satisfied on the adaptive update law (i.e., an input-to-state stability (ISS) result). In lieu of the class of ND-based controllers, Part I of this paper [11] described how a new feedback control strategy called the Robust Integral of the Sign of the Error (RISE) could be coupled with a generic update law to yield a new class of modular adaptive controllers.

In comparison with the outcomes of Part I, the focus of this (Part II) paper is to extend the class of RISE-based modular adaptive controllers to include uncertain dynamic systems that do not satisfy the LP assumption. For non-LP dynamic systems, the use of a model-based adaptive feedforward term is not possible. Neural networks (NNs) have gained popularity as a feedforward adaptive control method that can compensate for non-LP uncertainty in nonlinear systems. A limiting factor in previous NN-based feedforward control results is that a residual function approximation error exists that limits the steady-state performance to a uniformly ultimately bounded result, rather than an asymptotic result. Some results (cf. [12]–[18]) have been developed to augment the NN feedforward component with a discontinuous feedback element to achieve asymptotic tracking. Motivated by the practical limitations of discontinuous feedback, a multilayer NN-based controller was augmented by RISE feedback in [19] to yield the first asymptotic tracking result using a continuous controller. However, in all previous NN-based controllers, the NN adaptation is governed by a gradient update law to facilitate the Lyapunov-based stability analysis.

Since multilayer NNs are nonlinear in the weights, it is challenging to derive weight tuning laws in closed-loop feedback control systems that yield stability as well as bounded weights. The development in the current work illustrates how to extend the class of modular adaptive controllers in Part I for NNs. Specifically, the results in this paper allow the NN weight tuning laws to be determined from a developed generic update law (rather than be restricted to a gradient update law). We are not aware of any modular NN-based controller in literature with modularity in the tuning laws/controller. The NN feedforward structure adaptively compensates for the non-LP uncertain dynamics, thereby extending the results in Part I where only LP dynamics could be compensated by using a model-based feedforward term.

For the tuning laws that could be used in this result, the NN weights can be initialized randomly, and no off-line training is required.

II. Dynamic Model and Properties

The class of nonlinear dynamic systems considered in this paper is assumed to be modeled by the following Euler-Lagrange formulation:

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) + \tau_d(t) = \tau(t). \quad (1)$$

In (1), $M(q) \in \mathbb{R}^{n\times n}$ denotes the inertia matrix, $V_m(q,\dot{q}) \in \mathbb{R}^{n\times n}$ denotes the centripetal-Coriolis matrix, $G(q) \in \mathbb{R}^n$ denotes the gravity vector, $F(\dot{q}) \in \mathbb{R}^n$ denotes friction, $\tau_d(t) \in \mathbb{R}^n$ denotes a general nonlinear disturbance (e.g., unmodeled effects), $\tau(t) \in \mathbb{R}^n$ represents the torque input

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control vector, and $q(t)$, $\dot{q}(t)$, $\ddot{q}(t) \in \mathbb{R}^n$ denote the link position, velocity, and acceleration vectors, respectively. The subsequent development is based on the assumption that $q(t)$ and $\dot{q}(t)$ are measurable and that $M(q)$, $V_m(q,\dot{q})$, $G(q)$, $F(\dot{q})$ and $\tau_d(t)$ are unknown and do not have to satisfy the LP assumption. Moreover, the following properties and assumptions will be exploited in the subsequent development.

Property 1: The inertia matrix $M(q)$ is symmetric, positive definite, and satisfies the following inequality $\forall \xi(t) \in \mathbb{R}^n$:

$$m_1 \|\xi\|^2 \leq \xi^T M(q)\xi \leq \bar{m}(q) \|\xi\|^2 \tag{2}$$

where $m_1 \in \mathbb{R}$ is a known positive constant, $\bar{m}(q) \in \mathbb{R}$ is a known positive function, and $\|\cdot\|$ denotes the standard Euclidean norm.

Property 2: If $q(t)$, $\dot{q}(t) \in \mathcal{L}_\infty$, then $V_m(q,\dot{q})$, $F(\dot{q})$ and $G(q)$ are bounded. Moreover, if $q(t)$, $\dot{q}(t) \in \mathcal{L}_\infty$, then the first and second partial derivatives of the elements of $M(q)$, $V_m(q,\dot{q})$, $G(q)$ with respect to $q(t)$ exist and are bounded, and the first and second partial derivatives of the elements of $V_m(q,\dot{q})$, $F(\dot{q})$ with respect to $\dot{q}(t)$ exist and are bounded.

Property 3: The nonlinear disturbance term and its first two time derivatives, i.e. $\tau_d(t)$, $\dot{\tau}_d(t)$, $\ddot{\tau}_d(t)$ are bounded by known constants.

Property 4: The desired trajectory is assumed to be designed such that $q_d^{(i)}(t) \in \mathbb{R}^n$ $(i = 0, 1, ..., 4)$ exist, and are bounded.

III. Control Objective

The objective is to design a continuous modular adaptive controller which ensures that the system tracks a desired time-varying trajectory, denoted by $q_d(t) \in \mathbb{R}^n$, despite uncertainties in the dynamic model. To quantify this objective, a position tracking error, denoted by $e_1(t) \in \mathbb{R}^n$, is defined as

$$e_1 \triangleq q_d - q. \tag{3}$$

To facilitate the subsequent analysis, filtered tracking errors [20], denoted by $e_2(t)$, $r(t) \in \mathbb{R}^n$, are also defined as

$$e_2 \triangleq \dot{e}_1 + \alpha_1 e_1 \tag{4}$$

$$r \triangleq \dot{e}_2 + \alpha_2 e_2 \tag{5}$$

where α_1, $\alpha_2 \in \mathbb{R}$ denote positive constants. The filtered tracking error $r(t)$ is not measurable since the expression in (5) depends on $\ddot{q}(t)$.

IV. Feedforward NN Estimation

NN-based estimation methods are well suited for control systems where the dynamic model contains unstructured nonlinear disturbances as in (1). The main feature that empowers NN-based controllers is the universal approximation property. Let $\mathbb{S}$ be a compact simply connected set of $\mathbb{R}^{N_1+1}$. With map $f : \mathbb{S} \to \mathbb{R}^n$, define $\mathbb{C}^n(\mathbb{S})$ as the space where f is continuous. There exist weights and thresholds such that some function $f(x) \in \mathbb{C}^n(\mathbb{S})$ can be represented by a three-layer NN as [21], [22]

$$f(x) = W^T \sigma\left(V^T x\right) + \varepsilon(x) \tag{6}$$

for some given input $x(t) \in \mathbb{R}^{N_1+1}$. In (6), $V \in \mathbb{R}^{(N_1+1)\times N_2}$ and $W \in \mathbb{R}^{(N_2+1)\times n}$ are bounded constant weight matrices for the first-to-second and second-to-third layers respectively, where N_1 is the number of neurons in the input layer, N_2 is the number of neurons in the hidden layer, and n is the number of neurons in the third layer. The activation function[1] in (6) is denoted by $\sigma(\cdot) \in \mathbb{R}^{N_2+1}$, and $\varepsilon(x) \in \mathbb{R}^n$ is the functional reconstruction error. Note that, augmenting the input vector $x(t)$ and activation function $\sigma(\cdot)$ by "1" allows us to have thresholds as the first columns of the weight matrices [21], [22]. Thus, any tuning of W and V then includes tuning of thresholds as well. For more details on the NN structure, see [21], [22].

Remark 1: If $\varepsilon = 0$, then $f(x)$ is in the functional range of the NN. In general for any positive constant real number $\varepsilon_N > 0$, $f(x)$ is within ε_N of the NN range if there exist finite hidden neurons N_2, and constant weights so that for all inputs in the compact set, the approximation holds with $\|\varepsilon\| < \varepsilon_N$. For various activation functions, results such as the Stone-Weiserstrass theorem indicate that any sufficiently smooth function can be approximated by a suitable large network. Therefore, the fact that the approximation error ε is bounded follows from the *Universal Approximation Property* of the NNs (see [23], [24], and [25]).

Based on (6), the typical three-layer NN approximation for $f(x)$ is given as [21], [22]

$$\hat{f}(x) \triangleq \hat{W}^T \sigma(\hat{V}^T x) \tag{7}$$

where $\hat{V}(t) \in \mathbb{R}^{(N_1+1)\times N_2}$ and $\hat{W}(t) \in \mathbb{R}^{(N_2+1)\times n}$ are subsequently designed estimates of the ideal weight matrices. The estimate mismatches for the ideal weight matrices, denoted by $\tilde{V}(t) \in \mathbb{R}^{(N_1+1)\times N_2}$ and $\tilde{W}(t) \in \mathbb{R}^{(N_2+1)\times n}$, are defined as

$$\tilde{V} \triangleq V - \hat{V}, \quad \tilde{W} \triangleq W - \hat{W},$$

and the mismatch for the hidden-layer output error for a given $x(t)$, denoted by $\tilde{\sigma}(x) \in \mathbb{R}^{N_2+1}$, is defined as

$$\tilde{\sigma} \triangleq \sigma - \hat{\sigma} = \sigma\left(V^T x\right) - \sigma(\hat{V}^T x). \tag{8}$$

The NN has several properties that facilitate the subsequent development. These properties are described as follows.

Property 5: (*Boundedness of the Ideal Weights*) The ideal weights are assumed to exist and be bounded by known positive values so that

$$\|V\|_F^2 = tr\left(V^T V\right) = vec(V)^T vec(V) \leq \bar{V}_B \tag{9}$$

$$\|W\|_F^2 = tr\left(W^T W\right) = vec(W)^T vec(W) \leq \bar{W}_B, \tag{10}$$

where $\|\cdot\|_F$ is the Frobenius norm of a matrix, $tr(\cdot)$ is the trace of a matrix, and the operator $vec(\cdot)$ stacks the columns of a matrix $A \in \mathbb{R}^{m\times n}$ to form a vector $vec(A) \in \mathbb{R}^{mn}$ as

$$vec(A) \triangleq \begin{bmatrix} A_{11} & A_{21}... & A_{m1} & A_{12} \\ & A_{22}... & A_{1n}... & A_{mn} \end{bmatrix}^T.$$

[1]A variety of activation functions (e.g., sigmoid, hyperbolic tangent or radial basis) could be used for the control development in this paper.

Property 6: (*Convex Regions*) Based on (9) and (10), convex regions (e.g., see Section 4.3 of [26]) can be defined. Specifically, the convex region Λ_V can be defined as

$$\Lambda_V \triangleq \left\{v : v^T v \leq \bar{V}_B\right\}, \tag{11}$$

where $\bar{V}_B$ was given in (9). In addition, the following definitions concerning the region Λ_V and the parameter estimate vector $vec(\hat{V}) \in \mathbb{R}^{(N_1+1)N_2}$ (i.e., the dynamic estimate of $vec(V) \in \Lambda_V$) are provided as follows: $int(\Lambda_V)$ denotes the interior of the region Λ_V, $\partial(\Lambda_V)$ denotes the boundary for the region Λ_V, $vec(\hat{V})^{\perp} \in \mathbb{R}^{(N_1+1)N_2}$ is a unit vector normal to $\partial(\Lambda_V)$ at the point of intersection of the boundary surface $\partial(\Lambda_V)$ and $vec(\hat{V})$, where the positive direction for $vec(\hat{V})^{\perp}$ is defined as pointing away from $int(\Lambda_V)$ (note that $vec(\hat{V})^{\perp}$ is only defined for $vec(\hat{V}) \in \partial(\Lambda_V)$), $P_r^t(\psi)$ is the component of the vector $\psi \in \mathbb{R}^{(N_1+1)N_2}$ that is tangent to $\partial(\Lambda_V)$ at the point of intersection of the boundary surface $\partial(\Lambda_V)$ and the vector $vec(\hat{V})$, and

$$P_r^{\perp}(\psi) = \psi - P_r^t(\psi) \in \mathbb{R}^{(N_1+1)N_2} \tag{12}$$

is the component of the vector $\psi \in \mathbb{R}^{(N_1+1)N_2}$ that is perpendicular to $\partial(\Lambda_V)$ at the point of intersection of the boundary surface $\partial(\Lambda_V)$ and the vector $vec(\hat{V})$. Similar to (11), the convex region Λ_W is defined as

$$\Lambda_W \triangleq \left\{v : v^T v \leq \bar{W}_B\right\}, \tag{13}$$

where $\bar{W}_B$ was given in (10).

V. RISE Feedback Control Development

The contribution of this paper is modular control development and stability analysis that illustrate how the aforementioned textbook (e.g., [22]) NN feedforward estimation strategy can be fused with a RISE feedback control method as a means to achieve asymptotic stability for general Euler-Lagrange systems described by (1) while using generic NN weight update laws. In this section, the open-loop and closed-loop tracking error is developed for the combined control system.

A. Open-Loop Error System

The open-loop tracking error system can be developed by premultiplying (5) by $M(q)$ and utilizing the expressions in (1), (3), and (4) to obtain the following expression:

$$M(q)r = f_d + S + \tau_d - \tau \tag{14}$$

where the auxiliary function $f_d\left(q_d, \dot{q}_d, \ddot{q}_d\right) \in \mathbb{R}^n$ is defined as

$$f_d \triangleq M(q_d)\ddot{q}_d + V_m(q_d, \dot{q}_d)\dot{q}_d + G(q_d) + F\left(\dot{q}_d\right), \tag{15}$$

and the auxiliary function $S\left(q, \dot{q}, q_d, \dot{q}_d, \ddot{q}_d\right) \in \mathbb{R}^n$ is defined as

$$\begin{aligned} S \triangleq\ & M\left(q\right)\left(\alpha_1 \dot{e}_1 + \alpha_2 e_2\right) + M\left(q\right)\ddot{q}_d - M(q_d)\ddot{q}_d \\ & + V_m(q, \dot{q})\dot{q} - V_m(q_d, \dot{q}_d)\dot{q}_d \\ & + G(q) - G(q_d) + F\left(\dot{q}\right) - F\left(\dot{q}_d\right). \end{aligned} \tag{16}$$

The expression in (15) can be represented by a three-layer NN as

$$f_d = W^T \sigma\left(V^T x_d\right) + \varepsilon\left(x_d\right). \tag{17}$$

In (17), the input $x_d(t) \in \mathbb{R}^{3n+1}$ is defined as $x_d(t) \triangleq [1 \;\; q_d^T(t) \;\; \dot{q}_d^T(t) \;\; \ddot{q}_d^T(t)]^T$ so that $N_1 = 3n$ where N_1 was introduced in (6). Based on the assumption that the desired trajectory is bounded, the following inequalities hold

$$\begin{gathered} \left\|\varepsilon\left(x_d\right)\right\| \leq \varepsilon_{b_1} \qquad \left\|\dot{\varepsilon}\left(x_d, \dot{x}_d\right)\right\| \leq \varepsilon_{b_2} \\ \left\|\ddot{\varepsilon}\left(x_d, \dot{x}_d, \ddot{x}_d\right)\right\| \leq \varepsilon_{b_3} \end{gathered} \tag{18}$$

where $\varepsilon_{b_1}, \varepsilon_{b_2}, \varepsilon_{b_3} \in \mathbb{R}$ are known positive constants.

B. Closed-Loop Error System

Based on the open-loop error system in (14), the control torque input is composed of a three-layer NN feedforward term plus the RISE feedback term as

$$\tau \triangleq \hat{f}_d + \mu. \tag{19}$$

Specifically, $\mu(t) \in \mathbb{R}^n$ denotes the RISE feedback control term defined as [27]–[30]

$$\begin{aligned} \mu(t) \triangleq\ & (k_s+1)e_2(t) - (k_s+1)e_2(0) \\ & + \int_0^t [(k_s+1)\alpha_2 e_2(\sigma) + \beta_1 sgn(e_2(\sigma))]d\sigma \end{aligned} \tag{20}$$

where $k_s,\ \beta_1 \in \mathbb{R}$ are positive constant control gains. The feedforward NN component in (19), denoted by $\hat{f}_d(t) \in \mathbb{R}^n$, is generated as

$$\hat{f}_d \triangleq \hat{W}^T \sigma\left(\hat{V}^T x_d\right). \tag{21}$$

The estimates for the NN weights in (21) are generated on-line (there is no off-line learning phase) as

$$\dot{\hat{W}} \triangleq \begin{cases} \varrho_1 & \text{if } vec(\hat{W}) \in int\left(\Lambda_W\right) \\ \varrho_1 & \begin{cases} \text{if } vec(\hat{W}) \in \partial\left(\Lambda_W\right) \text{ and} \\ vec(\varrho_1)^T vec(\hat{W})^{\perp} \leq 0 \end{cases} \\ P_{Mr}^t\left(\varrho_1\right) & \begin{cases} \text{if } vec(\hat{W}) \in \partial\left(\Lambda_W\right) \text{ and} \\ vec(\varrho_1)^T vec(\hat{W})^{\perp} > 0 \end{cases} \end{cases} \tag{22}$$

$$\dot{\hat{V}} \triangleq \begin{cases} \varrho_2 & \text{if } vec(\hat{V}) \in int\left(\Lambda_V\right) \\ \varrho_2 & \begin{cases} \text{if } vec(\hat{V}) \in \partial\left(\Lambda_V\right) \text{ and} \\ vec\left(\varrho_2\right)^T vec(\hat{V})^{\perp} \leq 0 \end{cases} \\ P_{Mr}^t\left(\varrho_2\right) & \begin{cases} \text{if } vec(\hat{V}) \in \partial\left(\Lambda_V\right) \text{ and} \\ vec\left(\varrho_2\right)^T vec(\hat{V})^{\perp} > 0 \end{cases} \end{cases} \tag{23}$$

where

$$vec(\hat{W}\left(0\right)) \in int\left(\Lambda_W\right), \quad vec(\hat{V}\left(0\right)) \in int\left(\Lambda_V\right),$$

and the auxiliary terms $\varrho_1(t) \in \mathbb{R}^{(N_2+1)\times n}$, $\varrho_2(t) \in \mathbb{R}^{(N_1+1)\times N_2}$ are of the general forms that satisfy the following norm bounds:

$$\begin{aligned} \varrho_1 &= w_1\left(t\right) + \Xi_W(q, \dot{q}, e_1, e_2, r, t) \\ \varrho_2 &= v_1\left(t\right) + \Xi_V(q, \dot{q}, e_1, e_2, r, t). \end{aligned} \tag{24}$$

In (24), $w_1(t) \in \mathbb{R}^{(N_2+1)\times n}$ and $v_1(t) \in \mathbb{R}^{(N_1+1)\times N_2}$ are known functions such that

$$\begin{aligned} \|w_1(t)\| &\leq \gamma_1 \\ \|\dot{w}_1(t)\| &\leq \gamma_2 + \gamma_3 \|e_1\| + \gamma_4 \|e_2\| + \gamma_5 \|r\| \end{aligned} \tag{25}$$

$$\begin{aligned} \|v_1(t)\| &\leq \delta_1 \\ \|\dot{v}_1(t)\| &\leq \delta_2 + \delta_3 \|e_1\| + \delta_4 \|e_2\| + \delta_5 \|r\| \end{aligned} \tag{26}$$

and $\Xi_W \in \mathbb{R}^{(N_2+1)\times n}$ and $\Xi_V \in \mathbb{R}^{(N_1+1)\times N_2}$ satisfy the following bounds:

$$\begin{aligned} \|\Xi_W(t)\| &\leq \gamma_6 \|e_1\| + \gamma_7 \|e_2\| + \gamma_8 \|r\| \\ \|\Xi_V(t)\| &\leq \delta_6 \|e_1\| + \delta_7 \|e_2\| + \delta_8 \|r\| \end{aligned} \tag{27}$$

where $\gamma_i, \delta_i \in \mathbb{R}$, $i = 1, 2, ..., 8$ are known non-negative constants (i.e., the constants can be set to zero for different update laws). In (22) and (23), $P^t_{Mr}(A) = devec(P^t_r(vec(A)))$ for a matrix A, where the operation $devec(\cdot)$ is the reverse of $vec(\cdot)$.

Remark 2: The use of the projection algorithm in (22) and (23) is to ensure that $\hat{W}(t)$ and $\hat{V}(t)$ remain bounded inside the convex regions defined in (11) and (13). This fact will be exploited in the subsequent stability analysis

Remark 3: It is assumed that only the NN adaptation rules depend on the unmeasurable signal r(t) but the corresponding weight estimate obtained after integration is independent of r(t).

The closed-loop tracking error system can be developed by substituting (19) into (14) as

$$M(q)r = f_d - \hat{f}_d + S + \tau_d - \mu. \tag{28}$$

To facilitate the subsequent stability analysis, the time derivative of (28) is determined as

$$M(q)\dot{r} = -\dot{M}(q)r + \dot{f}_d - \dot{\hat{f}}_d + \dot{S} + \dot{\tau}_d - \dot{\mu}. \tag{29}$$

Using (17) and (21) the closed-loop error system in (29) can be expressed as

$$\begin{aligned} M(q)\dot{r} \;=\; & -\dot{M}(q)r + W^T\sigma'\left(V^T x_d\right)V^T\dot{x}_d \\ & -\dot{\hat{W}}^T\sigma(\hat{V}^T x_d) - \hat{W}^T\sigma'(\hat{V}^T x_d)\dot{\hat{V}}^T x_d \\ & -\hat{W}^T\sigma'(\hat{V}^T x_d)\hat{V}^T\dot{x}_d + \dot{\varepsilon} + \dot{S} + \dot{\tau}_d - \dot{\mu}. \end{aligned} \tag{30}$$

After adding and subtracting the term $W^T\hat{\sigma}'\hat{V}^T\dot{x}_d + \hat{W}^T\hat{\sigma}'\tilde{V}^T\dot{x}_d$ to (30), the following expression can be obtained:

$$\begin{aligned} M(q)\dot{r} \;=\; & -\dot{M}(q)r + \hat{W}^T\hat{\sigma}'\tilde{V}^T\dot{x}_d + \tilde{W}^T\hat{\sigma}'\hat{V}^T\dot{x}_d \\ & +W^T\sigma'V^T\dot{x}_d - W^T\hat{\sigma}'\hat{V}^T\dot{x}_d - \hat{W}^T\hat{\sigma}'\tilde{V}^T\dot{x}_d \\ & -\dot{\hat{W}}^T\hat{\sigma} - \hat{W}^T\hat{\sigma}'\dot{\hat{V}}^T x_d + \dot{S} + \dot{\tau}_d + \dot{\varepsilon} - \dot{\mu} \end{aligned} \tag{31}$$

where the notations $\hat{\sigma}$ and $\tilde{\sigma}$ are introduced in (8). Substituting the NN weight adaptation laws in (22), (23); (31) can be rewritten as

$$\begin{aligned} M(q)\dot{r} \;=\; & -\frac{1}{2}\dot{M}(q)r + \tilde{N} + N_B - e_2 \\ & -(k_s+1)r - \beta_1 sgn(e_2) \end{aligned} \tag{32}$$

where the fact that the time derivative of (20) is given as

$$\dot{\mu} = (k_s+1)r + \beta_1 sgn(e_2) \tag{33}$$

was utilized, and where the unmeasurable auxiliary terms $\tilde{N}(e_1, e_2, r, t)$, $N_B(\hat{W}, \hat{V}, x_d, \dot{x}_d, t) \in \mathbb{R}^n$ are defined as

$$\begin{aligned} \tilde{N} \;\triangleq\; & -\frac{1}{2}\dot{M}(q)r - proj(\Xi_W)^T\hat{\sigma} \\ & -\hat{W}^T\hat{\sigma}' proj(\Xi_V)^T x_d + \dot{S} + e_2 \end{aligned} \tag{34}$$

$$N_B \triangleq N_{B_1} + N_{B_2}. \tag{35}$$

In (35), $N_{B_1}(t)$, $N_{B_2}(\hat{W}, \hat{V}, x_d, \dot{x}_d, t) \in \mathbb{R}^n$ are given by

$$N_{B_1} = W^T\sigma' V^T\dot{x}_d + \dot{\varepsilon} + \dot{\tau}_d \tag{36}$$

$$\begin{aligned} N_{B_2} \;=\; & \hat{W}^T\hat{\sigma}'\tilde{V}^T\dot{x}_d + \tilde{W}^T\hat{\sigma}'\hat{V}^T\dot{x}_d \\ & -proj(w_1)^T\hat{\sigma} - \hat{W}^T\hat{\sigma}' proj(v_1)^T x_d \end{aligned} \tag{37}$$

where $proj(\cdot)$ is the projection operator. In a similar manner as in [27], the Mean Value Theorem can be used to develop the following upper bound

$$\left\|\tilde{N}(t)\right\| \leq \rho(\|z\|)\|z\| \tag{38}$$

where $z(t) \in \mathbb{R}^{3n}$ is defined as

$$z(t) \triangleq [e_1^T \;\; e_2^T \;\; r^T]^T, \tag{39}$$

and the bounding function $\rho(\|z\|) \in \mathbb{R}$ is a positive globally invertible nondecreasing function. The following inequalities can be developed based on Properties 2 and 3, (18), (24)-(26), and (35)-(37):

$$\|N_B\| \leq \zeta_1 \qquad \left\|\dot{N}_{B_1}\right\| \leq \zeta_2 \tag{40}$$

$$\left\|\dot{N}_{B_2}\right\| \leq \zeta_3 + \zeta_4\|e_1\| + \zeta_5\|e_2\| + \zeta_6\|r\| \tag{41}$$

where $\zeta_i \in \mathbb{R}$ $(i = 1, 2, ..., 6)$ are known positive constants.

VI. Stability Analysis

Theorem: The combined NN and RISE controller given in (19)-(23) ensures that all system signals are bounded under closed-loop operation and that the position tracking error is regulated in the sense that

$$\|e_1(t)\| \to 0 \qquad \text{as} \qquad t \to \infty$$

provided the control gain k_s introduced in (20) is selected sufficiently large (see the subsequent proof), α_1, α_2 are selected according to the following sufficient conditions:

$$\begin{aligned} \alpha_1 &> \frac{\beta_2}{4} + \frac{1}{2} \\ \alpha_2 &> \frac{\beta_2}{2} + \beta_3 + \frac{\beta_4}{2} + 1 \end{aligned} \tag{42}$$

and β_i $(i = 1, 2, 3, 4)$ are selected according to the following sufficient conditions:

$$\begin{aligned} \beta_1 > \zeta_1 + \frac{1}{\alpha_2}\zeta_2 + \frac{1}{\alpha_2}\zeta_3 \\ \beta_2 > \zeta_4 \qquad \beta_3 > \zeta_5 \qquad \beta_4 > \zeta_6 \end{aligned} \tag{43}$$

where β_1 was introduced in (20), and β_2-β_4 are introduced in (46).

Proof: Let $\mathcal{D} \subset \mathbb{R}^{3n+1}$ be a domain containing $y(t) = 0$, where $y(t) \in \mathbb{R}^{3n+1}$ is defined as

$$y(t) \triangleq [z^T(t) \quad \sqrt{P(t)}]^T. \tag{44}$$

In (44), the auxiliary function $P(t) \in \mathbb{R}$ is defined as

$$P(t) \triangleq \beta_1 \|e_2(0)\| - e_2(0)^T N_B(0) - \int_0^t L(\tau) d\tau \tag{45}$$

where the auxiliary function $L(t) \in \mathbb{R}$ is defined as

$$\begin{aligned} L(t) \quad \triangleq \quad & r^T(N_B(t) - \beta_1 sgn(e_2)) \\ & -\beta_2 \|e_1(t)\| \|e_2(t)\| - \beta_3 \|e_2(t)\|^2 \\ & -\beta_4 \|e_2(t)\| \|r(t)\| \end{aligned} \tag{46}$$

where $\beta_i \in \mathbb{R}$ $(i = 1, 2, 3, 4)$ are positive constants chosen according to the sufficient conditions in (43). Provided the sufficient conditions introduced in (43) are satisfied, the following inequality can be obtained in a similar manner as in [30]:

$$\int_0^t L(\tau) d\tau \leq \beta_1 \|e_2(0)\| - e_2(0)^T N_B(0). \tag{47}$$

Hence, (47) can be used to conclude that $P(t) \geq 0$.

Let $V_L(y, t) : \mathcal{D} \times [0, \infty) \rightarrow \mathbb{R}$ be a continuously differentiable positive definite function defined as

$$V_L(y, t) \triangleq e_1^T e_1 + \frac{1}{2} e_2^T e_2 + \frac{1}{2} r^T M(q) r + P, \tag{48}$$

which satisfies the following inequalities:

$$U_1(y) \leq V_L(y, t) \leq U_2(y) \tag{49}$$

provided the sufficient conditions introduced in (43) are satisfied. In (49), the continuous positive definite functions $U_1(y)$, and $U_2(y) \in \mathbb{R}$ are defined as $U_1(y) \triangleq \lambda_1 \|y\|^2$, and $U_2(y) \triangleq \lambda_2(q) \|y\|^2$, where λ_1, $\lambda_2(q) \in \mathbb{R}$ are defined as

$$\lambda_1 \triangleq \frac{1}{2} \min \{1, m_1\} \qquad \lambda_2(q) \triangleq \max \left\{ \frac{1}{2} \bar{m}(q), 1 \right\}$$

where m_1, $\bar{m}(q)$ are introduced in (2). After taking the time derivative of (48), $\dot{V}_L(y, t)$ can be expressed as

$$\begin{aligned} \dot{V}_L(y, t) \quad = \quad & r^T M(q) \dot{r} + \frac{1}{2} r^T \dot{M}(q) r \\ & + e_2^T \dot{e}_2 + 2 e_1^T \dot{e}_1 + \dot{P}. \end{aligned}$$

The derivative $\dot{P}(t) \in \mathbb{R}$ can be expressed as

$$\begin{aligned} \dot{P}(t) \quad = \quad & -L(t) = -r^T(N_B(t) - \beta_1 sgn(e_2)) \\ & + \beta_2 \|e_1(t)\| \|e_2(t)\| + \beta_3 \|e_2(t)\|^2 \\ & + \beta_4 \|e_2(t)\| \|r(t)\|. \end{aligned} \tag{50}$$

After utilizing (4), (5), (32), (33), and (50), $\dot{V}(y, t)$ can be simplified as follows:

$$\begin{aligned} \dot{V}_L(y, t) \quad = \quad & r^T \tilde{N}(t) - (k_s + 1) \|r\|^2 - \alpha_2 \|e_2\|^2 \\ & -2\alpha_1 \|e_1\|^2 + 2 e_2^T e_1 + \beta_2 \|e_1\| \|e_2\| \\ & + \beta_3 \|e_2\|^2 + \beta_4 \|e_2\| \|r\|. \end{aligned} \tag{51}$$

Based on the fact that

$$2 e_2^T e_1 \leq \|e_1\|^2 + \|e_2\|^2$$

$\dot{V}_L(y, t)$ can be upper bounded using the squares of the components of $z(t)$ as follows:

$$\begin{aligned} \dot{V}_L(y, t) \quad \leq \quad & r^T \tilde{N}(t) - (k_s + 1) \|r\|^2 - \alpha_2 \|e_2\|^2 \\ & -2\alpha_1 \|e_1\|^2 + \|e_1\|^2 + \|e_2\|^2 \\ & + \frac{\beta_2}{2} \|e_1\|^2 + \frac{\beta_2}{2} \|e_2\|^2 + \beta_3 \|e_2\|^2 \\ & + \frac{\beta_4}{2} \|e_2\|^2 + \frac{\beta_4}{2} \|r\|^2. \end{aligned} \tag{52}$$

By using (38), the expression in (52) can be rewritten as follows:

$$\dot{V}_L(y, t) \leq -\lambda_3 \|z\|^2 - \left[\left(k_s - \frac{\beta_4}{2} \right) \|r\|^2 - \rho(\|z\|) \|r\| \|z\| \right] \tag{53}$$

where $\lambda_3 \triangleq \min\{2\alpha_1 - \frac{\beta_2}{2} - 1, \alpha_2 - \frac{\beta_2}{2} - \beta_3 - \frac{\beta_4}{2} - 1, 1\}$; hence, α_1, and α_2 must be chosen according to the sufficient condition in (42). After completing the squares for the terms inside the brackets in (53), the following expression can be obtained:

$$\dot{V}_L(y, t) \leq -\lambda_3 \|z\|^2 + \frac{\rho^2(\|z\|) \|z\|^2}{4 \left(k_s - \frac{\beta_4}{2} \right)} \leq -U(y) \tag{54}$$

where $U(y) = c \|z\|^2$, for some positive constant c, is a continuous, positive semi-definite function that is defined on the following domain:

$$\mathcal{D} \triangleq \left\{ y \in \mathbb{R}^{3n+1} \mid \|y\| \leq \rho^{-1} \left(2 \sqrt{\lambda_3 \left(k_s - \frac{\beta_4}{2} \right)} \right) \right\}.$$

The inequalities in (49) and (54) can be used to show that $V_L(y, t) \in \mathcal{L}_\infty$ in $\mathcal{D}$; hence, $e_1(t)$, $e_2(t)$, and $r(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Given that $e_1(t)$, $e_2(t)$, and $r(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, standard linear analysis methods can be used to prove that $\dot{e}_1(t)$, $\dot{e}_2(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$ from (4) and (5). Since $e_1(t)$, $e_2(t)$, $r(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, the assumption that $q_d(t)$, $\dot{q}_d(t)$, $\ddot{q}_d(t)$ exist and are bounded can be used along with (3)-(5) to conclude that $q(t)$, $\dot{q}(t)$, $\ddot{q}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Since $q(t)$, $\dot{q}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, Property 2 can be used to conclude that $M(q)$, $V_m(q, \dot{q})$, $G(q)$, and $F(\dot{q}) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Thus from (1) and Property 3, we can show that $\tau(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Given that $r(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, (33) can be used to show that $\dot{\mu}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Since $\dot{q}(t)$, $\ddot{q}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, Property 2 can be used to show that $\dot{V}_m(q, \dot{q})$, $\dot{G}(q)$, $\dot{F}(q)$ and $\dot{M}(q) \in \mathcal{L}_\infty$ in $\mathcal{D}$; hence, (32) can be used to show that $\dot{r}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Since $\dot{e}_1(t)$, $\dot{e}_2(t)$, $\dot{r}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, the definitions for $U(y)$ and $z(t)$ can be used to prove that $U(y)$ is uniformly continuous in $\mathcal{D}$.

Let $\mathcal{S} \subset \mathcal{D}$ denote a set defined as follows:

$$\begin{aligned} \mathcal{S} \quad \triangleq \quad & \{ y(t) \subset \mathcal{D} \mid U_2(y(t)) \\ & < \lambda_1 \left(\rho^{-1} \left(2 \sqrt{\lambda_3 \left(k_s - \frac{\beta_4}{2} \right)} \right) \right)^2 \}. \end{aligned} \tag{55}$$

The region of attraction in (55) can be made arbitrarily large to include any initial conditions by increasing the control gain k_s (i.e., a semi-global type of stability result) [27]. Theorem 8.4 of [31] can now be invoked to state that

$$c\left\|z(t)\right\|^2 \to 0 \qquad \text{as} \qquad t \to \infty \qquad \forall y(0) \in \mathcal{S}. \tag{56}$$

Based on the definition of $z(t)$, (56) can be used to show that

$$\|e_1(t)\| \to 0 \qquad \text{as} \qquad t \to \infty \qquad \forall y(0) \in \mathcal{S}. \tag{57}$$

VII. Conclusion

Modularity in controller/NN weight tuning law was shown for a class of non-LP uncertain Euler-Lagrange systems with additive bounded disturbances. Specifically, a nonlinear multilayer NN was used in conjunction with the RISE feedback term (see [19]), and a generic form of the NN weight tuning law was derived. A new closed-loop error system was developed, and the typical RISE stability analysis was modified. New sufficient gain conditions were derived to show asymptotic tracking of the desired link position. The generic form of the tuning law allows for the use of any existing NN weight tuning law (e.g., gradient or Hebbian etc.).

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