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Bilateral Control of Nonlinear Teleoperation with Time Varying Communication Delays

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Abstract— **This paper addresses the bilateral control of nonlinear teleoperation with time varying communication delays. The proposed methods are two types of simple PD-type controllers which consists of D-controls depending on (the upper bound of) the rate of change of delay and P-controls depending on the upper bound of round-trip delay. Using Lyapunov-Krasovskii function, the delay-dependent stability of the origin is shown for the ranges of gains. Furthermore the proposed strategies also achieve master-slave position coordination and bilateral static force reflection. Several experimental results show the effectiveness of our proposed methods.**

I. Introduction

Teleoperation is the extension of a person's sensing and manipulation capability to a remote location and it has been tackled by researchers in control theory and robotics over the last few decades. A teleoperator is a dual robot system in which a remote slave robot tracks the motion of a master robot, which is, in turn, commanded by a human operator. To improve the task performance, information about the remote environment is needed. In particular, force feedback from the slave to the master, representing contact information, provides a more extensive sense of telepresence. When this is done, the teleoperator is said to be controlled bilaterally [1].

In bilateral teleoperation, the master and the slave are coupled via a communication network, and time delay is incurred in transmission of data between the master and slave site. It is well known that the delay in a closed-loop system can destabilize an otherwise stable system. Recently, essential research interest has been attracted by using the Internet as a communication network for teleoperation [2]-[6]. Using the Internet for communication network provides obvious benefits in terms of low cost and availability. However, at the present time, for teleoperation over the Internet the delays varies with such factors as congestion, bandwidth, or distance, and these varying delays may severely degrade performance or even result in an unstable system.

Stabilization for a teleoperation with constant communication delays has been achieved by the scattering transformation based on the idea of passivity [7] (This is equivalent wave variable formulation [8]). Then, the additional structure with position feedforward/feedback controls has proposed to improve the position coordination and force reflection performance [9], [10]. In [11]-[13], the PD-type controller without scattering transformation has been proposed which guarantees the stability for the constant communication delays. In these methods, the position coordination and force reflection have also been achieved by explicit position feedback/feedforward control. In [7]-[13], however, the time varying communication delays has not been treated.

Several researchers have addressed a problem of the teleoperation with time varying delays and several control methods based on scattering transformation have been reported. Some preliminary results are contained in [2], [3]. An interesting result has been obtained in [4]. A simple modification to the scattering transformation has been proposed, that inserts a time varying gain into the communication block which guarantees passivity for arbitrary time varying delays provided a bound on the rate of change of the time delays. In [2]-[4], however, it is insufficient for the performance of force reflection and/or positional coordination due to the lack of the explicit position feedback/feedforward controls. In [5], [6], they have proposed control methods without the scattering transformation. However, there are problems that the model of robots, the environment and the human operator are required by the controllers. Then robustness for parameter uncertainties has not been guaranteed and the controllers have become complex.

In this paper, we address the bilateral control of nonlinear teleoperation with time varying delays. Our proposed control strategies are two types of simple PD-type controllers which directly connects the master and slave robots by position and velocity signals over the delayed communication. For the velocity control, the first controller has a time varying D-gains which depend on the rate of change of delays and the second one has constant D-gains which are designed under stability condition. Moreover the both controllers have explicit position feedback/feedforward control. The proposed control strategies are independent of parameter uncertainties of the robot models, the human operator and the remote environment. Using Lyapunov-Krasovskii function, the delay-depend stability of the origin is shown for the ranges of the gains. Moreover the proposed framework enforces master-slave position coordination and static force reflection. Several experimental results show the effectiveness of our proposed framework.

II. Dynamics of Teleoperation System

In this paper, we consider a pair of nonlinear robotic system coupled via communication lines with time varying delays as shown in Fig. 1. Assuming absence of friction and other disturbances, the master and slave dynamics with n-

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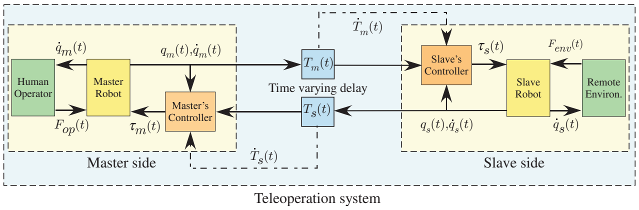

Fig. 1. Teleoperation system

DOF are described as [15]

$$\begin{cases} \boldsymbol{M_m}(q_m)\ddot{\boldsymbol{q}}_{\boldsymbol{m}}(t) + \boldsymbol{C_m}(q_m, \dot{q}_m)\dot{\boldsymbol{q}}_{\boldsymbol{m}}(t) = \boldsymbol{\tau_m}(t) + \boldsymbol{F_{op}}(t) \\ \boldsymbol{M_s}(q_s)\ddot{\boldsymbol{q}}_{\boldsymbol{s}}(t) + \boldsymbol{C_s}(q_s, \dot{q}_s)\dot{\boldsymbol{q}}_{\boldsymbol{s}}(t) = \boldsymbol{\tau_s}(t) - \boldsymbol{F_{env}}(t), \end{cases} \tag{1}$$

where the subscript "m" and "s" denote the master and the slave indexes, $\boldsymbol{q}_m,\ \boldsymbol{q_s} \in R^n$ are the joint angle vectors, $\dot{\boldsymbol{q}}_{\boldsymbol{m}},\ \dot{\boldsymbol{q}}_{\boldsymbol{s}}, \in R^n$ are the joint velocity vectors, $\ddot{\boldsymbol{q}}_{\boldsymbol{m}},\ \ddot{\boldsymbol{q}}_{\boldsymbol{s}}, \in R^n$ are the joint acceleration vectors, $\boldsymbol{\tau_m},\ \boldsymbol{\tau_s} \in R^n$ are the input torque vectors, $\boldsymbol{F_{op}} \in R^n$ is the operational force vectors applied to the master by human operator, $\boldsymbol{F_{env}} \in R^n$ is the environmental force vectors applied to the environment by the slave, $\boldsymbol{M_m},\ \boldsymbol{M_s} \in R^{n\times n}$ are the inertia matrices, $\boldsymbol{C_m}\dot{\boldsymbol{q}}_{\boldsymbol{m}},\ \boldsymbol{C_s}\dot{\boldsymbol{q}}_{\boldsymbol{s}} \in R^n$ are the centrifugal and Coriolis torque vectors, respectively. We assume that the gravity terms are either pre-compensated by the local control. It is well known that the dynamics (1) have several fundamental properties as follows.

Property 1: The inertia matrices $\boldsymbol{M_i}$ is symmetric and positive definite and both $\boldsymbol{M_i}$ and $\boldsymbol{M_i}^{-1}$ are uniformly bounded.

Property 2: Under an appropriate definition of the matrices $\boldsymbol{C_i}$, the matrices $\boldsymbol{N_i} = \dot{\boldsymbol{M}}_{\boldsymbol{i}} - 2\boldsymbol{C_i}$ is skew symmetric.

For the human operator and the remote environment, we assume as follows [12].

Assumption 1: The human operator can be modeled as non-passive system that applies any constant force on the master robot. The remote environment can be modeled as passive system that is any linear spring–damper system.
Under above assumption, the human operator is described as follows

$$\boldsymbol{F_{op}}(t) = \bar{\boldsymbol{F}}_{\boldsymbol{op}}, \tag{2}$$

where $\bar{\boldsymbol{F}}_{\boldsymbol{op}} \in R^n$ is any finite constant vector. The remote environment is described as follows

$$\boldsymbol{F_{env}}(t) = \boldsymbol{B_e}\dot{\boldsymbol{q}}_{\boldsymbol{s}}(t) + \boldsymbol{K_e}\boldsymbol{q_s}(t), \tag{3}$$

where $\boldsymbol{B_e} \in R^{n\times n}$ is any positive semi-definite environmental damper matrix and $\boldsymbol{K_e} \in R^{n\times n}$ is any positive semi-definite environmental spring matrix.

The communication structure are assumed as shown in Fig.1, where the forward and backward communications are delayed by the functions of time varying delay $T_m(t)$ and $T_s(t)$ as follows

Assumption 2: $T_m(t)$ and $T_s(t)$ are continuously differentiable functions and satisfy as follows

$$0 \le T_i(t) \le T_i^+ < \infty,\ |\dot{T}_i(t)| < 1, \quad i = m, s, \tag{4}$$

where $T_i^+ \in R$ are upper bounds of the communication delays. Moreover, the upper bound of the round trip communication delay $T_{ms}^+ = T_m^+ + T_s^+$ is known preliminarily, $\dot{T}_m$ can be measured at the slave site and $\dot{T}_s$ can be measured at the master site.

In addition, we assume for stability analysis as follows

Assumption 3: The velocities $\dot{\boldsymbol{q}}_{\boldsymbol{m}}$ and $\dot{\boldsymbol{q}}_{\boldsymbol{s}}$ equal zero for $t < 0$.

III. Control Objectives

We would like to design the control inputs $\boldsymbol{\tau_m}$ and $\boldsymbol{\tau_s}$ to achieve as follows

Control Objective 1: (Stability) The teleoperation system as shown in Fig. 1 is stable under the time varying communication delay, any constant operational inputs (2) and any environment (3).

Control Objective 2: (Static Force Reflection) The static contact force in slave side are accurately transmitted to the human operator in the master side as follows

$$\boldsymbol{F_{env}} = \boldsymbol{F_{op}} \quad as \quad t \to \infty. \tag{5}$$

Control Objective 3: (Master-Slave Position Coordination) If $\boldsymbol{F_{op}} = \boldsymbol{F_{env}} = 0$, the position coordination error $\boldsymbol{q_E}$ goes to zero as

$$\boldsymbol{q_E}(t) := \boldsymbol{q_m}(t) - \boldsymbol{q_s}(t) \to 0 \quad as \quad t \to \infty, \tag{6}$$

and the master and slave positions are coordinated.

Note that the *Control Objectives 2* and *3* mean achievement at minimal level of ideal transparency [14].

IV. Control Designs

To achieve above control objectives, we proposed two types of controllers for the teleoperation. One of them is a PD-type controller with time varying gains and another one is PD-type controller without time varying gains.

A. Control Law with Time Varying Gains

The proposed control law with time varying gains is now given as

Control Law 1

$$\begin{cases} \boldsymbol{\tau_m}(t) = \boldsymbol{K_{md}}(t)\left\{\dot{\boldsymbol{q}}_{\boldsymbol{s}}(t - T_s(t)) - \dot{\boldsymbol{q}}_{\boldsymbol{m}}(t)\right\} \\ \quad - \left\{\boldsymbol{D_{md}}(t) + \boldsymbol{D_p}\right\}\dot{\boldsymbol{q}}_{\boldsymbol{m}}(t) + \boldsymbol{K_p}\left\{\boldsymbol{q_s}(t - T_s(t)) - \boldsymbol{q_m}(t)\right\} \\ \boldsymbol{\tau_s}(t) = \boldsymbol{K_{sd}}(t)\left\{\dot{\boldsymbol{q}}_{\boldsymbol{m}}(t - T_s(t)) - \dot{\boldsymbol{q}}_{\boldsymbol{s}}(t)\right\} \\ \quad - \left\{\boldsymbol{D_{sd}}(t) + \boldsymbol{D_p}\right\}\dot{\boldsymbol{q}}_{\boldsymbol{s}}(t) + \boldsymbol{K_p}\left\{\boldsymbol{q_m}(t - T_m(t)) - \boldsymbol{q_s}(t)\right\}, \end{cases} \tag{7}$$

where $\boldsymbol{K_{md}}(t), \boldsymbol{K_{sd}}(t), \boldsymbol{D_{md}}(t), \boldsymbol{D_{sd}}(t)$ are time varying gain matrices depending on $\dot{T}_m(t)$ and $\dot{T}_s(t)$ as follows

$$\begin{cases} \boldsymbol{K_{md}}(t) = (1 - \dot{T}_s(t))\boldsymbol{K_d} \\ \boldsymbol{K_{sd}}(t) = (1 - \dot{T}_m(t))\boldsymbol{K_d}, \end{cases} \quad \begin{cases} \boldsymbol{D_{md}}(t) = \frac{\dot{T}_s(t)}{2}\boldsymbol{K_d} \\ \boldsymbol{D_{sd}}(t) = \frac{\dot{T}_m(t)}{2}\boldsymbol{K_d}, \end{cases} \tag{8}$$

and $\boldsymbol{D_p}, \boldsymbol{K_p}, \boldsymbol{K_d} \in R^{n\times n}$ are positive diagonal constant matrices. Our proposed *Control Law 1* (7) is using simple PD type controller with P-control gain $\boldsymbol{K_p}$ and time varying D-control gains $\boldsymbol{K_{md}}(t), \boldsymbol{K_{sd}}(t)$. $\boldsymbol{D_p}$ is the dissipation gain used to stabilize the P-control with time varying delay and it is designed from later stability analysis. $\boldsymbol{D_{md}}(t)$ and $\boldsymbol{D_{sd}}(t)$

are the time varying quasi-dissipation gains used to stabilize the D-control with time varying delay.

Note that the *Control Law 1* (7) requires the position, velocity and the rate of change of the delay signals (The rate of change of the delay can be detected as shown in Appendix A). The explicit position control improves the position coordination and force reflecting performance in comparison with the conventional scattering-based teleoperation [2]-[4]. The time varying gains which depend on the rate of change of the delay have been proposed in [4] stabilizing the time varying delay. However, our proposed control law does not use the scattering based approach. This is the main characteristic of my research.

To facilitate the stability analysis of the system, the closed loop system is now derived. The equilibrium points of the positions of the master and the slave are defined as $\bar{\boldsymbol{q}}_{\boldsymbol{m}} \in R^n$ and $\bar{\boldsymbol{q}}_{\boldsymbol{s}} \in R^n$ such that

$$\begin{cases} \bar{\boldsymbol{F}}_{\boldsymbol{op}} = \boldsymbol{K_p}(\bar{\boldsymbol{q}}_{\boldsymbol{m}} - \bar{\boldsymbol{q}}_{\boldsymbol{s}}) \\ 0 = \boldsymbol{K_e}\bar{\boldsymbol{q}}_{\boldsymbol{s}} - \boldsymbol{K_p}(\bar{\boldsymbol{q}}_{\boldsymbol{m}} - \bar{\boldsymbol{q}}_{\boldsymbol{s}}). \end{cases} \tag{9}$$

The new position variables with the origin of above equilibrium points are defined as follows

$$\begin{cases} \tilde{\boldsymbol{q}}_{\boldsymbol{m}}(t) = \boldsymbol{q}_{\boldsymbol{m}}(t) - \bar{\boldsymbol{q}}_{\boldsymbol{m}} \\ \tilde{\boldsymbol{q}}_{\boldsymbol{s}}(t) = \boldsymbol{q}_{\boldsymbol{s}}(t) - \bar{\boldsymbol{q}}_{\boldsymbol{s}}. \end{cases} \tag{10}$$

Then substituting (2), (3), (7) and (9) into (1) and assembling by (10), the closed loop systems can be described as

$$\begin{cases} \boldsymbol{M_m}\ddot{\boldsymbol{q}}_{\boldsymbol{m}} + \boldsymbol{C_m}\dot{\boldsymbol{q}}_{\boldsymbol{m}} = \boldsymbol{K_{md}}(t)\left\{\dot{\boldsymbol{q}}_{\boldsymbol{s}}(t-T_s(t)) - \dot{\boldsymbol{q}}_{\boldsymbol{m}}\right\} \\ \quad - \left\{\boldsymbol{D_{md}}(t) + \boldsymbol{D_p}\right\}\dot{\boldsymbol{q}}_{\boldsymbol{m}} + \boldsymbol{K_p}\left\{\tilde{\boldsymbol{q}}_{\boldsymbol{s}}(t-T_s(t)) - \tilde{\boldsymbol{q}}_{\boldsymbol{m}}\right\} \\ \boldsymbol{M_s}\ddot{\boldsymbol{q}}_{\boldsymbol{s}} + \boldsymbol{C_s}\dot{\boldsymbol{q}}_{\boldsymbol{s}} = \boldsymbol{K_{sd}}(t)\left\{\dot{\boldsymbol{q}}_{\boldsymbol{m}}(t-T_m(t)) - \dot{\boldsymbol{q}}_{\boldsymbol{s}}\right\} \\ \quad - \left\{\boldsymbol{D_{sd}}(t) + \boldsymbol{D_p}\right\}\dot{\boldsymbol{q}}_{\boldsymbol{s}} + \boldsymbol{K_p}\left\{\tilde{\boldsymbol{q}}_{\boldsymbol{m}}(t-T_m(t)) - \tilde{\boldsymbol{q}}_{\boldsymbol{s}}\right\} \\ \qquad\qquad - \boldsymbol{B_e}\dot{\boldsymbol{q}}_{\boldsymbol{s}} - \boldsymbol{K_e}\tilde{\boldsymbol{q}}_{\boldsymbol{s}}. \end{cases} \tag{11}$$

The following theorem describes stability properties of the closed loop teleoperation (11) with *Control Law 1*.

Theorem 1: Consider the nonlinear teleoperation described by (11) with *Assumptions 1-3*. Then for range of the control gain $\boldsymbol{K_p}$ as follows

$$\boldsymbol{K_p} < \frac{2}{T_{ms}^{+}}\boldsymbol{D_p}, \tag{12}$$

the origin of the system $\dot{\boldsymbol{q}}_{\boldsymbol{m}}, \dot{\boldsymbol{q}}_{\boldsymbol{s}}, \tilde{\boldsymbol{q}}_{\boldsymbol{m}}, \tilde{\boldsymbol{q}}_{\boldsymbol{s}}$ are asymptotically stable and $\lim_{t\to\infty}\boldsymbol{q}_{\boldsymbol{m}}(t) = \bar{\boldsymbol{q}}_{\boldsymbol{m}}$, $\lim_{t\to\infty}\boldsymbol{q}_{\boldsymbol{s}}(t) = \bar{\boldsymbol{q}}_{\boldsymbol{s}}$. Therefore the *Control Objective 1* is achieved.

Proof: Define a positive definite function (Lyapunov-Krasovskii function) for the system as

$$\begin{aligned} V_1(x) &= \dot{\boldsymbol{q}}_{\boldsymbol{m}}^T(t)\boldsymbol{M_m}(q_m)\dot{\boldsymbol{q}}_{\boldsymbol{m}}(t) + \dot{\boldsymbol{q}}_{\boldsymbol{s}}^T(t)\boldsymbol{M_s}(q_s)\dot{\boldsymbol{q}}_{\boldsymbol{s}}(t) \\ &+ \{\tilde{\boldsymbol{q}}_{\boldsymbol{m}}(t) - \tilde{\boldsymbol{q}}_{\boldsymbol{s}}(t)\}^T\boldsymbol{K_p}\{\tilde{\boldsymbol{q}}_{\boldsymbol{m}}(t) - \tilde{\boldsymbol{q}}_{\boldsymbol{s}}(t)\} + \tilde{\boldsymbol{q}}_{\boldsymbol{s}}^T(t)\boldsymbol{K_e}\tilde{\boldsymbol{q}}_{\boldsymbol{s}}(t) \\ &+ \int_{t-T_m(t)}^{t}\dot{\boldsymbol{q}}_{\boldsymbol{m}}^T(\xi)\boldsymbol{K_d}\dot{\boldsymbol{q}}_{\boldsymbol{m}}(\xi)d\xi + \int_{t-T_s(t)}^{t}\dot{\boldsymbol{q}}_{\boldsymbol{s}}^T(\xi)\boldsymbol{K_d}\dot{\boldsymbol{q}}_{\boldsymbol{s}}(\xi)d\xi, \end{aligned} \tag{13}$$

where $\boldsymbol{x}(t) = \left[\dot{\boldsymbol{q}}_{\boldsymbol{m}}^T(t) \;\; \dot{\boldsymbol{q}}_{\boldsymbol{s}}^T(t) \;\; (\tilde{\boldsymbol{q}}_{\boldsymbol{m}}(t) - \tilde{\boldsymbol{q}}_{\boldsymbol{s}}(t))^T \;\; \tilde{\boldsymbol{q}}_{\boldsymbol{s}}^T(t)\right]^T$. The derivative of the above Lyapunov function along trajectory of the system is given by

$$\begin{aligned} \dot{V}_1 = &- \{\dot{\boldsymbol{q}}_{\boldsymbol{m}}^T\boldsymbol{K_{md}}(t)\dot{\boldsymbol{q}}_{\boldsymbol{m}} - 2\dot{\boldsymbol{q}}_{\boldsymbol{m}}^T\boldsymbol{K_{md}}(t)\dot{\boldsymbol{q}}_{\boldsymbol{s}}(t-T_s(t)) \\ &\qquad + \dot{\boldsymbol{q}}_{\boldsymbol{s}}^T(t-T_s(t))\boldsymbol{K_{md}}(t)\dot{\boldsymbol{q}}_{\boldsymbol{s}}(t-T_s(t))\} \\ &- \{\dot{\boldsymbol{q}}_{\boldsymbol{s}}^T\boldsymbol{K_{sd}}(t)\dot{\boldsymbol{q}}_{\boldsymbol{s}} - 2\dot{\boldsymbol{q}}_{\boldsymbol{s}}^T\boldsymbol{K_{sd}}(t)\dot{\boldsymbol{q}}_{\boldsymbol{m}}(t-T_m(t)) \\ &\qquad + \dot{\boldsymbol{q}}_{\boldsymbol{m}}^T(t-T_m(t))\boldsymbol{K_{sd}}(t)\dot{\boldsymbol{q}}_{\boldsymbol{m}}(t-T_m(t))\} \\ &- 2\dot{\boldsymbol{q}}_{\boldsymbol{m}}^T\boldsymbol{D_p}\dot{\boldsymbol{q}}_{\boldsymbol{m}} + 2\dot{\boldsymbol{q}}_{\boldsymbol{m}}^T\boldsymbol{K_p}(\tilde{\boldsymbol{q}}_{\boldsymbol{s}}(t-T_s(t)) - \tilde{\boldsymbol{q}}_{\boldsymbol{s}}) \\ &- 2\dot{\boldsymbol{q}}_{\boldsymbol{s}}^T\boldsymbol{D_p}\dot{\boldsymbol{q}}_{\boldsymbol{s}} + 2\dot{\boldsymbol{q}}_{\boldsymbol{s}}^T\boldsymbol{K_p}(\tilde{\boldsymbol{q}}_{\boldsymbol{m}}(t-T_m(t)) - \tilde{\boldsymbol{q}}_{\boldsymbol{m}}) \\ &- 2\dot{\boldsymbol{q}}_{\boldsymbol{s}}^T\boldsymbol{B_e}\dot{\boldsymbol{q}}_{\boldsymbol{s}}. \end{aligned} \tag{14}$$

Completing the square for first and second terms in above equation, we have that

$$\begin{aligned} \dot{V}_1 = &- \dot{\boldsymbol{e}}_{\boldsymbol{m}}^T\boldsymbol{K_{md}}(t)\dot{\boldsymbol{e}}_{\boldsymbol{m}} - \dot{\boldsymbol{e}}_{\boldsymbol{s}}^T\boldsymbol{K_{sd}}(t)\dot{\boldsymbol{e}}_{\boldsymbol{s}} - 2\dot{\boldsymbol{q}}_{\boldsymbol{s}}^T\boldsymbol{B_e}\dot{\boldsymbol{q}}_{\boldsymbol{s}} \\ &- 2\dot{\boldsymbol{q}}_{\boldsymbol{m}}^T\boldsymbol{D_p}\dot{\boldsymbol{q}}_{\boldsymbol{m}} + 2\dot{\boldsymbol{q}}_{\boldsymbol{m}}^T\boldsymbol{K_p}(\tilde{\boldsymbol{q}}_{\boldsymbol{s}}(t-T_s(t)) - \tilde{\boldsymbol{q}}_{\boldsymbol{s}}) \\ &- 2\dot{\boldsymbol{q}}_{\boldsymbol{s}}^T\boldsymbol{D_p}\dot{\boldsymbol{q}}_{\boldsymbol{s}} + 2\dot{\boldsymbol{q}}_{\boldsymbol{s}}^T\boldsymbol{K_p}(\tilde{\boldsymbol{q}}_{\boldsymbol{m}}(t-T_m(t)) - \tilde{\boldsymbol{q}}_{\boldsymbol{m}}), \end{aligned} \tag{15}$$

where $\dot{\boldsymbol{e}}_{\boldsymbol{m}} = \dot{\boldsymbol{q}}_{\boldsymbol{s}}(t-T_s(t)) - \dot{\boldsymbol{q}}_{\boldsymbol{m}}, \dot{\boldsymbol{e}}_{\boldsymbol{s}} = \dot{\boldsymbol{q}}_{\boldsymbol{m}}(t-T_m(t)) - \dot{\boldsymbol{q}}_{\boldsymbol{s}}$, Using the fact that

$$\tilde{\boldsymbol{q}}_{\boldsymbol{i}}(t-T_i(t)) - \tilde{\boldsymbol{q}}_{\boldsymbol{i}} = -\int_0^{T_i(t)}\dot{\boldsymbol{q}}_{\boldsymbol{i}}(t-\xi)d\xi, \quad i = m, s \tag{16}$$

fixing the final times as t_f, and integrating the above equation, we obtain

$$\begin{aligned} \int_0^{t_f}\dot{V}_1 dt = &-\int_0^{t_f}\dot{\boldsymbol{e}}_{\boldsymbol{m}}^T\boldsymbol{K_{md}}(t)\dot{\boldsymbol{e}}_{\boldsymbol{m}}dt - \int_0^{t_f}\dot{\boldsymbol{e}}_{\boldsymbol{s}}^T\boldsymbol{K_{sd}}(t)\dot{\boldsymbol{e}}_{\boldsymbol{s}}dt \\ &- 2\int_0^{t_f}\dot{\boldsymbol{q}}_{\boldsymbol{s}}^T\boldsymbol{B_e}\dot{\boldsymbol{q}}_{\boldsymbol{s}}dt - 2\int_0^{t_f}\dot{\boldsymbol{q}}_{\boldsymbol{m}}^T\boldsymbol{D_p}\dot{\boldsymbol{q}}_{\boldsymbol{m}}dt - 2\int_0^{t_f}\dot{\boldsymbol{q}}_{\boldsymbol{s}}^T\boldsymbol{D_p}\dot{\boldsymbol{q}}_{\boldsymbol{s}}dt \\ &- 2\int_0^{t_f}\dot{\boldsymbol{q}}_{\boldsymbol{m}}^T\boldsymbol{K_p}\int_0^{T_s(t)}\dot{\boldsymbol{q}}_{\boldsymbol{s}}(t-\xi)d\xi dt \\ &- 2\int_0^{t_f}\dot{\boldsymbol{q}}_{\boldsymbol{s}}^T\boldsymbol{K_p}\int_0^{T_m(t)}\dot{\boldsymbol{q}}_{\boldsymbol{m}}(t-\xi)d\xi dt. \end{aligned} \tag{17}$$

The sixth term in (17) can be rewritten as follows

$$\begin{aligned} &-2\int_0^{t_f}\{\dot{\boldsymbol{q}}_{\boldsymbol{m}}^T(t)\boldsymbol{K_p}\int_0^{T_s(t)}\dot{\boldsymbol{q}}_{\boldsymbol{s}}^T(t-\xi)d\xi\}dt \\ &\qquad = -\sum_{j=1}^{n}K_{pj}2\int_0^{t_f}\{\dot{q}_{mj}\int_0^{T_s(t)}\dot{q}_{sj}(t-\xi)d\xi\}dt, \end{aligned} \tag{18}$$

where $\dot{q}_{mj}$ is a jth joint velocity of the master, $\dot{q}_{sj}$ is jth joint velocity of the slave and K_{pj} is jth P-control gain for jth joint. Using the fact that $-2a^Tb \le a^2 + b^2, \; a, b \in R$, Schwarz inequality and $0 < T_s(t) \le T_s^+$ in *Assumption 2*, above equation is easily transformed into

$$\begin{aligned} &-2\int_0^{t_f}\{\dot{q}_{mj}\int_0^{T_s(t)}\dot{q}_{sj}(t-\xi)d\xi\}dt \\ &\quad \le T_s^+\int_0^{t_f}\dot{q}_{mj}^2 dt + \frac{1}{T_s^+}\int_0^{t_f}\{\int_0^{T_s(t)}\dot{q}_{sj}(t-\xi)d\xi\}^2 dt \\ &\quad \le T_s^+\int_0^{t_f}\dot{q}_{mj}^2 dt + \frac{1}{T_s^+}\int_0^{t_f}T_s(t)\int_0^{T_s(t)}\dot{q}_{sj}^2(t-\xi)d\xi dt \\ &\quad \le T_s^+\int_0^{t_f}\dot{q}_{mj}^2(t)dt + \int_0^{T_s^+}\int_0^{t_f}\dot{q}_{sj}^2 dt d\xi \\ &\quad \le T_s^+\int_0^{t_f}\dot{q}_{mj}^2(t)dt + T_s^+\int_0^{t_f}\dot{q}_{sj}^2 dt. \end{aligned} \tag{19}$$

Then the sixth and seventh terms in (17) can be rewritten as follows

$$\begin{aligned}-2\int_0^{t_f}\{\dot{\boldsymbol{q}}_{\boldsymbol{m}}^T\boldsymbol{K_p}\int_0^{T_s(t)}\dot{\boldsymbol{q}}_{\boldsymbol{s}}(t-\xi)d\xi\}dt \\ \leq T_s^+\int_0^{t_f}\dot{\boldsymbol{q}}_{\boldsymbol{m}}^T\boldsymbol{K_p}\dot{\boldsymbol{q}}_{\boldsymbol{m}}dt+T_s^+\int_0^{t_f}\dot{\boldsymbol{q}}_{\boldsymbol{s}}^T\boldsymbol{K_p}\dot{\boldsymbol{q}}_{\boldsymbol{s}}dt \quad (20)\end{aligned}$$

$$\begin{aligned}-2\int_0^{t_f}\{\dot{\boldsymbol{q}}_{\boldsymbol{s}}^T\boldsymbol{K_p}\int_0^{T_m(t)}\dot{\boldsymbol{q}}_{\boldsymbol{m}}(t-\xi)d\xi\}dt \\ \leq T_m^+\int_0^{t_f}\dot{\boldsymbol{q}}_{\boldsymbol{s}}^T\boldsymbol{K_p}\dot{\boldsymbol{q}}_{\boldsymbol{s}}dt+T_m^+\int_0^{t_f}\dot{\boldsymbol{q}}_{\boldsymbol{m}}^T\boldsymbol{K_p}\dot{\boldsymbol{q}}_{\boldsymbol{m}}dt. \quad (21)\end{aligned}$$

Therefore, integral inequality (17) reduces to

$$\begin{aligned}\int_0^{t_f}\dot{V}_1dt \leq &-\lambda_m(2\boldsymbol{B_e})||\dot{\boldsymbol{q}}_{\boldsymbol{s}}||_2^2 \\ &-\lambda_m(\boldsymbol{K_{md}}(t))||\dot{\boldsymbol{e}}_{\boldsymbol{m}}||_2^2-\lambda_m(\boldsymbol{K_{sd}}(t))||\dot{\boldsymbol{e}}_{\boldsymbol{s}}||_2^2 \\ &-\lambda_m(2\boldsymbol{D_p}-T_{ms}^+\boldsymbol{K_p})||\dot{\boldsymbol{q}}_{\boldsymbol{m}}||_2^2 \\ &-\lambda_m(2\boldsymbol{D_p}-T_{ms}^+\boldsymbol{K_p})||\dot{\boldsymbol{q}}_{\boldsymbol{s}}||_2^2. \quad (22)\end{aligned}$$

where $\lambda_m(A)$ indicates the smallest eigenvalue of A and the notation $||*||_2$ denotes the $\mathcal{L}_2$ norm of a signal on the interval $[0,t_f]$. Note that $\boldsymbol{K_{md}}(t)$ and $\boldsymbol{K_{sd}}(t)$ are positive definite from $1-\dot{T}_i(t)>0$. Letting $\lim t_f=\infty$, $\int_0^\infty \dot{V}_1(x)\leq 0$ under the condition (12), we conclude that the signals $\dot{\boldsymbol{q}}_{\boldsymbol{m}},\ \dot{\boldsymbol{q}}_{\boldsymbol{s}},\ \tilde{\boldsymbol{q}}_{\boldsymbol{m}},\ \tilde{\boldsymbol{q}}_{\boldsymbol{s}}\in\mathcal{L}_\infty$, and $\dot{\boldsymbol{q}}_{\boldsymbol{m}},\ \dot{\boldsymbol{q}}_{\boldsymbol{s}},\ \dot{\boldsymbol{e}}_{\boldsymbol{m}},\ \dot{\boldsymbol{e}}_{\boldsymbol{s}}\in\mathcal{L}_2$. From the closed loop dynamics (11), $\ddot{\boldsymbol{q}}_{\boldsymbol{m}},\ddot{\boldsymbol{q}}_{\boldsymbol{s}}\in\mathcal{L}_\infty$. This implies that $\lim_{t\to\infty}\dot{\boldsymbol{q}}_{\boldsymbol{m}}=\lim_{t\to\infty}\dot{\boldsymbol{q}}_{\boldsymbol{s}}=\lim_{t\to\infty}\dot{\boldsymbol{e}}_{\boldsymbol{m}}=\lim_{t\to\infty}\dot{\boldsymbol{e}}_{\boldsymbol{s}}=0$ (See [16]).

It is easy to see that $\dddot{\boldsymbol{q}}_{\boldsymbol{m}},\dddot{\boldsymbol{q}}_{\boldsymbol{s}}\in\mathcal{L}_\infty$. therefore the signal $\ddot{\boldsymbol{q}}_{\boldsymbol{m}},\ddot{\boldsymbol{q}}_{\boldsymbol{s}}$ are uniformly continuous. Also as previously established, $\lim_{t\to\infty}\dot{\boldsymbol{q}}_{\boldsymbol{m}}=\lim_{t\to\infty}\dot{\boldsymbol{q}}_{\boldsymbol{s}}=0$, Invoking Barbalat's Lemma [17], $\lim_{t\to\infty}\ddot{\boldsymbol{q}}_{\boldsymbol{m}}=\lim_{t\to\infty}\ddot{\boldsymbol{q}}_{\boldsymbol{s}}=0$.

Consequently, the closed loop system dynamics (11) implies that

$$\begin{cases}\lim_{t\to\infty}|\tilde{\boldsymbol{q}}_{\boldsymbol{s}}(t-T_s(t))-\tilde{\boldsymbol{q}}_{\boldsymbol{m}}|=0\\ \lim_{t\to\infty}|\tilde{\boldsymbol{q}}_{\boldsymbol{m}}(t-T_m(t))-\tilde{\boldsymbol{q}}_{\boldsymbol{s}}|=\boldsymbol{K_p}^{-1}\boldsymbol{K_e}\lim_{t\to\infty}\tilde{\boldsymbol{q}}_{\boldsymbol{s}}.\end{cases} \quad (23)$$

Using the fact that $\tilde{\boldsymbol{q}}_{\boldsymbol{i}}(t-T_i(t))=\tilde{\boldsymbol{q}}_{\boldsymbol{i}}-\int_{t-T_i(t)}^t\dot{\boldsymbol{q}}_{\boldsymbol{i}}\ dt$ and $\lim_{t\to\infty}\dot{\boldsymbol{q}}_{\boldsymbol{m}}=\lim_{t\to\infty}\dot{\boldsymbol{q}}_{\boldsymbol{s}}=0$, we have that

$$\begin{cases}\lim_{t\to\infty}|\tilde{\boldsymbol{q}}_{\boldsymbol{s}}-\tilde{\boldsymbol{q}}_{\boldsymbol{m}}|=0\\ \lim_{t\to\infty}|\tilde{\boldsymbol{q}}_{\boldsymbol{m}}-\tilde{\boldsymbol{q}}_{\boldsymbol{s}}|=\boldsymbol{K_p}^{-1}\boldsymbol{K_e}\lim_{t\to\infty}\tilde{\boldsymbol{q}}_{\boldsymbol{s}}.\end{cases} \quad (24)$$

The above equations imply that $\lim_{t\to\infty}\tilde{\boldsymbol{q}}_m=0$, $\lim_{t\to\infty}\tilde{\boldsymbol{q}}_{\boldsymbol{s}}=0$. Then the origin of the system $\dot{\boldsymbol{q}}_{\boldsymbol{m}},\ \dot{\boldsymbol{q}}_{\boldsymbol{s}},\ \tilde{\boldsymbol{q}}_{\boldsymbol{m}},\ \tilde{\boldsymbol{q}}_{\boldsymbol{s}}$ are asymptotically stable and $\lim_{t\to\infty}\boldsymbol{q}_{\boldsymbol{m}}(t)=\bar{\boldsymbol{q}}_{\boldsymbol{m}}$, $\lim_{t\to\infty}\boldsymbol{q}_{\boldsymbol{s}}(t)=\bar{\boldsymbol{q}}_{\boldsymbol{s}}$. ■

The above result only guarantees stability of the teleoperation system. In the next result, we discuss the force reflection and position coordination abilities.

Corollary 1: Consider the nonlinear teleoperation described by (11) with *Assumptions 1-3*. Then for range of the control gain (12), we have following items,

1) The static force reflection is achieved as follows

$$\boldsymbol{F_{op}}=\boldsymbol{K_p}(\bar{\boldsymbol{q}}_{\boldsymbol{m}}-\bar{\boldsymbol{q}}_{\boldsymbol{s}})=\boldsymbol{K_e}\bar{\boldsymbol{q}}_{\boldsymbol{s}}=\boldsymbol{F_{env}}. \quad (25)$$

Therefore *Control Objective 2* is achieved.

2) If $\boldsymbol{F_{op}}=\boldsymbol{F_{env}}=0$, the position coordination error $\boldsymbol{q_E}$ in (6) goes to zero. Therefore *Control Objective 3* is achieved.

Proof:

1) From *Theorem 1*, $\lim_{t\to\infty}\dot{\boldsymbol{q}}_{\boldsymbol{m}}=\lim_{t\to\infty}\dot{\boldsymbol{q}}_{\boldsymbol{s}}=0$, $\lim_{t\to\infty}\boldsymbol{q}_{\boldsymbol{m}}=\bar{\boldsymbol{q}}_{\boldsymbol{m}}$, $\lim_{t\to\infty}\boldsymbol{q}_{\boldsymbol{s}}=\bar{\boldsymbol{q}}_{\boldsymbol{s}}$. Thus the operational force (2) and environmental force (3) can be rewritten as

$$\begin{cases}\boldsymbol{F_{op}}=\boldsymbol{K_p}(\bar{\boldsymbol{q}}_{\boldsymbol{m}}-\bar{\boldsymbol{q}}_{\boldsymbol{s}})=\boldsymbol{K_e}\bar{\boldsymbol{q}}_{\boldsymbol{s}}\\ \boldsymbol{F_{env}}=\boldsymbol{K_e}\bar{\boldsymbol{q}}_{\boldsymbol{s}}=\boldsymbol{K_p}(\bar{\boldsymbol{q}}_{\boldsymbol{m}}-\bar{\boldsymbol{q}}_{\boldsymbol{s}}).\end{cases} \quad (26)$$

The above equation can be written as (25).

2) If $\boldsymbol{F_{op}}=\boldsymbol{F_{env}}=0$, the equations (25) can be written as $\bar{\boldsymbol{q}}_{\boldsymbol{m}}-\bar{\boldsymbol{q}}_{\boldsymbol{s}}=0$. This implies that the equilibrium points of the master and slave are identical. Then position coordination error $\boldsymbol{q_E}$ go to zero as $\lim_{t\to\infty}\boldsymbol{q_E}(t)=\lim_{t\to\infty}(\boldsymbol{q}_{\boldsymbol{m}}(t)-\boldsymbol{q}_{\boldsymbol{s}}(t))=0$.

■

B. Control Law without Time Varying Gain

The proposed *Control Law 1* in previous section has the time varying gains depending on $\dot{T}_m$ and $\dot{T}_s$, and requires their measurement. Here we assume for the delay function $T_m(t)$ and $T_s(t)$ as follows

Assumption 4: $T_m(t)$ and $T_s(t)$ is continuously differentiable functions and satisfy as follows

$$0\leq T_i(t)\leq T_i^+<\infty, |\dot{T}_i(t)|<T^*<1,\quad i=m,s \quad (27)$$

where $T^*\in R$ are upper bounds of the rate of change of the communication delays. Moreover, the upper bound of the round-trip communication delay $T_{ms}^+=T_m^++T_s^+$ and the upper bound of the rate of the communication delay T^* are known preliminarily.

Under above assumption we propose new control law which not require the measurements of $\dot{T}_i$ as follows

Control Law 2

$$\begin{cases}\boldsymbol{\tau_m}(t)=\boldsymbol{K_d}\{\dot{\boldsymbol{q}}_{\boldsymbol{s}}(t-T_s(t))-\dot{\boldsymbol{q}}_{\boldsymbol{m}}(t)\}\\ \quad-\{\boldsymbol{D_d}+\boldsymbol{D_p}\}\dot{\boldsymbol{q}}_{\boldsymbol{m}}(t)+\boldsymbol{K_p}\{\boldsymbol{q_s}(t-T_s(t))-\boldsymbol{q_m}(t)\}\\ \boldsymbol{\tau_s}(t)=\boldsymbol{K_d}\{\dot{\boldsymbol{q}}_{\boldsymbol{m}}(t-T_s(t))-\dot{\boldsymbol{q}}_{\boldsymbol{s}}(t)\}\\ \quad-\{\boldsymbol{D_d}+\boldsymbol{D_p}\}\dot{\boldsymbol{q}}_{\boldsymbol{s}}(t)+\boldsymbol{K_p}\{\boldsymbol{q_m}(t-T_m(t))-\boldsymbol{q_s}(t)\}\end{cases} \quad (28)$$

where $\boldsymbol{K_d},\boldsymbol{D_d},\boldsymbol{D_p},\in R^{n\times n}$ are positive diagonal constant matrices. Then substituting (2), (3), (28) and (9) into (1) and assembling by (10), the closed loop systems can be described as

$$\begin{cases}\boldsymbol{M_m}\ddot{\boldsymbol{q}}_{\boldsymbol{m}}+\boldsymbol{C_m}\dot{\boldsymbol{q}}_{\boldsymbol{m}}=\boldsymbol{K_d}\{\dot{\boldsymbol{q}}_{\boldsymbol{s}}(t-T_s(t))-\dot{\boldsymbol{q}}_{\boldsymbol{m}}\}\\ \quad-\{\boldsymbol{D_d}+\boldsymbol{D_p}\}\dot{\boldsymbol{q}}_{\boldsymbol{m}}+\boldsymbol{K_p}\{\tilde{\boldsymbol{q}}_{\boldsymbol{s}}(t-T_s(t))-\tilde{\boldsymbol{q}}_{\boldsymbol{m}}\}\\ \boldsymbol{M_s}\ddot{\boldsymbol{q}}_{\boldsymbol{s}}+\boldsymbol{C_s}\dot{\boldsymbol{q}}_{\boldsymbol{s}}=\boldsymbol{K_d}\{\dot{\boldsymbol{q}}_{\boldsymbol{m}}(t-T_m(t))-\dot{\boldsymbol{q}}_{\boldsymbol{s}}\}\\ \quad-\{\boldsymbol{D_d}+\boldsymbol{D_p}\}\dot{\boldsymbol{q}}_{\boldsymbol{s}}+\boldsymbol{K_p}\{\tilde{\boldsymbol{q}}_{\boldsymbol{m}}(t-T_m(t))-\tilde{\boldsymbol{q}}_{\boldsymbol{s}}\}\\ \quad\quad-\boldsymbol{B_e}\dot{\boldsymbol{q}}_{\boldsymbol{s}}-\boldsymbol{K_e}\tilde{\boldsymbol{q}}_{\boldsymbol{s}}\end{cases} \quad (29)$$

The following theorem describe stability properties of the closed loop teleoperation with *Control Law 2* as (29).

Theorem 2: Consider the nonlinear teleoperation described by (11) with *Assumptions 1,3* and *4*. Then for range of the control gain $\boldsymbol{K_p}$ and $\boldsymbol{K_d}$ as follows

$$\boldsymbol{K_p}<\frac{2}{T_{ms}^+}\boldsymbol{D_p},\qquad \boldsymbol{K_d}\leq\frac{2(1-T^*)}{T^*}\boldsymbol{D_d} \quad (30)$$

the origin of the system $\dot{\boldsymbol{q}}_{\boldsymbol{m}}, \dot{\boldsymbol{q}}_{\boldsymbol{s}}, \tilde{\boldsymbol{q}}_{\boldsymbol{m}}, \tilde{\boldsymbol{q}}_{\boldsymbol{s}}$ are asymptotically stable and $\lim_{t\to\infty} \boldsymbol{q}_{\boldsymbol{m}}(t) = \bar{\boldsymbol{q}}_{\boldsymbol{m}}$, $\lim_{t\to\infty} \boldsymbol{q}_{\boldsymbol{s}}(t) = \bar{\boldsymbol{q}}_{\boldsymbol{s}}$. therefore the *Control Objective 1* is achieved.

Proof: This result is shown in the same arguments as in the proof of *Theorem 1* by using the following positive function.

$$
\begin{aligned}
V_2(x) &= \dot{\boldsymbol{q}}_{\boldsymbol{m}}^T(t)\boldsymbol{M}_{\boldsymbol{m}}(q_m)\dot{\boldsymbol{q}}_{\boldsymbol{m}}(t) + \dot{\boldsymbol{q}}_{\boldsymbol{s}}^T(t)\boldsymbol{M}_{\boldsymbol{s}}(q_s)\dot{\boldsymbol{q}}_{\boldsymbol{s}}(t) \\
&+ \{\tilde{\boldsymbol{q}}_{\boldsymbol{m}}(t) - \tilde{\boldsymbol{q}}_{\boldsymbol{s}}(t)\}^T \boldsymbol{K}_{\boldsymbol{p}} \{\tilde{\boldsymbol{q}}_{\boldsymbol{m}}(t) - \tilde{\boldsymbol{q}}_{\boldsymbol{s}}(t)\} + \tilde{\boldsymbol{q}}_{\boldsymbol{s}}^T(t)\boldsymbol{K}_{\boldsymbol{e}}\tilde{\boldsymbol{q}}_{\boldsymbol{s}}(t) \\
&+ \frac{1}{1-T^*}\int_{t-T_m(t)}^{t} \dot{\boldsymbol{q}}_{\boldsymbol{m}}^T(\xi)\boldsymbol{K}_{\boldsymbol{d}}\dot{\boldsymbol{q}}_{\boldsymbol{m}}(\xi)d\xi \\
&+ \frac{1}{1-T^*}\int_{t-T_s(t)}^{t} \dot{\boldsymbol{q}}_{\boldsymbol{s}}^T(\xi)\boldsymbol{K}_{\boldsymbol{d}}\dot{\boldsymbol{q}}_{\boldsymbol{s}}(\xi)d\xi \qquad (31)
\end{aligned}
$$

■

It is to be noted that the position coordination abilities in free space and static force reflection abilities are easy to show by following *Corollary 1*. Then *Control Objective 3* and *Control Objective 2* are also achieved by using Control Law 2.

Remark 1: From 1) of *Corollary 1*, the P-control gain $\boldsymbol{K}_{\boldsymbol{p}}$ in (7) and (28) determines the (static) force-reflection performance where it specifies how much force is generated for a given master-slave position errors. Furthermore (9) can be rewritten as

$$
\begin{cases} \bar{\boldsymbol{q}}_{\boldsymbol{m}} = (\boldsymbol{K}_{\boldsymbol{e}}^{-1} + \boldsymbol{K}_{\boldsymbol{p}}^{-1})\bar{\boldsymbol{F}}_{\boldsymbol{op}} \\ \bar{\boldsymbol{q}}_{\boldsymbol{s}} = \boldsymbol{K}_{\boldsymbol{e}}^{-1}\bar{\boldsymbol{F}}_{\boldsymbol{op}}. \end{cases} \qquad (32)
$$

This imply that the P-control gain $\boldsymbol{K}_{\boldsymbol{p}}$ also determines the (static) position error in contact with the environment. Note also that large dissipation gain $\boldsymbol{D}_{\boldsymbol{p}}$ in (7) and (28) would make the system response sluggish and deteriorates the operationability. Then the control gain should be designed with considering trade-off between "the operationability" and "the force reflection and the position error".

Remark 2: The *Control Law 1* and *2* in (7) and (28) are independent of the robot models (1), the operator (2) and the environment (3). Furthermore the stability of the system is also independent of the environmental parameter $\boldsymbol{B}_{\boldsymbol{e}}$ and $\boldsymbol{K}_{\boldsymbol{e}}$ and the operator input $\bar{\boldsymbol{F}}_{\boldsymbol{op}}$ from *Theorem 1* and *2*. Thus, our proposed control strategies guarantee robustness for the parameter uncertainties of the above mentioned parameter.

V. Evaluation by Control Experiments

In this section, we verify the efficacy of the proposed teleoperation methodology. The experiments were carried out on a pair of identical direct-drive planar 2 links revolute-joint robots as shown in Fig. 2. The inertia matrices and the Coriolis matrices are identified

$$
\boldsymbol{M}_{\boldsymbol{m}} = \boldsymbol{M}_{\boldsymbol{s}} = \begin{bmatrix} \theta_1 + 2\theta_3\cos(q_2) & \theta_2 + \theta_3\cos(q_2) \\ \theta_2 + \theta_3\cos(q_2) & \theta_2 \end{bmatrix},
$$

$$
\boldsymbol{C}_{\boldsymbol{m}} = \boldsymbol{C}_{\boldsymbol{s}} = \begin{bmatrix} -\theta_3\sin(q_2)\dot{q}_2 & -\theta_3\sin(q_2)(\dot{q}_1 + \dot{q}_2) \\ \theta_3\sin(q_2)\dot{q}_1 & 0 \end{bmatrix},
$$

where $\theta_1 = 0.3657[\mathtt{kgm}^2]$, $\theta_2 = 0.0291[\mathtt{kgm}^2]$ and $\theta_3 = 0.0227[\mathtt{kgm}]$. A remote environment is using a hard aluminum wall covered by a rubber on the slave side as shown in Fig. 2. We also measure the operational and the environmental torque (i.e. $\boldsymbol{F}_{\boldsymbol{env}}, \boldsymbol{F}_{\boldsymbol{op}}$ in (1)) using the force sensors. For implementation of the controllers and

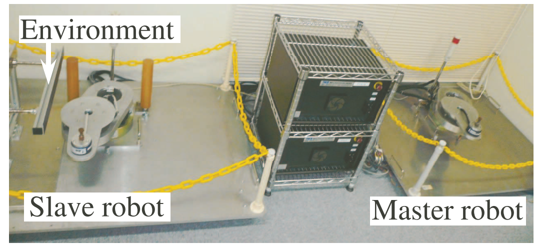

Fig. 2. Experimental setup

communication line, we use a dSPACE system (dSPACE Inc.) and 2.5 [ms] sampling rate is obtained. All experiments have been done with artificial time varying communication delays as

$$
T_m(t) = 0.1\sin 4t + 0.3, \quad \dot{T}_m(t) = 0.4\cos t
$$
$$
T_s(t) = 0.2\sin 4t + 0.3, \quad \dot{T}_s(t) = 0.8\cos t
$$

Hence, the upper bound of the round-trip delay in communication is $T_{ms}^+ = 0.9[s]$ and upper bound of the rate of change of delay is $T^* = 0.8$. In this paper, we verify only the *Control Law 1* due to space constraints. The controller parameters $\boldsymbol{K}_{\boldsymbol{d}}$, $\boldsymbol{D}_{\boldsymbol{p}}$ and $\boldsymbol{K}_{\boldsymbol{p}}$ are selected as

$$
\boldsymbol{K}_{\boldsymbol{d}} = \begin{bmatrix} 3 & 0 \\ 0 & 1.5 \end{bmatrix}, \quad \boldsymbol{K}_{\boldsymbol{p}} = \begin{bmatrix} 13.3 & 0 \\ 0 & 5.5 \end{bmatrix}, \quad \boldsymbol{D}_{\boldsymbol{p}} = \begin{bmatrix} 6 & 0 \\ 0 & 2.5 \end{bmatrix}.
$$

Two kind of experimental conditions are given as follows.

Case 1: The slave moves without any contact.

Case 2: The slave moves in contact with environment.

All experimental results show that the stability is guaranteed for time varying communication delays and any human inputs as Figs. 3-5. Fig. 3 shows the results of Case 1. The joint angles of the slave accurately track those of the master and the master-slave position coordination is achieved. Figs. 4-5 show the results of Case 2. When the slave robot is pushing the environment (5-28 [sec]), the contact torque is faithfully reflected to the operator. The operator can perceive the environment through the torque reflection. When the slave does not contact with environment and the operator force is negligible (30-40[sec]), the master-slave position coordination can be achieved. In Fig. 5, there are some small errors in the force responses, but it is seems to be due to the substantial device static friction of robots. These errors were not observed when a simulation without such a friction is performed.

VI. Conclusions

In this paper, we proposed the novel bilateral control strategies for nonlinear teleoperation with time varying delays. The proposed framework used the simple PD-type controllers with time varying gains and without time varying gains. Using Lyapunov-Krasovskii function, the delay-dependent stability of the origin was shown for the ranges of gains. Furthermore the proposed strategies achieved master-slave position coordination and bilateral static force reflection. Several experimental results showed the effectiveness of our proposed framework.

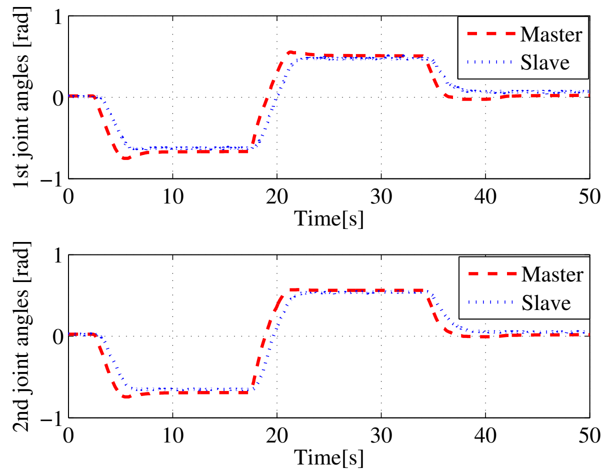

Fig. 3. Time responses in Case 1

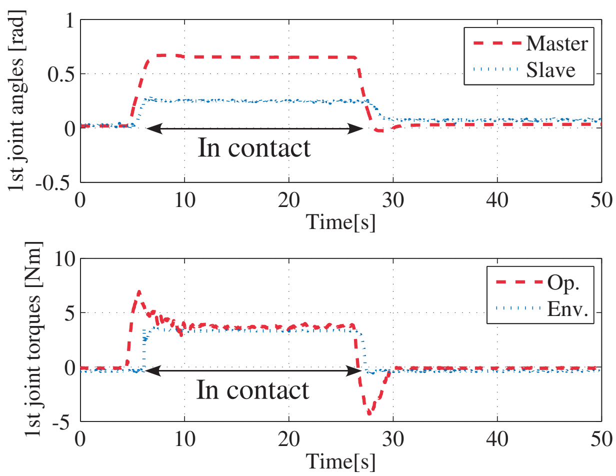

Fig. 4. Time responses at 1st joint in Case 2

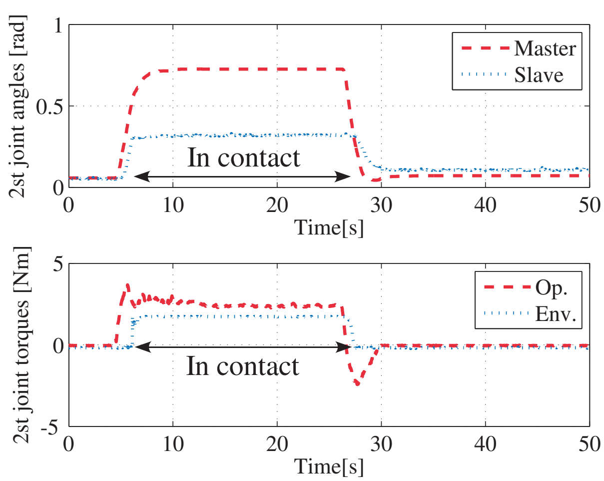

Fig. 5. Time responses at 2nd joint in Case 2

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APPENDIX

A. The detector of the rate of change of delay

The rate of change of delay can be detected as shown in Fig. 6. The ramp function as $r(t)(\ \dot{r}(t) = 1)$ is transmitted from sender to receiver through the communication line. On receiver side, delayed signal is differentiated. Thus it is easy to detect the rate of change of delay as follows,

$$\dot{T}(t) = -\left[\frac{d}{dt}\{r(t - T(t))\}\right] + 1. \tag{33}$$

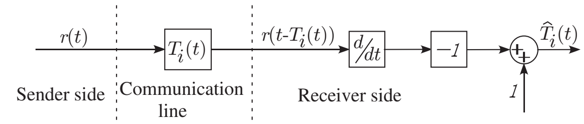

Fig. 6. The detector of the rate of change of delay