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Awareness Coverage Control Over Large Scale Domains with Intermittent Communications

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Abstract— **In this paper a novel dynamic awareness coverage model is proposed and applied to coverage control over a large-scale task domain for a decentralized multi-vehicle sensor network with intermittent communications and possibly faulty sensors. For each vehicle, an individual state of awareness is defined. The individual vehicle's state of awareness continuously evolves based on the vehicle's motion and is updated at discrete instants whenever the vehicle establishes a new range-based communication link with other vehicles. This information sharing update step aides in reducing the amount of redundant coverage. In this paper we first consider the simplifying assumption where no awareness loss occurs when no sensors are monitoring portions of the domain. This scenario is applicable to some search and rescue/retrieval problems (especially with static victims or objects of interests), domain monitoring, and low level surveillance. Under this assumption, a decentralized control strategy is proposed that guarantees that every point within the task domain will be covered with a satisfactory state of awareness even under intermittent communications and/or faulty sensors. We demonstrate the effectiveness of the novel awareness model and decentralized control law via numerical simulations.**

I. Introduction

This paper focuses on awareness coverage control for a mobile sensor network with intermittent communications and/or faulty sensors. Coverage control using multiple sensor-equipped vehicles has been of much interest recently due to its versatility in many aerial, terrestrial, and underwater applications such as surveillance, search and rescue/retrieval, and sampling. For a complete up-to-date overview of the literature on advances in the area of sensor networks, see [1], [2] and references therein.

Previous research on coverage control either focuses on optimizing the locations of immobile sensors, or redeployment of a fleet of mobile vehicles equipped with sensors to guarantee an improved coverage. The former class of problems is considered a problem in locational optimization [3], [4]. In such problems, the solution is a Voronoi partition [5]. For sensor redeployment problems, in [6], the authors apply a dynamic Voronoi partition, which is the dynamic version of the Lloyd algorithm [7]. In [8] and [9], the authors develop control strategies from the robotic and probabilistic network model, respectively.

An implicit assumption in the above-mentioned papers is that the task domain is small-scale. A small-scale task domain is one where the union of the sensory domains (assuming the best case scenario where sensor sensory ranges are disjoint) can cover the entire domain, otherwise the domain is said to be *large-scale* (i.e., a domain too large to be covered by a static limited-sensor-range sensor network). Large-scale domain problems arise when there are too few sensor vehicles, or when sensor ranges are too small relative to the domain size. For such problems, sensor mobility is necessary to be able to account for all locations contained in the domain of interest. Not being able to monitor parts of the domain continually for all time results in the requirement that the network be in a constant state of mobility with well-managed revisiting of locations in the domain to guarantee satisfactory awareness levels over all the domain.

Recent research on effective coverage control (see [10], [11], [12], [13]) has developed control strategies using vehicles with limited sensory capabilities over large-scale domains. The general assumption has been that the information of interest over task domain is stationary in nature during the entire process. This problem has been studied in [10] and [11] for motion planning for multiple spacecraft interferometric imaging systems. This problem is also closely related to the coverage path planning problem, see, for example [14], [15] and references therein. In [12], [13] and [16], a deterministic approach is pursued and a feedback control law is proposed to guarantee coverage of an entire task domain $\mathcal{D}$ using a multi-agent sensor network with flocking and guaranteed collision avoidance. In the stochastic setting, the author in [17] uses the Kalman filter for estimating a spatially-decoupled, time-independent field and using the prediction step of the filter for optimally guiding the vehicles to move in directions that improve the field estimate, and guarantees avoidance of local minima. A similar filtering-based problem is treated in [18] from an information theoretic perspective. Other dynamic coverage methods such as the literature on simultaneous localization and mapping (SLAM) are Kalman-filter based methods for dynamic surveillance of a domain $\mathcal{D}$. See [18], [19] and references therein.

Moreover, in most cases it is not economic or even possible for the vehicle fleet to maintain open communication channels during the entire mission. This is especially true for large-scale task domains, where vehicles may need to disperse (and, hence, loose connectivity with other vehicles) in order to cover the domain ([16], [20], [21], [22]). In [16], a flocking control strategy is developed for improved communication channels among sensors when the vehicles are carrying a coverage mission. Other flocking problems are studied in [20], [21], [22] aiming at maintaining communications under decentralized networks while guaranteeing

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system stability. In [23], the authors investigate distributed mobile robots in a wireless network under nearest neighbor communications. In [24], local undirected communication is used in fully distributed multi-agent systems. Both [23] and [24] demonstrate improvements in global behavior made by exchanging local sensing information.

In this paper a novel model for "dynamic awareness", which describes how "aware" the system vehicles are of events occurring over the task domain, is developed for a limited-sensory sensor network. Decentralized control laws developed in this paper will guarantee a satisfactory state of awareness over large-scale domains for the case where awareness loss is not possible (to be made precise in the sequel), and under an arbitrary dynamic communication structure and/or faulty sensors.

The organization of this paper is as follows. In Section II, we present a systems theoretic formulation for the awareness coverage control problem. A sensor model with a limited sensory range is introduced and a precise definition of the state of awareness is developed. In Section III, we extend the awareness model to a formulation applicable to decentralized control formulations. Assuming no awareness loss, in Section IV, we derive the awareness coverage control strategy, which guarantees that the entire fleet has a satisfactory state of awareness over the entire mission domain. Detailed numerical simulations are presented in Section V to illustrate the problem. This paper is concluded with a summary, and current and future research in Section VI.

II. Problem Formulation

A. Sensor Model

Before considering the sensor model used in this paper, we first introduce some notation. Let N be the number of vehicles in a mobile sensor network. Each vehicle is denoted by $\mathcal{V}_i, i \in \mathcal{I} = \{1, 2, 3, \cdots, N\}$. Define the configuration space of each vehicle as Q, and the task domain as $\mathcal{D}$ which the mobile sensor network is required to cover. Let the position of vehicle $\mathcal{V}_i$ be $\mathbf{q}_i \in Q$ and each point within the task domain be $\tilde{\mathbf{q}} \in \mathcal{D}$. We assume the following simple first-order kinematic equation of motion for each vehicle

$$\dot{\mathbf{q}}_i \quad = \quad \mathbf{u}_i, \ i \in \mathcal{I}, \tag{1}$$

where $\mathbf{u}_i$ is the control velocity of vehicle $\mathcal{V}_i$. In this paper, we employ the sensor model proposed in [12], [13] and [16], which is a fourth-order polynomial function of $s = \|\tilde{\mathbf{q}} - \mathbf{q}_i\|$ within the sensory domain and zero otherwise. Mathematically, the sensor model is given by

$$A_i(s) \quad = \quad \begin{cases} \frac{M_i}{r_i^4}\left(s^2 - r_i^2\right)^2 & \text{if } s \le r_i, \\ 0 & \text{if } s > r_i. \end{cases}$$

Note that each sensor has a peak sensory capability M_i exactly at the vehicle's position $\mathbf{q}_i$, a limited sensory domain $\mathcal{W}_i(t)$ with sensory range r_i, and decreasing sensory capability along with the sensory range. An example of the instantaneous coverage function $A_i(\|\tilde{\mathbf{q}} - \mathbf{q}_i\|)$ is illustrated by Figure 1. The most important property of the sensor that can be handled by the methods developed by the authors in this and former papers [12], [13], [16], [25] is their

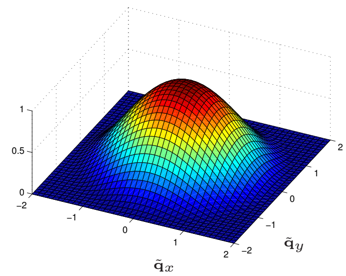


Fig. 1. Instantaneous coverage function A_i with $\mathbf{q}_i = 0$, $M_i = 1$ and $r_i = 2$.

limited sensory range. This models the practical difficulty in real implementation, especially for missions over *large-scale* domains. "Large-scale" means that for every $t > t_0 := 0$ there exists at least one point $\tilde{\mathbf{q}} \in \mathcal{D}$ such that $\tilde{\mathbf{q}} \notin \mathcal{W}_i$ for all $i \in \mathcal{I}$ under the best case scenario when all the sensory domains $\mathcal{W}_i$ are disjoint.

B. State of Awareness

We now introduce the notion of the state of awareness. An individual vehicle's state of awareness is a distribution $\mathbf{x}_i(\tilde{\mathbf{q}}, t)$. In practice, the domain is discretized into n cells, where $\tilde{\mathbf{q}}$ may represent, for example, the centroid of each cell. Hence, $\mathbf{x}_i(\tilde{\mathbf{q}}, t)$ can be written as a vector of dimension $2n$. The state of awareness $\mathbf{x}_i(\tilde{\mathbf{q}}, t)$ is defined to be a measure of how "aware" the vehicle is of events occurring at a specific location $\tilde{\mathbf{q}}$ at time t. Fixing a point $\tilde{\mathbf{q}} \in \mathcal{D}$, the state of awareness of a particular vehicle $\mathcal{V}_i$ at time t is assumed to satisfy the following differential equation

$$\begin{aligned} &\dot{\mathbf{x}}_i(\tilde{\mathbf{q}}, t) = -\left(A_i(\|\tilde{\mathbf{q}} - \mathbf{q}_i\|) - \alpha\right)\mathbf{x}_i(\tilde{\mathbf{q}}, t), \\ &\mathbf{x}_i(\tilde{\mathbf{q}}, 0) = \mathbf{x}_{i0} < 0, i \in \mathcal{I}, \end{aligned} \tag{2}$$

where the constant parameter α represents the *awareness loss bath*. If $\mathbf{x}_i(\tilde{\mathbf{q}}, t) < 0$, then we have insufficient awareness; If $\mathbf{x}_i(\tilde{\mathbf{q}}, t) > 0$, then we have excessive awareness. The initial state of awareness is negative, which reflects the fact that at the outset of the surveillance mission the fleet has poor awareness levels of events occurring over the domain $\mathcal{D}$. To reflect the difficulty of object detection one simply sets a more negative value for $\mathbf{x}_i(\tilde{\mathbf{q}}, 0)$. The easier the detection process is, the larger the negative value of $\mathbf{x}_i(\tilde{\mathbf{q}}, 0)$ can be set. The distribution $\mathbf{x}_i(\tilde{\mathbf{q}}, 0)$ may also be nonuniform to reflect regions where objects may be able to camouflage themselves better than in other regions of $\mathcal{D}$ (e.g., dense forests versus open fields). In this paper we will assume, without loss of generality, that $\mathbf{x}_{i0} = -1$.

The desired state of awareness is given by:

$$\mathbf{x}_i(\tilde{\mathbf{q}}, t) = 0, t > 0, i \in \mathcal{I}, \forall \tilde{\mathbf{q}} \in \mathcal{D}.$$

In this paper we develop control laws that will guarantee convergence of $\mathbf{x}_i(\tilde{\mathbf{q}}, t)$ to a neighborhood of 0: $\|\mathbf{x}_i(\tilde{\mathbf{q}}, t)\| < \epsilon$ for some $\epsilon > 0$. Note that under the awareness dynamics (2), the maximum value attainable by $\mathbf{x}_i$ is zero if the initial awareness level is negative. Inspecting equation (2), the system state of awareness is degrading except over regions where $A_i - \alpha$ has a positive value (i.e., $0 \le \alpha \le A_i$).

One can also define the overall state of awareness that satisfies the dynamics:

$$\dot{\mathbf{x}}(\tilde{\mathbf{q}},t) = -\sum_{i=1}^{N}\left(A_i(\|\tilde{\mathbf{q}}-\mathbf{q}_i\|)-\alpha\right)\mathbf{x}(\tilde{\mathbf{q}},t) \tag{3}$$

with negative initial conditions as discussed above for $\mathbf{x}_i(\tilde{\mathbf{q}},t)$. Note that for the case where all the vehicles are set to be fixed, if enough resources are available (i.e., enough vehicles and/or long enough sensor ranges), the awareness levels are everywhere increasing and converging to the desired value. To see why this is true, let us make the following simple analysis. For each given point $\tilde{\mathbf{q}}$, the term $\sum_{i=1}^{N}(A_i-\alpha)$ in equation (3) is a positive constant since each vehicle is assumed to be fixed. This means that, for each $\tilde{\mathbf{q}}$, the dynamics (3) is a linear differential equation in $\mathbf{x}(\tilde{\mathbf{q}},t)$, which leads to asymptotic convergence of $\mathbf{x}(\tilde{\mathbf{q}},t)$ to zero. For *large-scale* domains, a static sensor is guaranteed not meet the desired zero state of awareness because, by definition, there exists at least one $\tilde{\mathbf{q}}$ which is not covered by some sensor. The goal of this paper is to develop a decentralized control strategy that stabilize the zero state of awareness under intermittent communications and/or faulty sensors over a large-scale domain.

III. State of Awareness Dynamic Model

A. State of Awareness Updates

Now we assume that the vehicles are in a decentralized network and have intermittent communications when they are within a range of $\rho > 0$ of each other. Vehicles exchange their awareness coverage information during communications. Let $\mathcal{G}_i(t) = \{j \in \mathcal{I} : \|\mathbf{q}_j - \mathbf{q}_i\| < \rho\}, i \in \mathcal{I}$, be the set of vehicles that neighbor vehicle $\mathcal{V}_i$ (including vehicle $\mathcal{V}_i$ itself) at time t. Whenever new vehicles $\mathcal{V}_j$ are added to the set $\mathcal{G}_i$, vehicle $\mathcal{V}_i$ will instantaneously exchange all the available awareness information with *new* neighbors in a discrete awareness update step, where awareness information is updated based on the awareness achieved by other neighboring vehicles along with the vehicle's own awareness level. If no or more than one vehicle drop from $\mathcal{G}_i(t)$ (possibly faulty sensors), the individual state of awareness of vehicle $\mathcal{V}_i$ does not change. Let t_c be the time instant at which vehicles $\mathcal{V}_j,\ \mathcal{V}_k,\ \ldots$ become members of $\mathcal{G}_i$. That is $\mathcal{V}_j,\ \mathcal{V}_k,\ \ldots \notin \mathcal{G}_i(t_c^-)$ but $\mathcal{V}_j,\ \mathcal{V}_k,\ \ldots \in \mathcal{G}_i(t_c^+)$. Hence, we have the following update equation that takes place whenever a set of vehicles $\bar{\mathcal{G}}_i(t) \subset \mathcal{I}\backslash(\{i\}\cup\mathcal{G}_i(t))$ gets added to $\mathcal{G}_i(t)$ at time t:

$$\mathbf{x}_i(\tilde{\mathbf{q}},t^+) = (-1)^{\bar{n}_i(t)}\mathbf{x}_i(\tilde{\mathbf{q}},t)\cdot\prod_{j\in\bar{\mathcal{G}}_i(t)}\mathbf{x}_j(\tilde{\mathbf{q}},t), \tag{4}$$

where $\bar{n}_i(t)$ is the number of vehicles in $\bar{\mathcal{G}}_i(t)$. Hence, the state of awareness evolves according to the continuous dynamics given by equation (2) and undergoes a discrete update step given by equation (4) whenever new vehicle becomes $\mathcal{V}_i$'s neighbor. Figure 2 illustrates the awareness model for the continuous dynamics (2) and discrete awareness state update (4). Note that when the switching condition $\mathcal{G}_i(t) \neq \varnothing$ is satisfied, the initial condition of the system is reset according to the reset map (4).

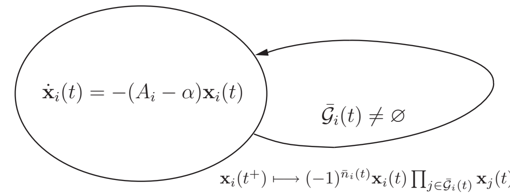


Fig. 2. Continuous and discrete awareness state update model.

If $\bar{\mathcal{G}}_i(t) = \varnothing$ (i.e., no new vehicles become neighbors of $\mathcal{V}_i$), then the awareness state of vehicle $\mathcal{V}_i$ obeys the continuous differential equation (2). This includes the case when vehicles drop from $\mathcal{G}_i(t)$ (e.g., faulty sensors) or when existing neighbors retain the neighbor status. If the number of new vehicles $\bar{n}_i(t)$ in $\bar{\mathcal{G}}_i(t)$ is nonzero at time t, then the value of the state of awareness of vehicle $\mathcal{V}_i$ will be discretely substituted with the product of the states of awareness of all the vehicles in $\bar{\mathcal{G}}_i(t)$. According to equation (4), the product reflects the improvement in the state of awareness of vehicle $\mathcal{V}_i$. For example, let us assume that all the vehicles in the mission fleet have an initial state of awareness of -1 and their coverage goal is to achieve an awareness value close to zero everywhere within the domain. If $\mathcal{V}_i$ has an awareness of -0.5 at some $\tilde{\mathbf{q}}$ at time t, and it updates its state of awareness based on the state of awareness of another neighbor vehicle of -0.5, then the new awareness at $\tilde{\mathbf{q}}$ is now -0.25 according to the update equation (4).

B. Awareness Metric

Let the awareness metric be given by

$$e_{gi}(t) = \int_{\mathcal{D}}\frac{1}{2}\mathbf{x}_i^2(\tilde{\mathbf{q}},t)\mathrm{d}\tilde{\mathbf{q}}, i \in \mathcal{I}, \tag{5}$$

which is the global error over the entire mission domain achieved by vehicle $\mathcal{V}_i$. It is said to be *global* since the integration is performed over the entire domain $\mathcal{D}$. The coverage goal of each vehicle is to guarantee that the above metric (5) decreases with time and ultimately converge to a small neighborhood of zero.

Consider the following *local awareness error function* associated with vehicle $\mathcal{V}_i$ to be

$$e_i(t) \quad = \quad \int_{\mathcal{W}_i(t)}\frac{1}{2}\mathbf{x}_i^2(\tilde{\mathbf{q}},t)\mathrm{d}\tilde{\mathbf{q}} \geq 0, i \in \mathcal{I}. \tag{6}$$

with $e_i(t) = 0$ if and only if $\mathbf{x}_i(\tilde{\mathbf{q}},t) = 0$ for every point $\tilde{\mathbf{q}}$ inside the sensory domain $\mathcal{W}_i(t)$. This metric will be used for the development of the control law. Note that the metric is a function of the position of the vehicle $\mathcal{V}_i$ because of the integral domain $\mathcal{W}_i(t)$.

IV. Decentralized Control with Intermittent Communications and Faulty Sensors

1) Nominal Control Law: Between switching instances, the vehicle kinematics equation (1) and state awareness equation (2) constitute two first order differential equations. In this section, these two equations together with the individual vehicle error function (6) are used to derive a control law that seeks to reduce the value of e_i for each vehicle. This control law will be called the *nominal control law*. The nominal control law itself does not guarantees convergence of $\mathbf{x}_i$ to a neighborhood of zero over the entire domain $\mathcal{D}$. Instead, it only guarantees that $\mathbf{x}_i \to 0$ within the sensory domain $\mathcal{W}_i$ for each vehicle. A *perturbation control law* (to be discussed in the next subsection) is need along with the nominal control to guarantee that $\|\mathbf{x}_i(\tilde{\mathbf{q}}, t)\| < \epsilon$ over the entire domain $\mathcal{D}$.

We will make, without any loss of generality, the following assumption for the initial state of awareness.

IC The initial state of awareness is given by:

$$\mathbf{x}_i(\tilde{\mathbf{q}}, 0) = \mathbf{x}_{i0} = -1, i \in \mathcal{I}.$$

Assumption IV.1. *Assume that there is no awareness loss. That is we have $\alpha = 0$.*

With Assumption IV.1, the ensuing results are applicable to problems in search and rescue/retrieval problems (especially with static victims or objects of interests), domain monitoring, and "low level" surveillance. Future research will focus on relaxing assumption IV.1.

Consider the following control law:

$$\bar{\mathbf{u}}_i(t) = \bar{k}_i \int_{\mathcal{W}_i(t)} \mathbf{x}_i^2(\tilde{\mathbf{q}}, t) \underbrace{\left(\int_0^t \frac{\partial(A_i(\tilde{\mathbf{q}}, \mathbf{q}_i(\sigma)))}{\partial \tilde{\mathbf{q}}} \mathrm{d}\sigma\right)}_{\text{memory term}} \mathrm{d}\tilde{\mathbf{q}}, \quad (7)$$

where $\bar{k}_i > 0$ is a feedback gain. We will prove that this control law (7) guarantees the convergence of $\mathbf{x}_i(\tilde{\mathbf{q}}, t)$ to zero at every point $\tilde{\mathbf{q}}$ in the sensory domain $\mathcal{W}_i(t)$. But first, we make the following remarks.

Remark. In the expression for $\bar{\mathbf{u}}_i(t)$, the time integral term under the spatial integration is an integration of historical data that translates into the reliance on past search history for decision making. Note that the memory term is multiplied by $\mathbf{x}_i^2(\tilde{\mathbf{q}}, t)$ before being integrated over the sensory domain at the current time t. This indicates that historical data as well as up-to-date awareness levels within the vehicle's sensor domain are compounded to decide on the motion. •

Remark on the computational efficiency of proposed control law. The search approach proposed herein requires computations at the order of $\mathcal{O}(\bar{n}^2 + 2)$ at each time step, where $\bar{n}$ is the number of cells in the discretized sensory domain $\mathcal{W}_i$. While alternative approaches, such as Voronoi-partitioning and stochastic-based SLAM methods, are computationally more burdensome. See [13] for more details.

We first consider the following Lemma (see [26] for a detailed exposition), which will be used shortly.

Lemma IV.1. For any function $F : \mathbb{R}^2 \times \mathbb{R} \to \mathbb{R}$ we have

$$\frac{d}{dt} \int_{\mathcal{W}_i(t)} F(\tilde{\mathbf{q}}, t) \mathrm{d}\tilde{\mathbf{q}} = \int_{\mathcal{W}_i(t)} \left[(\mathrm{grad}_{\tilde{\mathbf{q}}} F(\tilde{\mathbf{q}}, t)) \cdot \mathbf{u}_i + \frac{\partial F(\tilde{\mathbf{q}}, t)}{\partial t} \right] \mathrm{d}\tilde{\mathbf{q}},$$

where $\mathbf{u}_i$ is the velocity of vehicle $\mathcal{V}_i$ and $\mathrm{grad}_{\tilde{\mathbf{q}}}$ is the gradient operator with respect to $\tilde{\mathbf{q}}$.

Proof. This is a direct consequence of equation (3.3) in [26], where we note that $\mathbf{u}_i$ is the velocity of any point within the (rigid) domain $\mathcal{W}_i$ (including the boundary). ■

Next, consider the following condition, whose utility will also become obvious shortly.

Condition C. $\mathbf{x}_i(\tilde{\mathbf{q}}, t) = 0, \forall \tilde{\mathbf{q}} \in \mathcal{W}_i(t)$.

Lemma IV.2. For any $t \geq 0$, if Condition **C** holds for vehicle $\mathcal{V}_i$, then $e_i(t) = 0, i \in \mathcal{I}$. Conversely, if $e_i(t) = 0$ for some time $t \geq 0$, then Condition **C** holds for vehicle $\mathcal{V}_i$.

Proof. The proof follows directly from the definitions of $e_i(t)$ and $\mathbf{x}_i(\tilde{\mathbf{q}}, t)$. ■

Theorem IV.1. Under Assumption IV.1, the control law $\bar{\mathbf{u}}_i(t)$ given by equation (7) drives $e_i(t)$ asymptotically towards zero between state switches.

Proof. Consider the function $\bar{V}_i = e_i(t)$. From Lemma IV.2, $\bar{V}_i = 0$ if and only if Condition **C** holds for vehicle $\mathcal{V}_i$. According to Lemma IV.1,

$$\dot{\bar{V}}_i = \dot{e}_i(t) = \int_{\mathcal{W}_i(t)} \mathrm{grad}(\frac{1}{2}\mathbf{x}_i^2(\tilde{\mathbf{q}}, t)) \cdot \bar{\mathbf{u}}_i \mathrm{d}\tilde{\mathbf{q}} + \int_{\mathcal{W}_i(t)} \frac{\partial(\frac{1}{2}\mathbf{x}_i^2(\tilde{\mathbf{q}}, t))}{\partial t} \mathrm{d}\tilde{\mathbf{q}}. \quad (8)$$

First consider the spatial gradient term in (8):

$$\int_{\mathcal{W}_i(t)} \mathrm{grad}(\frac{1}{2}\mathbf{x}_i^2(\tilde{\mathbf{q}}, t)) \cdot \bar{\mathbf{u}}_i \mathrm{d}\tilde{\mathbf{q}} = \int_{\mathcal{W}_i(t)} \mathbf{x}_i(\tilde{\mathbf{q}}, t) \frac{\partial(\mathbf{x}_i(\tilde{\mathbf{q}}, t))}{\partial \tilde{\mathbf{q}}} \cdot \bar{\mathbf{u}}_i \mathrm{d}\tilde{\mathbf{q}}.$$

Next, we need to derive an expression for $\frac{\partial(\mathbf{x}_i(\tilde{\mathbf{q}}, t))}{\partial \tilde{\mathbf{q}}}$. From equation (2) and assuming $\alpha = 0$, we have

$$\mathbf{x}_i(\tilde{\mathbf{q}}, t) = e^{-\int_0^t A_i(\tilde{\mathbf{q}}, \mathbf{q}_i(\sigma)) \mathrm{d}\sigma} \mathbf{x}_{i0}.$$

Hence,

$$\frac{\partial \mathbf{x}_i}{\partial \tilde{\mathbf{q}}} = -\mathbf{x}_i(\tilde{\mathbf{q}}, t) \int_0^t \frac{\partial(A_i(\tilde{\mathbf{q}}, \mathbf{q}_i(\sigma)))}{\partial \tilde{\mathbf{q}}} \mathrm{d}\sigma.$$

Therefore,

$$\int_{\mathcal{W}_i(t)} \mathrm{grad}(\frac{1}{2}\mathbf{x}_i^2(\tilde{\mathbf{q}}, t)) \cdot \bar{\mathbf{u}}_i \mathrm{d}\tilde{\mathbf{q}} = -\int_{\mathcal{W}_i(t)} \mathbf{x}_i^2(\tilde{\mathbf{q}}, t) \left(\int_0^t \frac{\partial(A_i(\tilde{\mathbf{q}}, \mathbf{q}_i(\sigma)))}{\partial \tilde{\mathbf{q}}} \mathrm{d}\sigma\right) \cdot \bar{\mathbf{u}}_i \mathrm{d}\tilde{\mathbf{q}}.$$

Substitute $\bar{\mathbf{u}}_i(t)$ in equation (7) into the above equation, we obtain

$$\int_{\mathcal{W}_i(t)} \mathrm{grad}(\frac{1}{2}\mathbf{x}_i^2(\tilde{\mathbf{q}}, t)) \cdot \bar{\mathbf{u}}_i \mathrm{d}\tilde{\mathbf{q}} = -\bar{k}_i \left[\int_{\mathcal{W}_i(t)} \mathbf{x}_i^2(\tilde{\mathbf{q}}, t) \left(\int_0^t \frac{\partial(A_i(\tilde{\mathbf{q}}, \mathbf{q}_i(\sigma)))}{\partial \tilde{\mathbf{q}}} \mathrm{d}\sigma\right) \mathrm{d}\tilde{\mathbf{q}}\right]^2 \leq 0.$$

Next, let us consider the integral of the time derivation term in equation (8). According to equation (2) and assuming no information loss, that is, $\alpha = 0$,

$$\int_{\mathcal{W}_i(t)} \frac{\partial \frac{1}{2}(\mathbf{x}_i^2(\tilde{\mathbf{q}}, t))}{\partial t} \mathrm{d}\tilde{\mathbf{q}} = -\int_{\mathcal{W}_i(t)} \mathbf{x}_i^2(\tilde{\mathbf{q}}, t) A_i(\|\tilde{\mathbf{q}} - \mathbf{q}_i\|^2) \mathrm{d}\tilde{\mathbf{q}} \leq 0.$$

Therefore, $\dot{\bar{V}}_i \leq 0$. One can check that equality holds if and only if Condition **C** holds. ■

2) Perturbation Control Law: Using the nominal control law in equation (7) does not necessarily mean that the error $e_{gi}(t)$ of each vehicle over the entire domain given by equation (5) will converge to a neighborhood of zero. If Condition **C** holds but with $e_{gi}(t) \neq 0$, we consider a perturbation control law that perturbs the system away from the Condition **C**:

$$\bar{\bar{\mathbf{u}}}_i(t) = -\bar{\bar{k}}_i(\mathbf{q}_i(t) - \tilde{\mathbf{q}}_i^*(t_s)), \tag{9}$$

where t_s is the switching time, $\bar{\bar{k}}_i > 0$ is feedback controller gain, and $\tilde{\mathbf{q}}_i^* \in \mathcal{D}$ is chosen such that $\|\mathbf{x}_i(\tilde{\mathbf{q}}, t)\| > \epsilon$.

The choice of $\tilde{\mathbf{q}}_i^*$ by a vehicle can be made several ways. We provide one such choice that is more efficient than other possibilities. Let

$$\mathcal{D}_\epsilon^i(t) := \{\tilde{\mathbf{q}} \in \mathcal{D} : \|\mathbf{x}_i(\tilde{\mathbf{q}}, t)\| > \epsilon\},$$

which is an open set of all points $\tilde{\mathbf{q}}$ for which $\|\mathbf{x}_i(\tilde{\mathbf{q}}, t)\|$ is larger than ϵ. Let $\overline{\mathcal{D}}_\epsilon^i(t)$ be the closure of $\mathcal{D}_\epsilon^i(t)$, $\overline{\mathcal{D}}_{\epsilon,i}^i(t)$ be the set of points in $\overline{\mathcal{D}}_\epsilon^i(t)$ that minimize the distance between the position vector of vehicle $\mathcal{V}_i$ and the set $\overline{\mathcal{D}}_\epsilon^i(t)$:

$$\overline{\mathcal{D}}_{\epsilon,i}^i(t) = \left\{\tilde{\mathbf{q}}^* \in \overline{\mathcal{D}}_\epsilon^i(t) : \tilde{\mathbf{q}}^* = \operatorname{argmin}_{\tilde{\mathbf{q}} \in \overline{\mathcal{D}}_\epsilon^i(t)} \|\tilde{\mathbf{q}} - \mathbf{q}_i(t)\|\right\}.$$

Note that $\tilde{\mathbf{q}}_i^*$ is chosen based on coverage information available to vehicle $\mathcal{V}_i$ only, which is appropriate in the setting of this paper since the control law is decentralized.

Note that this perturbation control law was used in [12], [13] and [16], but for a different coverage control formulation.

3) Control Strategy: Under the control law (7), all vehicles move in the direction that improves the local (since integration is performed over the sensor domain $\mathcal{W}_i(t)$) awareness level and are in continuous motion as long as the state described in Condition **C** is avoided. Whenever the Condition **C** holds with nonzero error $e_{gi}(t), i \in \mathcal{I}$, the system has to be perturbed by switching to the perturbation control law (9). Once away from the Condition **C**, the controller is switched back to the nominal controller. Only when both Condition **C** and $\|\mathbf{x}(\tilde{\mathbf{q}}, t)\| < \epsilon$, for all $\tilde{\mathbf{q}} \in \mathcal{D}$, that the mission is said to be accomplished and no further switching is performed. The above discussions give us the following result.

Theorem IV.2. Under limited sensory range model and initial condition **IC**, the control law

$$\mathbf{u}_i^*(t) = \begin{cases} \bar{\mathbf{u}}_i & \text{if Condition } \mathbf{C} \text{ does not hold} \\ \bar{\bar{\mathbf{u}}}_i & \text{if Condition } \mathbf{C} \text{ holds} \end{cases}, \tag{10}$$

drives the error $e_{gi}(t), i \in \mathcal{I}$, to a neighborhood of the zero value.

Remarks.

1) Note that the neighborhood to which e_{gi} converges is given by the upper bound
$$\bar{\epsilon} = \left\|\int_{\mathcal{D}} \frac{1}{2}\mathbf{x}_i^2(\tilde{\mathbf{q}}, t)\mathrm{d}t\right\| \leq \frac{\epsilon^2 A_{\mathcal{D}}}{2},$$
where $A_{\mathcal{D}}$ is the area of $\mathcal{D}$.
2) Note that the fact that the state of awareness $\mathbf{x}_i$ undergoes a discrete update step whenever any new vehicle is added to the set of vehicles neighboring vehicle $\mathcal{V}_i$ does not introduce any instabilities. This is especially true since the update equation results in a discrete change from a continuous distribution $\mathbf{x}_i$ over $\mathcal{D}$ to another continuous distribution. Moreover, we have $\|\mathbf{x}_i(\tilde{\mathbf{q}}, t^+)\| \leq \|\mathbf{x}_i(\tilde{\mathbf{q}}, t)\|$ for each $\tilde{\mathbf{q}}$ at each switching instant. Hence, the resetting of $\mathbf{x}_i$ can not introduce instabilities that causes unbounded divergence.
3) Since the memory term in equation (7) is finite (since the coverage function A_i is C^∞), then when $\mathbf{x}_i$ undergoes a reset map, the control law $\bar{\mathbf{u}}_i$ undergoes a finite *drop* in magnitude (since $\mathbf{x}_i^2$ experiences a finite drop in magnitude, see equation (7), and since the memory term does not change across switches) and, hence, no infinite control inputs are encountered across awareness state switches. In between switches, the control $\bar{\mathbf{u}}_i$ is also finite (but, in this phase, continuous in time) because the memory term is finite and since the awareness state satisfies $\|\mathbf{x}_i\| < 1$ (assuming an awareness initial condition of -1).
4) Note that infinite switching between $\bar{\mathbf{u}}_i$ and $\bar{\bar{\mathbf{u}}}_i$ is impossible because (1) during the application of $\bar{\mathbf{u}}_i$ the value of e_{gi} decreases by an amount of non-zero measure, and (2) if Condition **C** occurs and the control law $\bar{\bar{\mathbf{u}}}_i$ is applied, once the vehicle is within a range less than r from $\tilde{\mathbf{q}}_i^*$, e_{gi} decreases by an amount of non-zero measure. This guarantees that a finite number of switches will be applied to guarantee that $e_{gi} < \bar{\epsilon}$ (which corresponds to $\|\mathbf{x}_i\| < \epsilon$).
5) If the condition for the reset map and the Condition **C** occur at the same instant, checking of the Condition **C** is performed *after* the reset map is performed. •

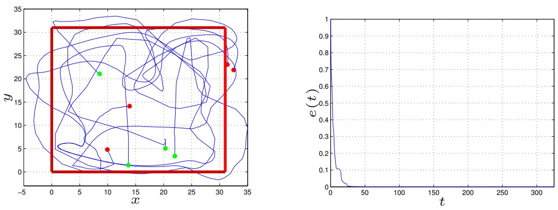
V. SIMULATIONS

In this section we provide a numerical simulation that illustrates the performance of the control strategy (10). We define the domain $\mathcal{D}$ as a square region whose size is 32×32 units length. Assume there are 4 vehicles ($N = 4$) with a randomly selected initial deployment as shown by the green dots in Figure 3(a). Let the initial state of awareness $\mathbf{x}_{i0}, i = 1, 2, 3, 4$, be -1 and the desired state of awareness $\mathbf{x}_i$ be 0. Here we use the nominal control law in equation (7) with control gain $\bar{k}_i = 3$ and the perturbation control law in equation (9) with control gain $\bar{\bar{k}}_i = 0.05$, $i = 1, \ldots, 4$. A vehicle is set to switch to the linear feedback control law whenever Condition **C** applies to it. For the sensor model, we have set $M_i = 1, r_i = 8$ for all $i = 1, \ldots, 4$. For the intermittent communication range, we set it the same as the sensory range $\rho = r_i = 8$. We used a simple trapezoidal method to compute integration over $\mathcal{W}_i(t)$ and a simple first order Euler scheme to integrate with respect to time. The global error

$$e(t) = \int_{\mathcal{D}} \frac{1}{2}\mathbf{x}^2(\tilde{\mathbf{q}}, t)\mathrm{d}\tilde{\mathbf{q}}, \tag{11}$$

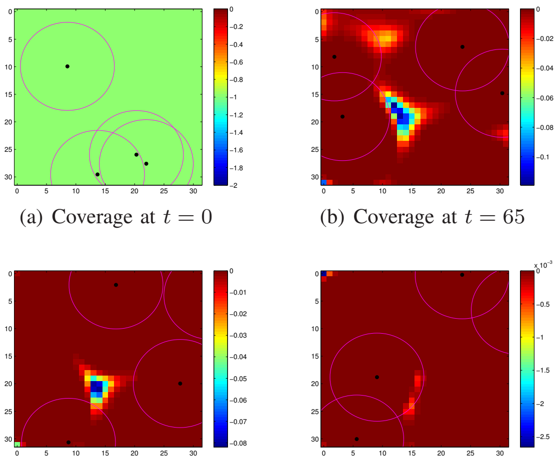
plotted in Figure 3(b) is the actual total performance achieved by the entire vehicle fleet and can be seen to converge to zero. Note that we have normalized the error so that the initial error is 1.

Figure 4 shows the variation of the state of awareness during the coverage mission. Note that the minimal state of awareness is about -2.6×10^{-3} over the entire domain and that the global error metric converges to a neighborhood of zero as predicted by Theorem (IV.2).



(a) (b)

Fig. 3. (a) Fleet motion in the plane (start at green dot and end at red dot), (b) actual global error $e(t)$.



(a) Coverage at $t = 0$

(b) Coverage at $t = 65$

(c) Coverage at $t = 130$

(d) Coverage at $t = 195$

Fig. 4. State of awareness (minimum value is -2.6$\times 10^{-3}$ at $t = 195$).

VI. Conclusion

In this paper, we proposed a novel state of awareness model that reflects how well a fleet of autonomous vehicles is aware of events taking place over a given task domain. A control strategy for networked sensors with intermittent communications and/or faulty sensors was developed to achieve satisfactory awareness levels over a domain. A detailed numerical simulation that shows the performance of the control strategy was provided. Current work includes extending the result to the case where awareness loss is taken into account, that is, $\alpha \neq 0$. Second-order nonlinear vehicle dynamics including model uncertainty and nonholonomic motion constrains will also be investigated by the authors. Moreover, an experimental test-bed composed of 4 autonomous underwater vehicles is currently under construction at Worcester Polytechnic Institute (WPI).

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