# Robust Stability Analysis for Uncertain State and Input Delay TCP/AQM Network Systems

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*Abstract*—In this paper, an active queue management (AQM) controller is designed for the problem of congestion control. In TCP networks, the packet-dropping probability function is considered as a control input. Thus, a TCP/AQM controller is modeled as a time-delay system with a saturated input. The robust observer-based controller is designed to achieve the desired queue size and guarantee the asymptotic stability of the operating point. For the particular TCP network mode, the Lyapunov-Krasovskii function is defined, and the system can be stabilize with the obtained observer and the state feedback control law by using linear matrix inequality (LMI). Simulation results show that the derived control strategy is validated.

#### I. INTRODUCTION

WITH the Internet scale expands rapidly, congestion control has become very important in network communications. So many researchers are seeking some methods to effectively control congestion. Transmission control protocol (TCP) congestion control mechanism is used to prevent congestion collapse in the past years. However, as amount of the traffic over the current Internet increases and the demand for quality of service (QoS) become stronger, it is no longer possible to exclusively rely on end hosts to perform end-toend congestion control. There has been a growing recognition within the Internet community that the network itself must participate in congestion control. Active Queue Management (AOM) schemes have been proposed to complement the TCP network congestion control. AQM is router-based control mechanism, which aims to reduce packet drops and improve network utilization. So the combination of TCP and AQM is the main approach to solve the problems of current Internet congestion control.

In the past few years, many AQM schemes have been studied in literatures. Random Early Detection (RED) [1] is regarded as the most famous AQM algorithm and has been recommended by IETF for deployment on the Internet [2]. It can prevent global synchronization, reduce packet loss ratios and minimize the bias against burst sources. However, studies such as [3] and [4] have shown that it is very difficult to tune

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RED parameters in order to perform well under different traffic condition. In order to eliminate the drawbacks associated with RED, some modified RED schemes have been proposed, such as ARED [5], FRED [6], SRED [7] and BLUE [8]. Most of these are heuristic algorithms and very few systematic and comparison were done until recently, and both high networks utilization and low packet loss can not be guaranteed.

Recently, some AQM algorithms have been proposed based on control-theoretic analysis and design. In [9], a fluid-flow model for TCP/AOM networks has been introduced. This model describes the evolution of the characteristic variables of the network, including the average TCP window size and the average queue length. It is shown that the TCP model accurately captured the qualitative behavior of TCP traffic flows. Hence, it is particularly useful for the design of innovative AQM control schemes using a control theory approach. Based on this method, a proportional (P) controller and a proportional-integral (PI) controller for AQM scheme are designed in [10]. They did not consider time delay and the uncertainties with respect to the number of active TCP sessions through the congestion AOM router, so a robust controller is required to design. [11,12] introduce sliding mode control that exists good performance and robustness with respect to the varied network parameters. As we known, in practical TCP networks, the packet-dropping probability function is considered as a control input. Therefore, the effect of a saturated actuator must be taken into account when designing a controller, which has been done in [13], but uncertainties are not considered.

In this paper, a robust AQM controller is designed for TCP dynamic network systems with input saturation, at the same time, considering time delay and uncertainties. Choosing Lyapunov-Krasovskii function method, an observer-based controller (OBC) has been developed for AQM on the basis of the LMI technique. We have shown that the proposed scheme have reliable asymptotic stability and robust in various network scenarios, and can accurately track the desired queue length, avoiding network congestion.

The remainders of this paper are organized as follows. Section II gives TCP network dynamics flow model. Section III presents the stability design method for AQM based on observer-based scheme, considering the effect of time-delay and uncertainties. Simulation results of the proposed scheme for various networks condition are shown in section IV. Finally, we conclude our brief work in section V.

#### II. TCP NETWORK DYNAMIC MODEL

In [9], a nonlinear dynamic model of TCP behavior is developed using fluid-flow and stochastic differential equation analysis. The simplified version that ignores the TCP timeout mechanism is as follows.

$$\begin{cases} \dot{W}(t) = \frac{1}{R(t)} - \frac{W(t)W(t - R(t))}{2R(t)}P(t - R(t)) \\ \dot{q}(t) = \frac{N(t)}{R(t)}W(t) - C(t) \end{cases}$$
(1)

where W(t) is TCP window size; q(t) is the instantaneous queue length on the router; R(t) is round-trip time (RTT), which satisfies  $R(t) = T_p + q(t)/C(t)$ ;  $T_p$  is the propagation delay; p(t) is the packet-dropping probability function, which is the control input used to reduce the sending rate and to maintain the bottleneck queue, which satisfies  $0 \le p(t) \le 1$ ; C(t) is the link capacity, N(t) is the number of the active TCP sessions.

For designing the AQM controller, it is assumed that  $R(t) = R_0$ , N(t) = N, C(t) = C to be the nominal values of R(t), N(t), C(t). Using linearization techniques at the operating point  $(W_0, q_0, p_0)$ , the nonlinear model could be expressed in the form of the following linear model [2].

$$\begin{cases} \delta \dot{W}(t) = -\frac{N}{R_0^2 C} \left( \delta W(t) + \delta W(t - R_0) \right) \\ -\frac{1}{R_0^2 C} \left( \delta q(t) - \delta q(t - R_0) \right) - \frac{R_0 C^2}{2N^2} \delta p(t) \quad (2) \\ \delta \dot{q}(t) = \frac{N}{R_0} \delta W(t) - \frac{1}{R_0} \delta q(t) \end{cases}$$

where  $\delta W \doteq W - W_0$ ,  $\delta q \doteq q - q_0$ ,  $\delta p \doteq p - p_0$ .

Let  $x_1 = \delta W(t)$ ,  $x_2 = \delta q(t)$ . The plant (2) can be described as

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-\tau) + Bu(t) \\ y(t) = Cx(t) \end{cases}$$
(3)

where  $x(t) = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$ ,  $u(t) = \delta p(t)$ , in which u(t) satisfies  $-p_0 \le u(t) \le 1 - p_0$ ,  $R_0 = \tau$ , A,  $A_d$ , B and C are constant matrices of appropriate dimensions expressed in the following forms

$$A = \begin{bmatrix} -\frac{N}{R_0^2 C} & -\frac{1}{R_0^2 C} \\ \frac{N}{R_0} & -\frac{1}{R_0} \end{bmatrix}, A_d = \begin{bmatrix} -\frac{N}{R_0^2 C} & \frac{1}{R_0^2 C} \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} -\frac{R_0 C^2}{2N^2} \\ 0 \end{bmatrix}$$

 $C = \begin{bmatrix} 0 & 1 \end{bmatrix}.$ 

The AQM controller is designed for the above model. The robust nature of the presented strategy here will not only reduce the sensitivity to network parameters, but also eliminate bad influence due to the use of the linear model with a saturated input.

#### III. DESIGN OF CONTROLLER FOR AQM

The objective of this section is to design an observer-based controller, which is capable of achieving asymptotic stability of robust performance.

For the TCP model defined by equation (3), the following time-delay system with saturated input can be derived

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-\tau) + Bsat(u(t)) \\ y(t) = Cx(t) \end{cases}$$
(4)

The design of controller should take into account the time-delay and uncertainty in the linear TCP model because of changing network parameter. So we consider the following form

$$\begin{cases} \dot{x}(t) = (A + \Delta A(t))x(t) + (A_d + \Delta A_d(t))x(t - \tau) \\ + Bsat(u(t)) \end{cases}$$
(5)  
$$y(t) = Cx(t)$$

where  $x(t) \in \mathbb{R}^2$ ,  $u(t) \in \mathbb{R}^1$ , and  $y(t) \in \mathbb{R}^1$  represent the system state, the control input, and the system output, respecttively.  $\Delta A(t)$  and  $\Delta A_d(t)$  are the uncertainties depending on network parameters.

In process of designing controller, the following assumptions are taken.

Assumption 1: The pairs (A, B) and (A, C) are controllable and observable, respectively.

Assumption2: The perturbed matrices  $\Delta A(t)$  and  $\Delta A_d(t)$ satisfy  $\begin{bmatrix} \Delta A(t) & \Delta A_d(t) \end{bmatrix} = DF(t) \begin{bmatrix} E_1 & E_2 \end{bmatrix}$ , where D,  $E_1$ and  $E_2$  are constant matrices with appropriate dimensions and the parameter F(t) satisfies  $||F(t)|| \le 1$ .

The saturated input is expressed by the following non-linearity

$$sat(u(t)) = \begin{cases} u_{\max} & \text{if } u(t) \ge u_{\max} \\ u(t) & \text{if } u_{\min} \le u(t) < u_{\max} \\ u_{\min} & \text{if } u(t) < u_{\min} \end{cases}$$
(6)

where  $u_{\min} = -p_0$  and  $u_{\max} = 1 - p_0$ . From equation (6), the saturation term in (5) can be rewritten as

$$sat(u(t)) = \beta(u(t))u(t)$$
(7)

where

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$$\beta(u(t)) = \begin{cases} u_{\max} / u(t) & \text{if } u(t) \ge u_{\max} \\ 1 & \text{if } u_{\min} \le u(t) < u_{\max} \\ u_{\min} / u(t) & \text{if } u(t) < u_{\min} \end{cases}$$
(8)

and

$$0 \le \beta(u(t)) \le 1 \text{ for all } t \ge 0 \tag{9}$$

Therefore, based on equations (6)-(9), the system (5) can be rewritten in an equivalent form as

$$\begin{cases} \dot{x}(t) = (A + \Delta A(t))x(t) + (A_d + \Delta A_d(t))x(t-\tau) \\ + B\beta(u(t))u(t) \\ y(t) = Cx(t) \end{cases}$$
(10)

The following lemma will be useful in designing a robust observer for the uncertain linear time-delay system (10).

**Lemma 1**: [14] (i) For any  $z, y \in \mathbb{R}^{n \times n}$ , we have

$$2z^{\mathrm{T}}y \le z^{\mathrm{T}}z + y^{\mathrm{T}}y$$

(ii) For any  $x, y \in \mathbb{R}^{n \times n}$  and F(t) being real matrices of appropriate dimensions with  $||F(t)|| \le 1$ , we have

$$2x^{\mathrm{T}}Fy \leq x^{\mathrm{T}}x + y^{\mathrm{T}}y$$

Since communication networks are large-scale complex systems, it is impossible to measure the size of the state variable window locally. A more particle approach is used to develop a robust observer-based controller (OBC).

The OBC for practical networks are considered in the following form

$$\begin{cases} \dot{z}(t) = Az(t) + B\beta(u(t))u(t) + L(y(t) - \hat{y}(t)) \\ \hat{y}(t) = Cz(t) \end{cases}$$
(11a)

$$u(t) = -Kz(t) \tag{11b}$$

where  $z(t) \in \mathbb{R}^2$  is the estimation of x(t),  $\hat{y}(t) \in \mathbb{R}^1$  is the observer output,  $L \in \mathbb{R}^{2\times 1}$  is the gain matrix of the observer,  $K \in \mathbb{R}^{1\times 2}$  is the control gain.

Define the error state

$$e(t) = x(t) - z(t) \tag{12}$$

The dynamics of the state error are then given by

$$\dot{e}(t) = \dot{x}(t) - \dot{z}(t)$$

$$= (A + \Delta A)x(t) + (A_d + \Delta A_d)x(t - \tau) +$$

$$B\beta(u(t))u(t) - Az(t) - B\beta(u(t))u(t) - (13)$$

$$L(y(t) - \dot{y}(t))$$

$$= (A - LC)e(t) + \Delta Ax(t) + (A_d + \Delta A_d)x(t - \tau)$$

By substituting (11b) into (10), it can be shown that

$$\dot{x}(t) = (A + \Delta A)x(t) + (A_d + \Delta A_d)x(t - \tau) + B\beta(-Kz(t))$$
  
=  $(A + \Delta A - B\beta K)x(t) + B\beta Ke(t) + (14)$   
 $(A_d + \Delta A_d)x(t - \tau)$ 

The objection of this section is to design observer gain matrix L and feedback gain matrix K, to guarantee that the augmented system (13) and (14) is asymptotically stable. Then, choosing the Lyapunov-Krasovskii function, and using the linear matrix inequality technology, the stability of the observer can be guaranteed and the effect of model uncertainties on the estimated state can be reduced.

The following theorem offers the theoretical basis for achieving the desired design goal.

**Theorem 1**: Consider the augmented system (13) and (14). The system is stabilized by OBC in (11) for any constant delay  $\tau$ , if the observer gain *L* is chosen such that  $L = Q^{-1}C^{T}$  and the feedback gain *K* is chosen such that  $K = \gamma_0 B^{T} P$  with  $\gamma_0 > 0$ , where *P* and *Q* are symmetric positive definite matrices and satisfy the following matrix inequality

$$\begin{bmatrix} A^{\mathrm{T}}P + PA + \Omega & PD & PB & P \\ D^{\mathrm{T}}P & -0.5I & 0 & 0 \\ B^{\mathrm{T}}P & 0 & -\varepsilon I & 0 \\ P & 0 & 0 & -I \end{bmatrix} < 0 \quad (15)$$
$$\begin{bmatrix} A^{\mathrm{T}}Q + QA & QD & PB & Q \\ D^{\mathrm{T}}Q & -0.5I & 0 & 0 \\ B^{\mathrm{T}}P & 0 & -\varepsilon I & 0 \\ Q & 0 & 0 & -I \end{bmatrix} < 0 \quad (16)$$

where  $\Omega = 2E_1^{T}E_1 + 2E_2^{T}E_2 + 2A_d^{T}A_d$ ,  $\varepsilon = \gamma_0^{-1}$ .

**Proof:** Choose the following Lyapunov-Krasovskii function

$$V(t) = V_1(t) + V_2(t) + V_3(t)$$
(17)

where

$$V_1(t) = x^{\mathrm{T}}(t) P x(t)$$
(18)

$$V_2(t) = e^{\mathrm{T}}(t)Qe(t)$$
<sup>(19)</sup>

$$V_{3}(t) = \int_{t-\tau}^{t} x^{\mathrm{T}}(s) \left(2E_{2}^{\mathrm{T}}E_{2} + 2A_{d}^{\mathrm{T}}A_{d}\right) x(s) \mathrm{d}s \quad (20)$$

*P* and *Q* are symmetry positive matrices.

Taking the time derivative of  $V_1$  along the trajectory of the system (13) yields

$$\dot{V_{1}} = \dot{x}^{\mathrm{T}}(t)Px(t) + x^{\mathrm{T}}(t)P\dot{x}(t)$$

$$= x^{\mathrm{T}}(t)(A^{\mathrm{T}}P + PA)x(t) + 2x^{\mathrm{T}}(t)P\Delta Ax(t) - 2x^{\mathrm{T}}(t)PB\beta Kx(t) + 2x^{\mathrm{T}}(t)PB\beta Ke(t) + 2x^{\mathrm{T}}(t)PA_{d}x(t-\tau) + 2x^{\mathrm{T}}(t)P\Delta A_{d}x(t-\tau)$$
(21)

Using Lemma 1, Assumption 2 and equation (9), setting  $K = \gamma_0 B^{T} P$  with  $\gamma_0 > 0$ , we have the following the forms of inequality

$$2x^{\mathrm{T}}(t)P\Delta Ax(t)$$
  
=  $2x^{\mathrm{T}}(t)PDF(t)E_{1}x(t)$  (22)  
 $\leq x^{\mathrm{T}}(t)PDD^{\mathrm{T}}Px(t) + x^{\mathrm{T}}(t)E_{1}^{\mathrm{T}}E_{1}x(t)$ 

$$2x^{T}(t)PB\beta Ke(t)$$

$$= 2x^{T}(t)PB\beta \gamma_{0}B^{T}Pe(t)$$

$$\leq \beta \gamma_{0}x^{T}(t)PBB^{T}Px(t) + \beta \gamma_{0}e^{T}(t)PBB^{T}Pe(t)$$

$$\leq \gamma_{0}x^{T}(t)PBB^{T}Px(t) + \gamma_{0}e^{T}(t)PBB^{T}Pe(t)$$
(23)

$$2x^{\mathrm{T}}(t)PA_{d}x(t-\tau)$$

$$\leq x^{\mathrm{T}}(t)PP^{\mathrm{T}}x(t)+x^{\mathrm{T}}(t-\tau)A_{d}^{\mathrm{T}}A_{d}x(t-\tau)$$
(24)

$$2x^{\mathrm{T}}(t)P\Delta A_{d}x(t-\tau)$$
  
=  $2x^{\mathrm{T}}(t)PDF(t)E_{2}x(t-\tau)$  (25)  
 $\leq x^{\mathrm{T}}(t)PDD^{\mathrm{T}}Px(t)+x^{\mathrm{T}}(t-\tau)E_{2}^{\mathrm{T}}E_{2}x(t-\tau)$ 

Substituting (22)-(25) into (21), we have the following inequality

$$\dot{V}_{1} \leq x^{\mathrm{T}}(t) \Big( A^{\mathrm{T}}P + PA + 2PDD^{\mathrm{T}}P + E_{1}^{\mathrm{T}}E_{1} + \gamma_{0}PBB^{\mathrm{T}}P + PP \Big) x(t) + \gamma_{0}e^{\mathrm{T}}(t)PBB^{\mathrm{T}}Pe(t) +$$
(26)  
$$x^{\mathrm{T}}(t-\tau) \Big( A_{d}^{\mathrm{T}}A_{d} + E_{2}^{\mathrm{T}}E_{2} \Big) x(t-\tau)$$

Using the same method, taking the time derivative of  $V_2$ along the trajectory of the system (14), choosing  $L = Q^{-1}C^{T}$ , the following of the form is obtained

$$\dot{V}_{2} = \dot{e}^{\mathrm{T}}(t)Qe(t) + e^{\mathrm{T}}(t)Q\dot{e}(t)$$

$$= e^{\mathrm{T}}(t)(A^{\mathrm{T}}Q + QA)e(t) + 2e^{\mathrm{T}}(t)Q\Delta Ax(t) - 2e^{\mathrm{T}}(t)QLCe(t) + 2e^{\mathrm{T}}(t)QA_{d}x(t-\tau) + 2e^{\mathrm{T}}(t)Q\Delta A_{d}x(t-\tau)$$

$$\leq e^{\mathrm{T}}(t)(A^{\mathrm{T}}Q + QA + 2QDD^{\mathrm{T}}Q + QQ)e(t) +$$
(27)

$$x^{\mathrm{T}}(t)E_{1}^{\mathrm{T}}E_{1}x(t)+x^{\mathrm{T}}(t-\tau)(A_{d}^{\mathrm{T}}A_{d}+E_{2}^{\mathrm{T}}E_{2})x(t-\tau)$$

The time derivative of  $V_3$  can be computed as

$$\dot{V}_{3} = x^{\mathrm{T}}(t) \Big( 2E_{2}^{\mathrm{T}}E_{2} + 2A_{d}^{\mathrm{T}}A_{d} \Big) x(t) - x^{\mathrm{T}}(t-\tau) \\ \Big( 2E_{2}^{\mathrm{T}}E_{2} + 2A_{d}^{\mathrm{T}}A_{d} \Big) x(t-\tau)$$
(28)

Then, by using (26)-(28), it can be shown that

$$\dot{V} = \dot{V}_{1} + \dot{V}_{2} + \dot{V}_{3}$$
  
$$\leq x^{\mathrm{T}}(t)H_{1}x(t) + e^{\mathrm{T}}(t)H_{2}e(t)$$
(29)

where

$$H_1 = A^{\mathrm{T}}P + PA + 2E_1^{\mathrm{T}}E_1 + 2E_2^{\mathrm{T}}E_2 + 2A_d^{\mathrm{T}}A_d + 2PDD^{\mathrm{T}}P + \gamma_0 PBB^{\mathrm{T}}P + PP$$

$$H_2 = A^{\mathrm{T}}Q + QA + 2QDD^{\mathrm{T}}Q + \gamma_0 PBB^{\mathrm{T}}P + QQ$$

By Schur complements, it can be shown that (15) and (16) imply  $\dot{V} < 0$ . Therefore, from the Lyapunov-Krasovskii stability theorem, it can be concluded that the system (13) and (14) is uniformly asymptotically stable. We can complete the proof.

## IV. SIMULATION RESULTS

In this section, computer simulations are carried out to confirm the validity of the proposed algorithm. For comparison purposes, we also simulate the PI-AQM scheme in [11]. The network parameters of simulation model are refers to scenario with a single bottleneck router running AQM schemes.

The choosing of the parameters are based on [11]. Let N = 50, C = 300 packets/s. The RTT is  $R_0 = 50$ ms, the desired queue size is  $q_0 = 100$  packets, the desired window size is  $W_0 = 2.5$  packets,  $P_0 = 2/2.5^2 = 0.32$ . Therefore,  $u_{\min} = -0.32$ ,  $u_{\max} = 0.68$ . To PI-AQM, the choosing of parameters are  $K_P = 0.0023$ ,  $K_I = 0.0004$ .  $\gamma_0 = 1$ . From Theorem 1, we use LMI toolbox in the matlab to solve matrices P and Q.

$$P = \begin{bmatrix} 0.0223 & -0.0022 \\ -0.0022 & 0.1408 \end{bmatrix}, \ Q = \begin{bmatrix} 0.0115 & -0.0015 \\ -0.0015 & 0.1037 \end{bmatrix}.$$

We can verify that matrices P and Q are symmetry positive, so they satisfy the demand of Theorem 1.

The following observer and feedback gains are obtained

 $K = \begin{bmatrix} -0.0017 & 0.1114 \end{bmatrix}, L = \begin{bmatrix} 1.2712 & 9.6663 \end{bmatrix}^{\mathrm{T}}.$ 

Fig.1-Fig.3 plot the simulation results of difference parameters of the network.

In Fig.1, we choose the parameters of networks as above. We can see that the OBC can get fast and stability responses, PI controller exhibits strongly oscillation and instability.



Fig1. Queue length responses with nominal values

In order to test the robust performance of PI and OBC for varied parameters, we vary N from 50 to 80, C from 300 to 250. The simulation results are given in Fig.2. The superior steady performance of OBC is observed when network parameters change.



Fig2. Queue length responses with varied network parameters



Fig3. Queue length responses with nominal value and bursting flows



Fig4. Queue length responses with varied network parameters disturbance and bursting flows

In Fig.3-Fig.4, we add to the TCP flows for UDP flows (transmitting on 1Mbit/s) from 5th second, respectively,

choosing fixed and varied network parameters. We can see that PI controller performance is bad if UDP flows go down. Again, the OBC controller shows better performance, with exhibiting faster responses and better regulation properties.

### V. CONCLUSION

In this paper, we analyzed a developed nonlinear model of TCP by using linearization technology. For TCP network model with the time-delayed uncertainty and saturated input, a robust OBC is designed. Based on the Lyapunov-Krasovskii functional approach, by solving two linear matrix inequalities, the corresponding congestion laws are developed to achieve asymptotic stability. The simulation results demonstrate that the proposed AQM congestion control schemes can obtain well performance in various networks conditions.

#### REFERENCES

- S. Floyd and V. Jacobson, "Random Early Detection gateway for congestion avoidance," *IEEE/ACM Transaction on Networking*, vol.1, no. 4, pp. 397-413, 1993.
- [2] B. Braden, D. Clark, J. Crowcroft, "Recommendations on queue management and congestion avoidance in the Internet," *IETF Request* for Comments, RFC 2309, April 1998.
- [3] T. Bonald, M. May, and J. C. Bolot, "Analytic evaluation of RED performance," *Nineteenth Annual Joint Conference of the IEEE Computer and Communications Societies*, SanFrancisco, CA, 2000, pp. 1415–1424.
- [4] M. Christiansen, K. Jeffay, D. Ott, and F. D. Smith, "Tuning RED for web traffic," *IEEE/ACM Transaction on Networking*, vol.9, pp. 249-264, June, 2001.
- [5] S. Floyd, R. Gummadi, S. Shenker, Adaptive RED: an algorithm for increasing the robustness of RED's active queue management, Technical Report, August 2001. Available from: <a href="http://www.icir.org/floyd/red.html">http://www.icir.org/floyd/red.html</a>.
- [6] D. Lin and R. Morris, "Dynamics of random early detection," In Proceeding of ACM SIGCOMM Conference on Application, Technologies, Architectures, and Protocol for Compter Communication. NewYork, USA, 1997, pp.127–137.
- [7] T. J. Ott, T. V. Lakshman, and L. H.Wong, "SRED: Stabilized RED," *Eighteenth Annual Joint Conference of the IEEE Computer and Communications Societies*, New York, USA, vol.3, pp. 1346–1355, March, 1999.
- [8] W. C. Feng, K. G. Shin, D. D. Kandlur, and D. Saha, "The blue active queue management algorithms," *IEEE Transaction on Networking*, vol. 10, no.4, pp. 513–528, August, 2002.
- [9] V. Misra, W.B.Gong, D.Towsley, "Fluid-based analysis of a network of AQM routers supporting TCP flows with an application to RED," *Proceedings of the ACM/SIGCOM*, Stockholm, pp. 152–160, 2000.
- [10] C. Hollot, V. Misra, D. Towsley, and W.-B. Gong, "On designing improved controllers for AQM routers supporting TCP flows," *In Proceedings of. IEEE/INFOCOM*, 2001, pp. 1726–1734.
- [11] F. J. Yin, G. M. Dimirovski, Y. W. Jing. "Robust stabilization of uncertain input delay for internet congestion control," *In Proceedings* of the 2006 American Control Conference Minneapolis, Minnesota, USA. 2006, pp. 14-16.
- [12] F. Y. Ren, C. Lin, X. H. Yin, "Design a congestion controller based on sliding mode variable structure control," *Computer Communications*, 1050-1061, 2005.
- [13] C. K. Chen, Y. C. Hung, T. L. Liao, et al, "Design of robust active queue management controllers for a class of TCP communication networks," *Information Sciences*, pp. 4059-4071,2007.
- [14] X Li, C. E. D. Souza, "Criteria for robust stability and stabilization of uncertain linear system with state delay," *Automatic*, vol.33, no.9, pp.1657-1662, 1997.