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Adaptive Control of Redundant Robot Manipulators With Sub-task Objectives

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Abstract: In this paper, adaptive control of kinematically redundant robot manipulators is considered. An end-effector tracking controller is designed and the manipulator's kinematic redundancy is utilized to integrate a general sub-task controller for self-motion control. The control objectives are achieved by designing a feedback linearizing controller that includes a least-squares estimation algorithm to compensate for the parametric uncertainties.

I. Introduction

When the number of joints of a robot manipulator is greater than the dimension of its task-space position vector then it is called a kinematically redundant robot manipulator. In many applications, robot manipulators with such additional degrees of freedom are preferred to execute complicated tasks. This kinematic redundancy can result in joint motion in the null space of the Jacobian matrix that does not affect the end-effector position, this phenomenon is commonly referred to as *self-motion*. There are generally an infinite number of solutions for the inverse kinematics of redundant robot manipulators [1], [2], [3], this complicates the control of kinematically redundant robot manipulators since it is difficult to select a reasonable desired joint trajectory for a given desired task-space trajectory.

In our previous work [4], an adaptive full-state feedback quaternion based controller developed in [5] was utilized and a general sub-task controller was designed. In [4], the sub-task controller was systematically integrated into the stability analysis and specific sub-task objectives (such as singularity avoidance, joint limit avoidance, bounding the impact forces and bounding the potential energy) were introduced to make use of the kinematic redundancy. In [6], configuration control of redundant robot manipulators was investigated. The proposed controller achieved task-space tracking and the redundancy was utilized to impose kinematic and dynamic constraints or posture control. In [7], Hsu *et al.* proposed a dynamic feedback linearizing control law that guarantees asymptotic tracking of a desired task-space trajectory. However, the controller in [7] required that the exact dynamic model of the robot manipulator be known. Zergeroglu *et al.* [8] used the controller in [7] as a basis and developed an adaptive controller to compensate for the parametric uncertainty in the dynamic model. In both [7] and [8], the researchers provided control of the redundant link velocities to accomplish desirable sub-task objectives. In [9], Peng *et al.* proposed two compliant motion controllers for redundant manipulators where the redundancy was utilized to realize additional constraints that optimize a user defined objective function. For a more detailed overview of the research on redundant robot manipulators, the reader is referred to [1], [5], [8], [10], [11], [12], [13] and the references therein.

The control objectives for a redundant robot manipulator can be classified as either task-space objectives or joint motion (sub-task) objectives. In this work, the task-space control objective is to guarantee end-effector tracking of a time-varying desired trajectory. The joint motion objectives are to track a null-space velocity vector and to incorporate a sub-task controller to make use of the kinematic redundancy. In this paper, the feedback linearizing controller in [7] is redesigned to compensate for parametric uncertainties present in the dynamic model. This work demonstrates a major improvement to our previous work [4] by proving that the null-space velocity tracking error goes to zero. By controlling the joint velocities in the null-space, we can integrate sub-task control objectives and achieve a stable system. In essence, the extra degrees-of-freedom are utilized to integrate sub-task objectives. The reader is referred to [4] and [14] for specific sub-task objectives. Review of the adaptive redundant robot control literature (such as [4] and [8]) suggests that researchers typically prefer gradient-type algorithms for parameter estimation. The design proposed here uses a least-squares algorithm in a seemingly novel departure from adaptive redundant robot control. Lyapunov-based stability analysis techniques are utilized in the design of the nonlinear control strategy.

II. Dynamic and Kinematic Model

The dynamic model for an n-joint ($n \geq 6$), revolute, direct drive robot manipulator is described by the following expression

$$M(\theta)\ddot{\theta} + N(\theta, \dot{\theta}) = \tau \qquad (1)$$

where $\theta(t), \dot{\theta}(t), \ddot{\theta}(t) \in \mathbb{R}^n$ denote the position, velocity, and acceleration in the joint-space, respectively. In (1), $M(\theta) \in \mathbb{R}^{n\times n}$ represents the inertia effects, $N(\theta, \dot{\theta}) \in \mathbb{R}^n$ represents other dynamic effects (centripetal-Coriolis effects,

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gravitational effects, dynamic frictional effects), and $\tau(t) \in \mathbb{R}^n$ represents the control input torque vector. The subsequent development is based on the following properties [15].

Property 1: The inertia matrix $M(\theta)$ is symmetric and positive-definite, and satisfies the following inequalities

$$m_1 \|\xi\|^2 \leq \xi^T M(\theta)\xi \leq m_2 \|\xi\|^2 \qquad \forall \xi \in \mathbb{R}^n \tag{2}$$

where $m_1,\ m_2 \in \mathbb{R}$ are positive constants and $\|\cdot\|$ denotes the standard Euclidean norm.

Property 2: The left-hand side of (1) can be linearly parameterized as

$$M(\theta)\ddot{\theta} + N(\theta, \dot{\theta}) = Y\left(\theta, \dot{\theta}, \ddot{\theta}\right)\phi \tag{3}$$

where $\phi \in \mathbb{R}^p$ contains the constant system parameters and the regression matrix $Y(\cdot) \in \mathbb{R}^{n \times p}$ contains known functions dependent on the signals $\theta(t)$, $\dot{\theta}(t)$, and $\ddot{\theta}(t)$.

The kinematic model for the robot manipulator is described by the following expression

$$\dot{x} = J(\theta)\dot{\theta} \tag{4}$$

where $x(t) \in \mathbb{R}^m$ is the task-space position and $J(\theta) \in \mathbb{R}^{m \times n}$ is the manipulator Jacobian matrix. The subsequent development is based on the assumption that $x(t)$, $\dot{x}(t)$, $\theta(t)$, and $\dot{\theta}(t)$ are measurable.

Remark 1: The dynamic and kinematic terms for a general revolute robot manipulator, denoted by $M(\theta)$, $N\left(\theta, \dot{\theta}\right)$, $J(\theta)$ and $J^+(\theta)$, are assumed to depend on $\theta(t)$ only as arguments of trigonometric functions, and hence, remain bounded for all possible $\theta(t)$. During the control development, the assumption will be made that if $x(t)$ is bounded then $\theta(t)$ is a bounded signal.

III. Pseudo-inverse and its properties

The pseudo-inverse of the Jacobian, denoted by $J^+(\theta) \in \mathbb{R}^{n \times m}$, is defined as follows

$$J^+ \triangleq J^T \left(JJ^T\right)^{-1}. \tag{5}$$

From (5), it is clear that $J^+(\theta)$ satisfies the following[1]

$$JJ^+ = I_m. \tag{6}$$

As shown in [1], the pseudo-inverse defined by (5) satisfies the Moore-Penrose conditions given below

$$JJ^+J = J \quad , \quad J^+JJ^+ = J^+, \tag{7}$$

$$(J^+J)^T = J^+J \quad , \quad (JJ^+)^T = JJ^+. \tag{8}$$

In addition to the above properties, the matrix $(I_n - J^+J)$ satisfies the following properties

$$\left(I_n - J^+J\right)\left(I_n - J^+J\right) = I_n - J^+J \tag{9}$$

$$\left(I_n - J^+J\right)^T = \left(I_n - J^+J\right) \tag{10}$$

$$J\left(I_n - J^+J\right) = 0_{n \times 1} \tag{11}$$

$$\left(I_n - J^+J\right)J^+ = 0_{n \times 1}. \tag{12}$$

[1] Throughout the paper, I_n and $0_{m \times r}$ will be used to represent an $n \times n$ standard identity matrix and an $m \times r$ zero matrix, respectively.

The following expression can be obtained for the time derivative of J^+J

$$\frac{d}{dt}\left\{J^+J\right\} = J_\phi + J^+\dot{J}\left(I_n - J^+J\right) \tag{13}$$

where $J_\phi(t) \in \mathbb{R}^{n \times n}$ is an auxiliary function defined as follows

$$J_\phi \triangleq \dot{J}^+J + J^+\dot{J}J^+J. \tag{14}$$

It should be noted that $J_\phi(t)$ satisfies the following property

$$\begin{aligned} JJ_\phi &= \left(J\dot{J}^+ + JJ^+\dot{J}J^+\right)J \\ &= \frac{d}{dt}\left\{JJ^+\right\}J \\ &= 0_{n \times n} \end{aligned} \tag{15}$$

where (6) was utilized. In addition, the following property will also be utilized throughout the subsequent analysis

$$\begin{aligned} \left(I_n - J^+J\right)J_\phi &= J_\phi - J^+JJ_\phi \\ &= J_\phi \end{aligned} \tag{16}$$

where (15) was utilized.

Remark 2: During the subsequent control development, the assumption is made that the minimum singular value of the manipulator Jacobian matrix, denoted by σ_m, is greater than a known small positive constant $\delta > 0$, such that $\max\{\|J^+(\theta)\|\}$ is known *a priori* and all kinematic singularities are always avoided.

IV. Task-Space Controller Development

The primary control design objective is to formulate a control input that ensures that the end-effector of the manipulator tracks a desired trajectory. The task-space tracking error denoted by $e(t) \in \mathbb{R}^m$ is defined as follows

$$e \triangleq x_d - x \tag{17}$$

where $x_d(t) \in \mathbb{R}^m$ is the task-space desired trajectory. In the subsequent development, it will be assumed that $x_d(t)$, $\dot{x}_d(t)$, and $\ddot{x}_d(t)$ are bounded signals.

Based on (4), the following expression can be obtained for the joint velocities

$$\dot{\theta} = J^+\dot{x} + \left(I_n - J^+J\right)\dot{\theta}. \tag{18}$$

To facilitate the task-space controller development, the time derivative of (18) is given as follows

$$\ddot{\theta} = J^+\ddot{x} + \dot{J}^+\dot{x} - \frac{d}{dt}\left\{J^+J\right\}\dot{\theta} + \left(I_n - J^+J\right)\ddot{\theta}. \tag{19}$$

After utilizing (4), the following simplified expression can be obtained for $\ddot{\theta}(t)$

$$\ddot{\theta} = J^+\left(\ddot{x} - \dot{J}\dot{\theta}\right) + \ddot{\theta}_N \tag{20}$$

where $\ddot{\theta}_N(t) \in \mathbb{R}^n$ is defined as follows

$$\ddot{\theta}_N \triangleq \left(I_n - J^+J\right)\ddot{\theta}. \tag{21}$$

The estimation form of (3) is defined as

$$\hat{M}(t)\ddot{\theta} + \hat{N}(t) = Y\left(\theta, \dot{\theta}, \ddot{\theta}\right)\hat{\phi} \tag{22}$$

where $\hat{\phi}(t) \in \mathbb{R}^p$, $\hat{M}(t)$, and $\hat{N}(t)$ are the estimates of ϕ, $M(\theta)$, and $N(\theta, \dot{\theta})$, respectively. After subtracting (22) from the manipulator's dynamics in (1), the following is obtained

$$Y\tilde{\phi} = \tau - \left(\hat{M}\ddot{\theta} + \hat{N}\right) \tag{23}$$

where $\tilde{\phi}(t) \in \mathbb{R}^p$ is the parameter estimation error defined as

$$\tilde{\phi} \triangleq \phi - \hat{\phi}. \tag{24}$$

After premultiplying (23) by $\hat{M}^{-1}(t)$, the following expression can be obtained

$$\hat{M}^{-1}Y\tilde{\phi} = \hat{M}^{-1}\tau - \hat{M}^{-1}\hat{N} - \ddot{\theta} \tag{25}$$

for the open-loop error system. To facilitate the subsequent analysis the control input $\tau(t)$ is designed as follows

$$\tau \triangleq \hat{M}\left[J^{+}u_1 + \phi_N\right] + \hat{N} + u_2 \tag{26}$$

where $u_1(t) \in \mathbb{R}^m$, $u_2(t) \in \mathbb{R}^n$ are auxiliary control inputs, and $\phi_N(t) \in \mathbb{R}^n$ is a vector in the null-space of $J(t)$. The auxiliary control input $u_1(t)$ is designed as

$$u_1 \triangleq \ddot{x}_d + k_v\dot{e} + k_p e - \dot{J}\dot{\theta} + u_{aux} \tag{27}$$

where k_v and k_p are positive constants, and $u_{aux}(t) \in \mathbb{R}^m$ is another auxiliary control input that will be designed subsequently. After substituting (26) and (27) into the open-loop error system in (25), the following expression is obtained

$$\begin{aligned}\hat{M}^{-1}Y\tilde{\phi} \;&=\; J^{+}\left(\ddot{e} + k_v\dot{e} + k_p e + u_{aux}\right)\\ &\quad +\hat{M}^{-1}u_2 + \phi_N - \ddot{\theta}_N.\end{aligned} \tag{28}$$

After premultiplying (28) by $J(t)$ and rearranging, the following expression can be obtained

$$\ddot{e} + k_v\dot{e} + k_p e + u_{aux} = J\hat{M}^{-1}\left(Y\tilde{\phi} - u_2\right) \tag{29}$$

where (6) and the following facts were utilized

$$J\phi_N = 0_{m\times 1} \quad , \quad J\ddot{\theta}_N = 0_{m\times 1} \; . \tag{30}$$

It should be noted that since $\ddot{\theta}(t)$ is an unmeasurable signal the regression matrix $Y\left(\theta, \dot{\theta}, \ddot{\theta}\right)$ introduced in (3) is unmeasurable. To tackle this issue, a filtered regression matrix $Y_f(t) \in \mathbb{R}^{n\times p}$ is introduced [16]

$$\dot{Y}_f \triangleq -\alpha Y_f + \alpha Y \quad , \quad Y_f(t_0) \triangleq 0_{n\times p} \tag{31}$$

where $\alpha \in \mathbb{R}$ is a positive constant. Notice that (31) cannot be implemented since $Y\left(\theta, \dot{\theta}, \ddot{\theta}\right)$ is unmeasurable. For an implementable form of the filtered regression matrix see Appendix II. A filtered control input is defined similarly [16]

$$\dot{\tau}_f \triangleq -\alpha\tau_f + \alpha\tau \quad , \quad \tau_f(t_0) \triangleq 0_{n\times 1}. \tag{32}$$

To facilitate the subsequent analysis a prediction error, denoted by $z(t) \in \mathbb{R}^n$, is defined as follows

$$z \triangleq \hat{M}^{-1}\left(\tau_f - Y_f\hat{\phi}\right). \tag{33}$$

After utilizing the development in Appendix I, the prediction error in (33) can be written as follows

$$z = \hat{M}^{-1}Y_f\tilde{\phi} \tag{34}$$

where (24) was also utilized. The auxiliary control input $u_2(t)$ is designed as

$$u_2 \triangleq \frac{1}{\alpha}Y_f\dot{\hat{\phi}} + \frac{1}{\alpha}\dot{\hat{M}}z. \tag{35}$$

After substituting $u_2(t)$ into (29), the following expression can be obtained

$$\ddot{e} + k_v\dot{e} + k_p e = J\left(\frac{1}{\alpha}\dot{z} + z\right) - u_{aux} \tag{36}$$

where (34) and its time derivative were utilized. A filtered tracking error, denoted by $r(t) \in \mathbb{R}^m$, is defined to be of the following form

$$r \triangleq \dot{e} + \sigma_1 e \tag{37}$$

where $\sigma_1 \in \mathbb{R}$ is a positive constant. After setting the constant control gains k_v and k_p, which were introduced in (27), as

$$k_v \triangleq \sigma_1 + \sigma_2 \quad , \quad k_p \triangleq \sigma_1\sigma_2 \tag{38}$$

then the left-hand-side of (36) can be written as

$$\ddot{e} + k_v\dot{e} + k_p e = \dot{r} + \sigma_2 r \tag{39}$$

where $\sigma_2 \in \mathbb{R}$ is a positive constant. After utilizing (39), the expression in (36) can be rewritten as

$$\dot{r} + \sigma_2 r = J\left(\frac{1}{\alpha}\dot{z} + z\right) - u_{aux}. \tag{40}$$

To facilitate the subsequent stability analysis, an auxiliary error signal, denoted by $y(t) \in \mathbb{R}^m$ is defined as

$$y \triangleq r - \frac{1}{\alpha}Jz. \tag{41}$$

The time derivative of $y(t)$ is given as

$$\dot{y} = -\sigma_2 y + \left(1 - \frac{\sigma_2}{\alpha}\right)Jz - \frac{1}{\alpha}\dot{J}z - u_{aux} \tag{42}$$

where (40) and (41) were utilized. The auxiliary control input $u_{aux}(t)$ is designed as

$$u_{aux} \triangleq \left(1 - \frac{\sigma_2}{\alpha}\right)Jz - \frac{1}{\alpha}\dot{J}z. \tag{43}$$

After substituting (43) into (42), the following simplified expression is obtained for the dynamics of $y(t)$

$$\dot{y} = -\sigma_2 y. \tag{44}$$

From (44), standard analysis techniques can be utilized to show that

$$y(t) = y(t_0)\exp(-\sigma_2 t) \tag{45}$$

from which we can conclude that $\|y(t)\| \to 0$ exponentially fast. Motivated by the subsequent stability analysis, the parameter estimate vector $\hat{\phi}(t)$ is generated by the following update law

$$\dot{\hat{\phi}} \triangleq \Gamma Y_f^T\hat{M}^{-T}z \tag{46}$$

where $\Gamma(t) \in \mathbb{R}^{p\times p}$ is a least-squares estimation gain matrix designed as follows

$$\frac{d}{dt}\left(\Gamma^{-1}\right) \triangleq Y_f^T\hat{M}^{-T}\hat{M}^{-1}Y_f. \tag{47}$$

Remark 3: It should be noted that when $\Gamma^{-1}(t_0)$ is selected to be positive definite and symmetric, then it is clear that $\Gamma(t_0)$ is also positive definite and symmetric. Therefore, it follows that both $\Gamma^{-1}(t)$ and $\Gamma(t)$ will remain positive definite and symmetric $\forall t$. From (47), the following expression can be obtained

$$\dot{\Gamma} = -\Gamma Y_f^T \hat{M}^{-T} \hat{M}^{-1} Y_f \Gamma. \tag{48}$$

From (48), it is easy to see that $\dot{\Gamma}(t)$ is negative semidefinite; therefore, the estimation gain matrix $\Gamma(t)$ is always constant or decreasing, and hence, $\Gamma(t)$ is bounded (for more details, the reader is referred to [16] and [17]).

Remark 4: The matrix inverse of the estimate of $M(\theta)$ (i.e., $\hat{M}(\theta)$) can be guaranteed to exist through the use of a projection algorithm as described in [18].

Theorem 1: The control law described in (26), (27), (35), and (43) and the adaptation law defined in (46) guarantee that $z(t)$, $r(t)$, and $e(t)$ are driven to zero as $t \to \infty$.

Proof: See Appendix III.

Remark 5: The proof of Theorem 1 requires the boundedness of $\dot{\theta}(t)$ and $\phi_N(t)$. In the subsequent sections, an auxiliary null-space control signal, denoted by $g(\theta)$, will be designed to meet these conditions.

V. Sub-Task Error System

In addition to the end-effector tracking objective, there may be sub-task objectives that are required for a particular redundant robot application. To integrate the sub-task objective into the controller, an auxiliary control signal, denoted by $g(\theta)$, will be introduced. The integration of this sub-task objective into the controller is completed by designing a framework that places preferences on desirable configurations based on the sub-task objective. The auxiliary null-space controller $g(\theta)$ is designed through the joint motion in the null-space of the Jacobian matrix (*i.e.*, *self-motion*).

The null-space velocity tracking error is defined as [7]

$$\dot{e}_N \triangleq \left(I_n - J^+ J\right)\left(g - \dot{\theta}\right) \tag{49}$$

where $g(t) \in \mathbb{R}^n$ is the subsequently designed null-space controller. The following expression can be obtained for the dynamics of $\dot{e}_N(t)$

$$\begin{aligned} \ddot{e}_N \;=\;& \left(I_n - J^+ J\right)\dot{g} + \left(I_n - J^+ J\right)\left(\frac{1}{\alpha}\dot{z} + z - \phi_N\right) \\ & - J_\phi\left(g - \dot{\theta}\right) - J^+ \dot{J} \dot{e}_N \end{aligned} \tag{50}$$

where (12), (13), (49) were utilized along with the following expression for $\ddot{\theta}(t)$

$$\ddot{\theta} = -\left(\frac{1}{\alpha}\dot{z} + z\right) + J^+ u_1 + \phi_N \tag{51}$$

where (20), (25), (26), (34), the time derivative of (34), and (35) were utilized. In order to facilitate the null-space control development, an auxiliary error signal, denoted by $p(t) \in \mathbb{R}^n$, is defined as follows

$$p \triangleq \dot{e}_N - \frac{1}{\alpha}\left(I_n - J^+ J\right) z. \tag{52}$$

The dynamics of $p(t)$ can be written as

$$\begin{aligned} \dot{p} \;=\;& \left(I_n - J^+ J\right)\dot{g} + \left(I_n - J^+ J\right)(z - \phi_N) \\ & - J_\phi\left(g - \dot{\theta} - \frac{1}{\alpha} z\right) - J^+ \dot{J} p \end{aligned} \tag{53}$$

where (13), (50), and (52) were utilized. The auxiliary null-space vector $\phi_N(t)$, introduced in (26), is now designed as follows

$$\begin{aligned} \phi_N \;\triangleq\;& \left(I_n - J^+ J\right)(\dot{g} + k_n p + z) \\ & + J_\phi\left(g - \dot{\theta} - \frac{1}{\alpha} z\right) \end{aligned} \tag{54}$$

where $k_n \in \mathbb{R}$ is a positive constant. After substituting $\phi_N(t)$ into (53) the following simplified expression is obtained for the dynamics of $p(t)$

$$\dot{p} = -k_n\left(I_n - J^+ J\right) p - J^+ \dot{J} p \tag{55}$$

where (9) and (16) were utilized.

Theorem 2: The auxiliary null-space vector described by (54) guarantees that $\dot{e}_N(t)$ is driven to zero as $t \to \infty$.

Proof: See Appendix IV.

VI. Sub-Task Controller

In this section, a general sub-task controller is developed. As proven in the subsequent stability analysis, the sub-task objective will be met if the Jacobian-related null-space matrix maintains full rank. Specifically, when the subsequently defined Jacobian-related null-space matrix loses rank, the sub-task objective will not be met.

An auxiliary positive function $y_a(t) \in \mathbb{R}$ is defined as

$$y_a \triangleq \exp(-k_y \beta(\theta)) \tag{56}$$

where $k_y \in \mathbb{R}$ is a positive constant, $\beta(\theta) \in \mathbb{R}$ is a non-negative function that is specific to each sub-task, and $\exp(\cdot)$ is the natural logarithmic exponential function. After taking the time derivative of (56), the following simplified expression is obtained for the dynamics of $y_a(t)$

$$\dot{y}_a = J_s \dot{\theta} \tag{57}$$

where $J_s(t) \in \mathbb{R}^{1\times n}$ is a Jacobian-type vector defined as follows

$$J_s = \frac{\partial y_a}{\partial \theta}. \tag{58}$$

After adding and subtracting the terms $J_s J^+ J \dot{\theta}$ and $J_s\left(I_n - J^+ J\right)\left(g - \dot{\theta}\right)$ to the right-hand-side of (57), we obtain the following for the time derivative of $y_a(t)$

$$\dot{y}_a = J_s J^+ \dot{x} + J_s\left(I_n - J^+ J\right) g - J_s \dot{e}_N \tag{59}$$

where (4) and (49) were utilized. Based on the subsequent stability analysis, the sub-task controller is designed as

$$g \triangleq -k_s J_s^T y_a \tag{60}$$

where $k_s \in \mathbb{R}$ is a positive constant. After substituting (60) into (59), we obtain the following expression

$$\begin{aligned} \dot{y}_a \;=\;& -k_s J_s\left(I_n - J^+ J\right)\left(I_n - J^+ J\right) J_s^T y_a \\ & + J_s J^+ \dot{x} - J_s \dot{e}_N \end{aligned} \tag{61}$$

where (9) was utilized.

Remark 6: The auxiliary signal $y_a(t)$ in (56) was preferred because of the useful properties of the logarithmic exponential function given that many different positive functions could also be utilized for the design of $y_a(t)$. From (56), it is clear that as $\beta(\theta)$ increases, $y_a(t)$ decreases and $y_a(t)$ satisfies these inequalities $0 < y_a(t) \leq 1$.

The following theorem is stated to show the performance of the sub-task controller.

Theorem 3: The control law described by (60) guarantees that $y_a(t)$ is practically regulated (*i.e.*, ultimately bounded) in the following sense

$$|y_a(t)| \leq \sqrt{|y_a^2(t_0)| \exp(-2\gamma t) + \frac{\varepsilon}{\gamma}} \tag{62}$$

provided the following sufficient conditions hold

$$\left\| J_s \left(I_n - J^+ J \right) \right\|^2 > \bar{\delta} \tag{63}$$

$$\left\| J_s \left(J^+ \dot{x} - \dot{e}_N \right) \right\| \leq \delta_1 \tag{64}$$

$$k_s > \frac{1}{\bar{\delta}\delta_2} \tag{65}$$

where ε, γ, $\bar{\delta}$, δ_1, $\delta_2 \in \mathbb{R}$ are positive constants.

Proof: See [19] (the reader is also referred to proof of Theorem 2 in [14] for a similar proof).

Remark 7: For specific sub-task objectives including singularity avoidance, joint-limit avoidance, bounding the impact forces, and bounding the potential energy, the reader is referred to [14].

Remark 8: The sub-task objective is met only if the sufficient conditions described by the inequalities in (63)-(65) are satisfied. From the result of Theorem 1, the task-space tracking objective is guaranteed and the sub-task objective is always secondary to it. When the sub-task controller forces the end-effector of the robot manipulator to take a path not allowed by the task-space tracking controller, the condition in (63) will not be satisfied; hence, the result of Theorem 3 will not hold. To meet the task-space tracking and sub-task objectives simultaneously, careful consideration is required in the design of the desired task-space trajectory and the sub-task objective.

VII. Conclusions

Lyapunov-based stability analysis techniques were utilized to design a feedback linearizing adaptive controller for kinematically redundant robot manipulators. The controller compensates for the parametric uncertainties in the dynamic model using a least-squares estimation algorithm. To our best knowledge, this is novel when compared to the previous adaptive redundant robot control literature. Task-space tracking was achieved and the kinematic redundancy was utilized to integrate a general sub-task controller.

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Appendix I
Filter Development

From (1), (3), (31), and (32) the following can be obtained

$$\dot{\tau}_f + \alpha \tau_f = \dot{Y}_f \phi + \alpha Y_f \phi. \tag{66}$$

The expression in (66) can be rewritten as

$$(s + \alpha) \tau_f = (s + \alpha) Y_f \phi \tag{67}$$

where s is the Laplace variable. From (67) the following can be obtained [16]

$$\tau_f = Y_f \phi \tag{68}$$

where the initial condition information defined in (31) and (32) were utilized.

APPENDIX II
IMPLEMENTABLE FORM OF THE REGRESSION MATRIX

In order to obtain an implementable form of (31) the entries of $Y\left(\theta,\dot{\theta},\ddot{\theta}\right)$ will be written in the following form

$$Y_{ij}\left(\theta,\dot{\theta},\ddot{\theta}\right) \triangleq B_{ij}^{T}(\theta)\ddot{\theta} + A_{ij}\left(\theta,\dot{\theta}\right) \tag{69}$$

where $B_{ij}^{T}(\theta) \in \mathbb{R}^{1\times n}$ and $A_{ij}\left(\theta,\dot{\theta}\right) \in \mathbb{R}$ for $\forall i = 1,..,n$ and $\forall j = 1,..,p$. An auxiliary filter signal, denoted by $P_{ij}(t) \in \mathbb{R}$, is designed as follows

$$\begin{aligned} \dot{P}_{ij}(t) &\triangleq -\alpha Y_{f_{ij}} - \dot{B}_{ij}^{T}(\theta)\dot{\theta} + A_{ij}\left(\theta,\dot{\theta}\right) && (70)\\ P_{ij}(t_0) &\triangleq -B_{ij}^{T}(\theta(t_0))\dot{\theta}(t_0) && (71) \end{aligned}$$

where $Y_{f_{ij}}(t)\ \forall i,j$ are defined as follows

$$Y_{f_{ij}} \triangleq P_{ij} + B_{ij}^{T}(\theta)\dot{\theta} \tag{72}$$

From (70)-(72), it is clear that (31) is satisfied and $Y_{f_{ij}}(t)$ defined in (72) can be implemented without measuring $\ddot{\theta}(t)$.

APPENDIX III
PROOF OF THEOREM 1

The following non-negative function is introduced to analyze the stability of the task-space controller

$$V_1 \triangleq \frac{1}{2}\tilde{\phi}^{T}\Gamma^{-1}\tilde{\phi}. \tag{73}$$

The time derivative of the Lyapunov function in (73) is given as follows

$$\begin{aligned} \dot{V}_1 &= \frac{1}{2}\tilde{\phi}^{T}\dot{\Gamma}^{-1}\tilde{\phi} - \tilde{\phi}^{T}\Gamma^{-1}\dot{\hat{\phi}} \\ &= \frac{1}{2}\tilde{\phi}^{T}Y_f^{T}\hat{M}^{-T}\hat{M}^{-1}Y_f\tilde{\phi} - \tilde{\phi}^{T}Y_f^{T}\hat{M}^{-T}z \\ &= -\frac{1}{2}z^{T}z && (74) \end{aligned}$$

where (34), (46) and (47) were utilized. After integrating (74) the following expression can be obtained

$$V_1(t_0) - V_1(\infty) = \frac{1}{2}\int_{t_0}^{\infty} z^{T}(\tau)\,z(\tau)\,d\tau. \tag{75}$$

From (73), it is clear that $V_1(t_0) > 0$, and from (75), it is easy to see that $V_1(t) \leq V_1(t_0)$, then we can conclude that $V(t)$ is bounded, hence $z(t) \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\tilde{\phi}(t) \in \mathcal{L}_\infty$. From (24), $\hat{\phi}(t) \in \mathcal{L}_\infty$ hence $\hat{M}(t)$ and $\hat{N}(t)$ are also bounded. Based on Remark 4, the matrix inverse of the estimate of $M(\theta)$ (i.e., $\hat{M}^{-1}(t)$) exists and is bounded. Remark 1 can be utilized to show that $J(\theta)$ and $J^{+}(\theta)$ are bounded. These boundedness statements can be utilized along with (41) to prove that $r(t)$ is bounded; hence, from (37), we can conclude that $e(t),\ \dot{e}(t) \in \mathcal{L}_\infty$. Since the desired trajectory and its time derivative are assumed to be bounded then from (17) and its time derivative we can prove that $x(t),\ \dot{x}(t) \in \mathcal{L}_\infty$. Based on Remark 1, $\theta(t)$ is bounded. The rest of the development requires the joint velocities to be bounded. From the proof of Theorem 2 (see Appendix IV), we know that $\dot{e}_N(t) \in \mathcal{L}_\infty$ and from the proof of Theorem 3 (see [19]), we know that $g(t) \in \mathcal{L}_\infty$. Based on these facts, we can show that $(I_n - J^{+}J)\dot{\theta} \in \mathcal{L}_\infty$. After utilizing this along with (4) and the fact that $\dot{x}(t) \in \mathcal{L}_\infty$, we can prove that $\dot{\theta}(t) \in \mathcal{L}_\infty$. After utilizing the facts that $\theta(t),\ \dot{\theta}(t) \in \mathcal{L}_\infty$, we can conclude that $M(\theta)$ and $N\left(\theta,\dot{\theta}\right)$ are bounded. By utilizing the above boundedness statements it is easy to show that $\dot{J}(\theta) \in \mathcal{L}_\infty$. From the development in Appendix II, we can show that $Y_f(t),\ \dot{Y}_f(t) \in \mathcal{L}_\infty$. Then from (46), it is clear that $\dot{\hat{\phi}}(t)$ is also bounded. Thus $\dot{\hat{M}}(t)$ and $\frac{d}{dt}\left(\hat{M}^{-1}(t)\right)$ can be shown to be bounded. We can utilize the the time derivative of (34) to prove that $\dot{z}(t) \in \mathcal{L}_\infty$. The above boundedness statements can be utilized along with (43) and (35) to show that $u_{aux}(t),\ u_2(t) \in \mathcal{L}_\infty$, thus, from (27), $u_1(t)$ is also bounded. From the proof of Theorem 3 (see [19]), we know that $g(t),\ \dot{g}(t) \in \mathcal{L}_\infty$. After utilizing these facts and the previous boundedness statements along with (54), then we can prove that $\phi_N(t) \in \mathcal{L}_\infty$. Then from (26) and (51), it is clear that $\tau(t),\ \ddot{\theta}(t) \in \mathcal{L}_\infty$. Since $z(t) \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\dot{z}(t) \in \mathcal{L}_\infty$ then we can conclude that $\|z(t)\| \to 0$ as $t \to \infty$. Then from (41), it is clear that $\|r(t)\| \to 0$ as $t \to \infty$; thus from (37), $\|e(t)\|,\ \|\dot{e}(t)\| \to 0$ as $t \to \infty$.

APPENDIX IV
PROOF OF THEOREM 2

Let $V_2(t) \in \mathbb{R}$ denote the following non-negative function

$$V_2 \triangleq \frac{1}{2}p^{T}p. \tag{76}$$

The time derivative of (76) is given as follows

$$\dot{V}_2 = -k_n p^{T}p + p^{T}J^{+}\left(k_n J - \dot{J}\right)p \tag{77}$$

where the dynamics of $\dot{p}(t)$ in (55) was utilized. To facilitate the subsequent development the following property is introduced

$$\begin{aligned} p^{T}J^{+} &= \left(g - \dot{\theta} - \frac{1}{\alpha}z\right)^{T}\left(I_n - J^{+}J\right)^{T}J^{+} \\ &= \left(g - \dot{\theta} - \frac{1}{\alpha}z\right)^{T}\left(I_n - J^{+}J\right)J^{+} \\ &= 0_{1\times m} && (78) \end{aligned}$$

where (49) and (52) were utilized. In view of (78), (77) can be written in the following simple form

$$\dot{V}_2 = -k_n p^{T}p. \tag{79}$$

From (76) and (79), standard linear analysis techniques can be utilized to show that

$$p(t) = p(t_0)\exp(-k_n t) \tag{80}$$

from which we can conclude that $\|p(t)\| \to 0$ exponentially fast. Then from (52) and the proof of Theorem 1, it is easy to see that $\|\dot{e}_N(t)\| \to 0$ as $t \to \infty$.