

# A Method For Collision-free Navigation of a Wheeled Mobile Robot Using The Range-only Information

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**Abstract**— We present a new guidance logic for the problem of collision-free navigation of wheeled mobile robot (WMR) towards an unknown stationary or maneuvering target based on just the relative distance between the robot and target, also known as Line-Of-Sight range (LOS-range). With constant robot linear velocity, we use the LOS-range variation as a measure for the angle at which the robot approaches the target. Having applied the proposed steering control law, termed Equiangular Navigation Guidance (ENG), the robot approaches the stationary or maneuvering target along a semi-equiaxial spiral and eventually goes into a circular trajectory around it. Using the sensory information, ENG is then modified to Augmented-ENG (AENG) in order to navigate the WMR towards the target and simultaneously avoid the obstacles on the way. AENG enables the robot to approach an unknown stationary or follow an unpredictable maneuvering target in a cramped environment, while keeping a certain distance from the target, and simultaneously preserving a safety margin from the obstacles. The performance of the guidance law and its effectiveness is confirmed with an extensive simulation study.

## I. INTRODUCTION

The research on wheeled mobile robot (WMR) navigation as a classical example of nonholonomic systems and obstacle avoidance as an essential part of developing every Autonomous Ground Vehicle (AGV) have gained a great deal of interest over the past few years. Many sophisticated approaches in control of a WMR for target following and trajectory tracking have been proposed in the literature; including nonlinear control [14], dynamic feedback linearizing [5], sliding mode control [16], Fuzzy control [13], neural network [3] and vision-based navigation [10]. [1] suggested nonlinear controllers based on control Lyapunov function, and [2] proposed a strategy in robot guidance based on proportional navigation guidance and [11] proposed a precision guidance law with impact angle constraint for a 2-D planar intercept. However, in most of current methods, target velocity, position, moving direction or line-of-sight angle (the angle between the reference line and the imaginary straight line starts at the robot's reference point and is directed towards the target's position) are considered given, which are not always available in practice. GPS systems can provide accurate location information in outdoor settings, however they fail in indoor navigation where GPS signals can not be reliably received. Video or IR based positioning systems,

similarly, are restricted to line-of-sight limitations or poor performance with fluorescent lighting, direct sunlight and lack of light situations. Furthermore, in some applications the target is either too small to appear in an image frame or located behind an obstacle in indoor applications or too far from the robot in outdoor applications.

On the other side, in order to safely navigate and reliably operate in populated environments, an autonomous vehicle should be able to detect and avoid the obstacles on the way towards the target. Several classical approaches such as potential field techniques [7], vector field histogram techniques [4], the curvature velocity method [15] and the dynamic window approach [6] have been discussed in the literature for the problem of collision-free navigation of WMR. Based on the current sensory input, these strategies determine the best next action in order to safely guide the robot towards the target. Potential field approach uses vector sums of attractive and repulsive forces from the goal and obstacles, respectively, in order to calculate the commanded steering control. Robot velocity is usually considered proportional to the magnitude of the potential vector. Potential field method suffers from oscillatory behavior in narrowly confined corridors and local minima, which usually arises due to the symmetry of the environment and concave obstacles, [8]. Vector Field Histogram (VFH) is a modification of potential field method by computing a one dimensional polar histogram, which is then processed to determine the steering direction towards the target among all open areas. To construct an occupancy grid from sensor readings and to drive the robot towards a specified goal, VFH uses the Histogram In Motion Mapping (HIMM). However, since the standard technique is computationally expensive, it is not amenable for real-time navigation. The Curvature Velocity Method (CVM), formulates the problem of local obstacle avoidance as one of constrained optimization in velocity space. Constraints are derived from physical limitations on the robot's velocities and accelerations, and the configuration of the obstacles. The velocity commands should satisfy all the constraints and maximize an objective function that trades off speed, safety and direction towards the goal. Although this approach yield very good results for obstacle avoidance at high velocities, it is not computationally efficient, as VFH. Dynamic Window Approach (DWA), similar to CVM, computes the optimal velocity of robot using the admissible velocity space. The admissible velocity space is the collection of velocity candidates which satisfy the kinematics and dynamic constraints of robot. The result should maximize a given evaluation function which typically measures the progress towards the

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goal, the forward velocity of the robot and the distance to the next obstacle on the trajectory.

In this paper, considering the dynamic constraints of the mobile robot, i.e. bounded linear and angular velocities, we propose a new algorithm, termed Equiangular Navigation Guidance (ENG), for the problem of robot guidance towards an unknown stationary or maneuvering target using just the relative distance between the robot and target, known also as Line-Of-Sight range (LOS-range). The LOS-range can be achieved by measuring the strength of the signal transmitted by the target and received at the robot position. Having applied the proposed idea, the robot approaches the unknown target in a semi-equiangular spiral, whose arc-length and curvature are subjects to change with a control parameter and eventually goes into a circular trajectory around the target. Using the sensory information from range finders, ENG is then modified to Augmented-ENG in order to navigate WMR towards the target and simultaneously detect and avoid the obstacles on the way. The AENG enables the robot to approach an unknown stationary or follow an unpredictable maneuvering target, while keeping a certain distance from the target, and simultaneously preserving a safety margin from the obstacles in a cramped environment.

## II. PROBLEM STATEMENT

Let us consider a three-wheeled, non-holonomic mobile robot of Dubin's car type, which moves in a horizontal plane and an unknown environment. In a two-dimensional space, the position of the robot can be represented by a triplet  $P_R = (X_R, Y_R, \theta_R)$  where  $(X_R, Y_R)$  is the location of the middle of the wheel base and  $\theta_R$  is the heading angle with respect to the reference line. Let  $V_R$  be the linear velocity and  $\omega_R$  the angular velocity of mobile robot. A rolling-without-slippage model is assumed for the robot. The evolution model is classically given by:

$$\begin{aligned}\dot{X}_R &= V_R \cos(\theta_R) \\ \dot{Y}_R &= V_R \sin(\theta_R) \\ \dot{\theta}_R &= \omega_R\end{aligned}\quad (1)$$

with  $U = [V_R \ \omega_R]^T$  as the control vector of the mobile robot,  $U \in V \times [-\omega_{max} \ \omega_{max}]$  with  $V, \omega_{max} > 0$ .

The target may be stationary or moving in any direction. We consider the moving target as another nonholonomic mobile robot and represent its position and orientation with  $P_T = (X_T, Y_T, \theta_T)$ , which has the same kinematic equation as (1) with  $(V_T, \omega_T)$  as linear and angular velocities, respectively. No information of the target motion or environment is available. We assume the following conditions are satisfied:

- The robot is faster than target
- The robot has a higher level of maneuverability than target
- The target path is smooth
- The robot is equipped with range sensors, which detect the distance from the robot to obstacles if they are in the detectable area

The only available information from the target is the LOS-range or the relative distance between the robot and target.

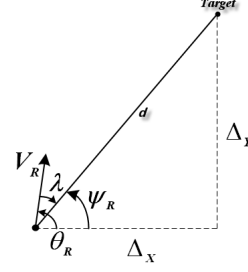


Fig. 1. Robot position and orientation with respect to target

The objective is to design a collision-free guidance law in relative coordinates, which allows the robot to approach the stationary target or follow a maneuvering target object, keeping a certain distance from it and simultaneously avoiding the obstacles on the way. We chose the relative coordinate system due to its simplicity in calculations and also nonnecessity to other sensors to measure the robot or target absolute positions.

## III. TRACKING PROBLEM

Given the robot position and orientation with respect to a stationary target position  $T$  in the polar coordination system, we define the relative distance between the robot and target,  $d$ , and the angle between the front-direction and the target direction,  $\lambda$ , as shown in fig. (1)

$$\begin{aligned}d &= \sqrt{\Delta_X^2 + \Delta_Y^2} \\ \lambda &= \psi_R - \theta_R\end{aligned}\quad (2)$$

where  $\theta_R$  is the robot's heading angle,  $\psi_R$  is the line-of-sight angle and  $|\lambda| \leq \pi$ . The robot motion is expressed by, [1]

$$\dot{d} = -V_R \cos(\lambda) \quad (3a)$$

$$\dot{\lambda} = -\omega_R + \frac{V_R}{d} \sin(\lambda) \quad (3b)$$

Note that the kinematic equations (3) are only valid for non-zero values of the LOS-range, since  $\lambda$  is undefined for  $d = 0$ .

Considering an obstacle-free environment, in the first step, we design a navigation law to guide the robot towards the target using the range-only information. In the second step, we modify the proposed algorithm to fulfil the mission in an unknown and cramped environment, i.e. approaching the target and simultaneously avoiding the obstacles.

To design a steering control law, having the LOS-range at each time step, we note that the target can be in any position on a circle centered on the robot position with the radius  $d$ . With constant robot's linear velocity, considering (3a), the angle  $\lambda$  has the main role to guide the robot towards the target. Since  $d$  is the only available information, we use the LOS-range variation as a measure for the angle at which the robot approaches the target. With a fixed  $\dot{d}$  smaller than  $V_R$ , given (3a), the robot approaches the target with a fixed  $\lambda = \lambda_o$  along the trajectory, where  $0 < |\lambda_o| < \frac{\pi}{2}$ . We propose

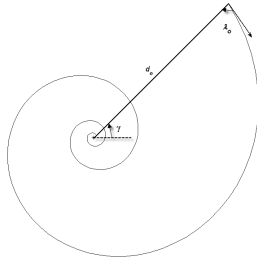


Fig. 2. An equiangular spiral

a new steering control law which drives the robot towards the target in a semi-equiangular spiral.

#### IV. EQUIANGULAR NAVIGATION GUIDANCE (ENG)

ENG has derived from geometry of robot movement combined with the kinematic equation of robot-target tracking system. Considering the target at the origin, the equiangular spiral is a spiral whose polar equation is given by

$$d = d_o e^{-b\gamma} \quad (4)$$

where  $d$  is the LOS-range and  $d_o$  is the initial LOS-range.  $b = \cot(\lambda_o)$ , where  $0 < |\lambda_o| < \frac{\pi}{2}$  is the approaching angle, and  $\gamma$  is the angle between the x-axis and the vector starts at the target position pointing to the robot position, shown in fig. (2)(see e.g. [9]). As  $\lambda_o \rightarrow \frac{\pi}{2}$ ,  $b \rightarrow 0$  and as a result the spiral approaches a circle.

With constant robot linear velocity, the LOS-range derivative can be considered as a measure for the angle  $\lambda_R$  at which the robot approaches the target. To approach a stationary target,  $\dot{d}$  should be negative and using (3a) we obtain  $|\lambda_R| < \frac{\pi}{2}$ .

The idea is to approach the target with a fixed  $\lambda_R = \lambda_o$  along an equiangular spiral where  $|\lambda_o| < \frac{\pi}{2}$ . Having fixed the angle  $\lambda_R$ , from (3a) the value of  $\dot{d}$  fluctuates around a positive constant  $L = V_R \cos(\lambda_o)$ , which is bounded to robot's linear velocity.

On the other hand, to prevent any dangerous settings of the controls, which would break the gears of the vehicle, the low level motor controllers apply a constraint on robot's angular velocity as  $|\omega| \leq \omega_{max}$ . So, given the maximum turn rate for the robot and based on the argument above the ENG's steering control would be a bang-bang solution, switching between minimum and maximum values of  $\omega$ . Equiangular Navigation Guidance (ENG), considering the dynamic constraints of the mobile robot, is introduced as follows:

$$\omega_R = -\omega_{max} \text{sgn}(L + \dot{d}) \quad (5)$$

where  $0 < L < V_R$  and  $\text{sgn}(\cdot) = +1, 0$  or  $-1$  according as the expression contained in brackets is positive, zero or negative, respectively.

*Remarks:* Due to the symmetry properties, the steering control law

$$\omega_R = \omega_{max} \text{sgn}(L + \dot{d}) \quad (6)$$

has similar performance and characteristics as control law (5).

#### A. Following A Maneuvering Target

For a moving target, we bound the positive constant  $L$  to difference of the robot and target linear velocities,  $L < (V_R - V_T)$ . As it mentioned before, along the trajectory  $\alpha < |\lambda|$  and  $V_R > V_T$ . As a result, with a constant linear velocity the robot inevitably goes into a circular trajectory while following a moving target.

The result is acceptable in applications like Unmanned Aerial Vehicles (UAVs) navigation, in which the vehicle's velocity should not goes below the stall speed to keep the altitude constant. However, in other applications like trajectory tracking or target following by a WMR, it is more desirable that the robot decreases the linear velocity to that of the target and follows it in a smooth trajectory, while preserving a certain distance  $d_t$  from the target. Given  $d_t$  less than one meter, we define the robot velocity as a function of LOS-range as follows

$$V_R = \begin{cases} V_{max} & d > 1 \\ \frac{V_{max}}{(1-d_t)^2} (d - d_t)^2 & d_t < d \leq 1 \\ 0 & d \leq d_t \end{cases} \quad (7)$$

The robot approaches the target with the maximum velocity. When the relative distance between the robot and target is less than one meter, the robot reduces the speed in order to avoid circling. To preserve the certain distance, the speed of the robot converges to zero as the LOS-range tends to  $d_t$ . Since the robot velocity decreases as it approaches the target, to have a smooth tracking with a constant approaching angle  $\lambda_o$ ,  $L$  should change continuously with  $V_R$ . We have,

$$L = 0.95(V_R - V_T) \quad (8)$$

#### V. CHARACTERISTICS OF THE GUIDANCE LOGIC

In this section, we discuss some properties of ENG, which will be used to extend the guidance law, in order to allowing the robot to approach the target in a populated environment.

#### A. Semi-equiangular spiral trajectory

Since the value of  $\lambda$  is nearly constant along the trajectory and equal to  $|\lambda_o| \approx \arccos(\frac{L}{V_R})$ , the robot trajectory towards the target is a semi-equiangular spiral. The parameter  $b$  and hence, the arc length and the curvature change with  $L$  where  $0 < L < V_R$ . Different paths can be generated for different values of  $L$  and since it is real, an infinite number of paths is possible. As  $L \rightarrow 0$ ,  $\lambda_o \rightarrow \frac{\pi}{2}$  and as a result  $b \rightarrow 0$ , the robot path becomes more curved and spiral approaches a circle. Fig.(3) shows the robot trajectory with different values of  $L$  when it goes towards a stationary target. As you see, with smaller values of  $L$ , the robot trajectory converges to a circle and as  $L \rightarrow V_R$ ,  $\lambda \rightarrow 0$  and the trajectory converges to a straight line.

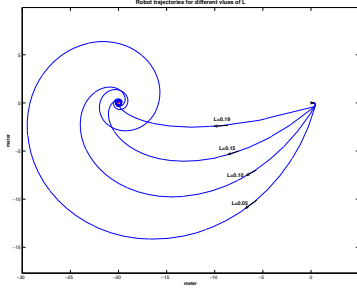


Fig. 3. Moving towards a stationary target with different values of  $L$

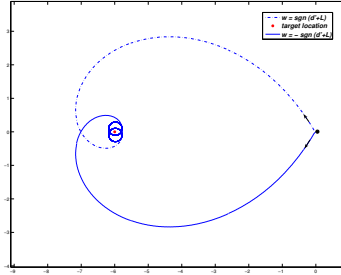


Fig. 4. Symmetry of the two trajectories

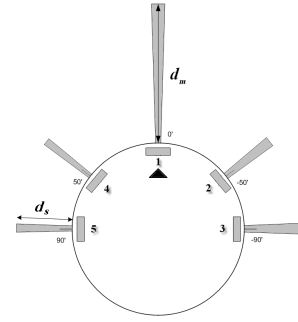


Fig. 5. Mobile robot range sensors topology

### B. Symmetry of the trajectories

Applying the proposed steering controls (5) and (6), the robot approaches the target in symmetric trajectories. Having applied (5), the robot approaches the target with a negative approaching angle  $\lambda_o$  and the spiral's turning direction is clockwise. While,  $\lambda_o$  is positive with (6) and the spiral turns counterclockwise. Applying (5) and (6) with the same control parameter  $L$ , the resulted trajectories shown in fig.(4). The current property, can be used to change the robot moving direction, in the beginning or in the middle of the experiment, when there is an obstacle on the way of robot towards the target.

## VI. AUGMENTED EQUIANGULAR NAVIGATION GUIDANCE (AENG)

Navigation of a mobile robot towards a stationary or maneuvering target in the presence of obstacles is more challenging and difficult. The robot has to operate in a real and unprepared environment without any priori information about the position of obstacles or their geometric distribution. We assume the robot is equipped with range sensors, sonar or IR sensors, which detect the distance from the robot to obstacles if they are in the detectable area. Fig. (5) shows the physical disposition and numeration of sensors in the ring.

Since the spiral turning direction is clockwise for (5) and counterclockwise for (6), to avoid the obstacles we use two different sets of sensors, each of which consist of three range finders; sensors 1, 2 and 3 with control law (5) and 1, 4 and 5 with (6).

In Augmented-ENG (AENG), the robot has the ability to change the trajectory in realtime to a more curved path in order to detour an obstacle as soon as the obstacle appears on the way. The first range finder, which is used for both control laws, has the role to change the robot moving direction, as soon as it detects an obstacle in the detectable zone and the other two in each set, preserve the safety distance from the obstacles.

In order to detour the obstacles, ENG's steering control law is slightly modified using the fact that reducing  $L$  forces the robot to travel along a more curved trajectory towards the target. As the robot approaches an obstacle, the relative distance between the robot and obstacle in the moving

direction  $d_o$  decreases and  $\dot{d}_o$  is negative. Adding  $L_o = \dot{d}_o$  to  $L$ , we obtain a steering control with a smaller value of  $L$ , which forces the robot to continue along a more curved path and as a result bypass the obstacle. We have

$$\omega_R = \pm \omega_{max} \text{sgn}(\dot{d} + L + L_o) \quad (9)$$

where

$$L_o = \begin{cases} \dot{d}_o & \text{if } d_o < d_m \\ 0 & \text{Otherwise} \end{cases} \quad (10)$$

where  $d_m$  is the maximum range of detectable area. When the front range finder detects an obstacle,  $L_o$  is added to  $L$ . Since  $L_o$  is negative as the robot approaches an obstacle, the approaching angle  $\lambda_o$  is increased,  $b \rightarrow 0$  and the spiral approaches a circle. The robot continues changing the direction until there is no obstacle in the moving direction. Continuing along this trajectory, which is tangent to obstacle's circumference, the robot will collide with obstacle at the intersection point.

In order to prevent any collision, depend on the robot's physical characteristics, a safety margin  $d_s$  is preserved to obstacles on each side. We consider  $d_s = 2D_p$ , where  $D_p$  is the robot's platform diameter. To hold  $d_s$ , we use two pairs of range finders, one in  $\pm 50^\circ$  to the front direction, sensors 4 and 2, and the other in  $\pm 90^\circ$  to the front direction of the mobile robot, sensors 5 and 3. Using (5), since the spiral turning direction is clockwise and the robot confronts the obstacles with the right side, sensors 2 and 3 are used to keep the safety margin. Similarly, we use 4 and 5 for the steering control (6). AENG is represented by pseudo code, shown in table (I).

In table (I),  $S_i$  is the measured range via  $i^{th}$  range finder,  $\sigma = -1$  for (5) and  $\sigma = 1$  for steering control (6). In case of detection of an obstacle on one side of the robot, while the robot approaches the target in AENG, it will turn to the other side to preserve the safety distance. As a result, the robot continues moving towards the target and at the same time holds the safety margin with obstacles. Using more range finders at more angular positions or switching between (5) and (6) at specific situations throughout the experiment, can make the algorithm even stronger in the presence of more challenging environments. Having applied the AENG for both offline and online navigation, in the next section,

TABLE I  
PSEUDO CODE FOR AUGMENTED-ENG

IF	$(d_o < d_m) \text{ and } (d_o < d)$	THEN
	$L_o = \dot{d}_o$	
ELSE		
	$L_o = 0$	
END		
IF	$(S_2 \text{ or } S_3) < 2D_p$	THEN
	$L_o = -L - \dot{d} + \text{sgn}(\sigma)$	
END		
IF	$(S_4 \text{ or } S_5) < 2D_p$	THEN
	$L_o = -L - \dot{d} - \text{sgn}(\sigma)$	
END		
	$\omega_R = \sigma \omega_{max} \text{sgn}(\dot{d} + L + L_o)$	

TABLE II  
SIMULATION PARAMETERS I

Parameter	Value	Comments
$t_s$	0.1s	Sampling Intervals
$V_R$	0.5 m/s	Robot linear velocity
$\omega_{max}$	1 rad/s	Maximum angular velocity
$D_p$	0.5 m	Robot platform diameter
$x_0$	(0, 0, $\pi$ )	Initial robot posture
$T$	(0, -20m)	Target position

we will show how the robot can find the way towards the target in different situations.

## VII. SIMULATION RESULTS

To study the performance and characteristics of AENG, we simulate a mobile robot moving in an unprepared environment. We apply the proposed algorithm for both stationary and maneuvering targets and in online and offline navigation. Simulation parameters throughout the experiment shown in Table II. For simulation purposes and testing we used Mobotsim 1.0 simulator, which is a 2D easy to use graphical mobile robot simulator [12].

### A. Approaching a stationary target

1) **Offline Robot Navigation:** In offline robot navigation, the position of target and obstacles are given. In order to guide the robot towards the target and at the same time avoid the obstacles on the way, the sign of  $\sigma$  and proper value of  $L$  are chosen offline. Fig. (6) shows different robot trajectories towards the target with different values of  $L$  for steering laws (5) and (6).

Although offline navigation is applicable in some situations, it seems not feasible and challenging in practice for many other applications, in which the position of target and obstacles are unknown.

2) **Online Robot Navigation:** In online navigation, the only available information from the target is the LOS-range  $d$ . The robot is equipped with some range finders who give the relative distance between the robot and obstacles which are inside the detectable area. In this strategy, the value

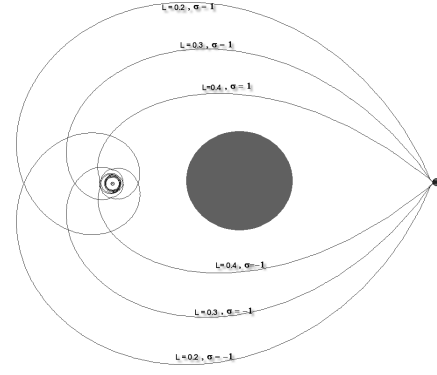


Fig. 6. Offline collision-free robot navigation towards the stationary target

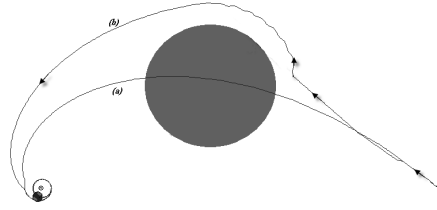


Fig. 7. Online robot navigation (a). robot trajectory without obstacle, (b). robot trajectory in the presence of obstacle

of  $L$  is fixed during the experiment. To have a better or smarter collision-free guidance, the sign of  $\sigma$  can change in some predefined conditions throughout the experiment. However, for the sake of simplicity, we fix  $\sigma$  to 1 and consider  $L = 0.35$ . When the first range finder detects an obstacle on the way towards the target, in online navigation, the robot changes the trajectory and continues along the line tangent to obstacle, shown in fig. (7). As soon as the relative distance between the robot and target is less than  $d_s$ , the robot turns to the other side while ENG tries to keep the robot on the trajectory towards the target. Hence, the robot bypasses the obstacle and turns around it with the minimum distance defined as safety margin.

Fig. (8) shows a U shape obstacle, in which the potential field method fails to find a trajectory and the problem of local minima happens, [8].

### B. Following a maneuvering target

In this experiment, the robot is supposed to follow a moving target with a smaller linear velocity and lower maneuverability. The target initially moves straight with a constant linear velocity  $V_T = 0.2 \text{ m/s}$  and starts maneuvering after a while with:

$$\omega_T = \begin{cases} 0 & t < 250 \\ 0.05 \cos(0.005t + 1) & t > 250 \end{cases}$$

Having applied the AENG with constant robot linear velocity and  $L = 0.25$ , the robot and target trajectories are shown in fig. (9(a)). The robot approaches the target while avoiding the obstacles on the way. Since the robot velocity is constant

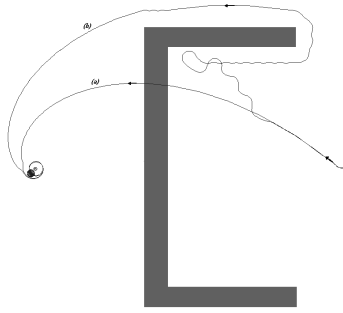


Fig. 8. Online robot navigation (a). robot trajectory without obstacle, (b). robot trajectory in the presence of obstacle

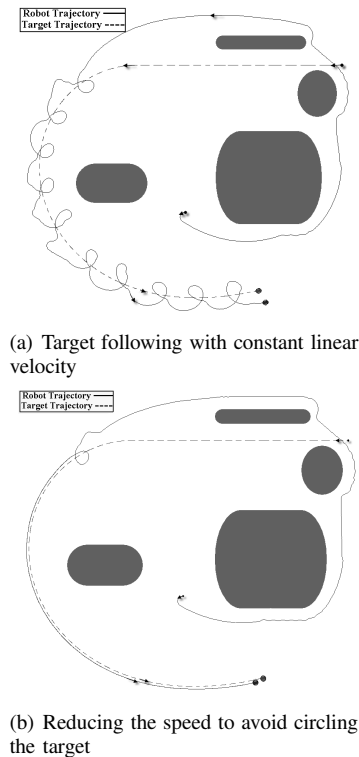


Fig. 9. Online maneuvering target following using AENG

and bigger than target speed, it goes into a circular trajectory around the target while following the target. To avoid circling the target, we take into account (7) and (8) in order to reduce the robot speed to that of the target as the robot goes towards it. Applying the AENG with adjustable speed, the robot moves towards the target with maximum velocity. When the relative distance between the robot and target is less than one meter, it reduces the speed and follows the target in a smoother trajectory, shown in fig. (9(b)).

### VIII. CONCLUSION

The problem of WMR navigation towards an unknown stationary or maneuvering target based on just the relative distance between the robot and target has been considered and a new guidance law- Equiangular Navigation Guidance (ENG)- has been proposed. Having applied ENG, the robot

approaches the target in a semi-equiangular spiral, whose approaching angle and consequently arrival time to the target can be adjusted using a control parameter, and eventually goes into a circular trajectory around the target. In an unknown and cramped environment, using the sensory information, the Augmented-ENG enables the robot to approach the target and simultaneously avoids the obstacles on the way. Apart from simplicity and ease of use for realtime applications, ENG is also applicable for other nonholonomic vehicles which in specific situations have the same kinematic equation as WMR, such as UAVs and space robots.

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