2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

High-Order Design of Adaptive Inverses for Signal-Dependent Actuator Nonlinearities

Dipankar Deb, Gang Tao, Jason O. Burkholder

Abstract— **In this paper, an adaptive compensation scheme is presented for controlling signal-dependent synthetic jet actuator nonlinearities, using higher order approximation of twin approximators for effective adaptive inversion of actuator nonlinearities. Approximation of a nonlinearly parametrized actuator model by a linearly parametrized function is performed. Adaptive inversion of actuator nonlinearities is implemented by another approximator in the feedforward path. The network reconstruction error is reduced in principle with usage of a second-order approximation, compared to a first-order approximation. A nonlinear state feedback control law is designed for controlling a nonlinear dynamic system. Parameter projection based adaptive laws ensure desired closed-loop stability.**

Keywords: Actuator nonlinearity, adaptive inverse, nonlinear aircraft dynamics, stability, synthetic jet actuators.

I. Introduction

The characteristics of synthetic jet actuators used for aircraft flight control are highly nonlinear and can change significantly with the aircraft's angle of attack [1], [2], [3]. An adaptive inversion approach [9] has been used for compensation of actuator nonlinearities, in controlling a dynamic system. The design of an adaptive inverse needs to be further developed for synthetic jet actuator nonlinearities, to handle their complex characteristics and signal-dependence and to improve the nonlinearity inversion accuracy.

Selmic et al. [8] have addressed the problem of compensating a nonparametrizable deadzone-like actuator nonlinearity, by using two coupled neural networks which are tuned such that an effective inversion is adaptively achievable. To handle signal-dependent actuator nonlinearities, an adaptive nonlinearity inverse needs to be designed with networks with increased dimensions [3]. Higher dimensional network based adaptive inverse schemes need enhancement for improved inversion accuracy, which will be studied in this paper. This work is motivated from the need of such an inverse to handle the intrinsically complicated nonlinear characteristics of actuators, for applications including aircraft flight control using synthetic jet actuators. Our objective is to develop a new adaptive inverse design with enhanced inversion accuracy.

As demonstrated in [1], [2], the input-output relationship of a synthetic jet significantly depends on the angle of attack α of the aircraft wing, with the input to the actuator being $v(t)$, that is, we observe a state dependent actuator control problem with an actuator nonlinearity function $u = f(v(t), \alpha)$ proceeded at the input of a dynamic system $\dot{x} = f_0(x) + g(x)u$. Such a nonlinear function $f(v(t), \alpha)$ is difficult to model. Approximately, for an actuator function $f_1(v)$ at a low angle ($\alpha \approx 3°$) of attack and an actuator function $f_2(v)$ at a high angle ($\alpha \approx 24°$) of attack, the actuator nonlinearity function $f(v, \alpha)$ may be expressed as

$$f(v, \alpha) = a(\alpha) f_1(v) + b(\alpha) f_2(v), \tag{1}$$

where $a(\alpha)$ and $b(\alpha)$ are certain functions that determine the dependency of the actuator nonlinearity on the signal α, and $f_1(v)$ and $f_2(v)$ are static nonlinearity functions dependent on the actuator input v. Such a signal-dependent actuator nonlinearity can be compensated for by using enhanced designs of the inverse scheme of [8].

We will develop an approximation based inverse for a signal-dependent actuator nonlinearity. There are several approaches for approximation available in literature, including neural network based models, fuzzy logic approximators, wavelets and splines. A neural network based design is used for demonstration and is based on the work by Selmic et al. [8]. The synthetic jet actuator application is studied for illustration of research motivation and demonstration of our new adaptive inverse control design.

The two main contributions of this work are:

- A new and improved higher-order inverse parametrization scheme in presence of signal dependent actuator nonlinearities, thereby improving accuracy of approximation, is presented.
- An application of this scheme to synthetic jet actuation in aircraft fight control systems, is presented.

The paper is organized as follows. In Section II, an adaptive compensation scheme for signal-dependent actuator nonlinearities is developed, using a high-order approximation based inverse. In Section III, we present a benchmark study of an adaptive inverse based feedback control scheme, by combining it with a feedback control law applied to a nonlinear aircraft dynamic model with synthetic jet actuators. In Section IV, we develop an adaptation scheme for parameter updates of the coupled nonlinearity approximation functions used to build the adaptive nonlinearity inverse, and show the closed-loop system stability and tracking properties.

Dipankar Deb and Gang Tao are with Charles L. Brown Department of Electrical and Computer Engineering, University of Virginia, Charlottesville, VA, 22904, (email: dd8v, gt9s@virginia.edu)

Jason O. Burkholder is with Barron Associates, Inc., 1410 Sachem Place, Suite 202, Charlottesville, VA 22901 (email: burkholder@bainet.com)

This research was partially supported by the US Air Force Grant FA9550-04-C-0065 and the NSF grant ECS0601475.

II. **Higher Order Parametrization of Inverse Schemes**

In this section, we present a modified parametrization scheme for adaptive inversion of signal dependent actuator nonlinearities, using second-order terms of the Taylor series functional expansion of the approximating functions.

As a motivating example, we consider a synthetic jet actuator nonlinearity characteristic $u = N(v, \alpha)$ which depends on the angle of attack α, and the actuator input v, where u is the lift coefficient (force) on airfoil surface.

It is well-known that approximators such as RBF networks or feedforward-type neural networks (NNs) and fuzzy logic approximators have a powerful *universal approximation* property, that is, for any continuous function $f(v,\alpha), \forall\ (v,\alpha) \in S \subset R^2$, one has

$$u = N(v,\alpha) = f(v,\alpha) = W^T\phi(v,\alpha) + \epsilon_1(v,\alpha),$$

where $\phi(v,\alpha)$ is a basis vector, W is a constant weighting vector (for $u \in R$) and $\epsilon_1(v,\alpha)$ is the approximation error.

As a demonstrative tool of this new parametrization scheme, we use a neural network approximator with the basis $\phi(v,\alpha) = \sigma(V^T x_1 + v_0)$, and the actuator output is

$$u = W^T\sigma(V^T x_1 + v_0) + \epsilon_1(v,\alpha), \tag{2}$$

where $x_1 = [v,\alpha]^T$, and $\sigma(\cdot) \in R^L$ is a hidden layer activation vector, and V and v_0 are the first-layer weighting matrix and vector of the neural network [7], [8].

The functional approximation properties of neural networks is used to convert an unknown nonlinear function into a set of unknown constant parameters W and V, and a bounded disturbance $\epsilon_1(v,\alpha)$. A neural network of this type is capable of approximating any smooth function to any desired accuracy. Note that W and V indicate constant parameter values that minimize $\epsilon_1(v,\alpha)$.

Using this desirable property, we propose to employ a neural network to estimate the characteristic $u = N(v,\alpha)$:

$$\hat{u} = \hat{W}^T\sigma(V^T x_1 + v_0), \tag{3}$$

where $\hat{W}$ is an estimate of the ideal NN weight W.

This neural network is used as an actuator nonlinearity estimator or observer. The output of this neural network will be used for tuning a second neural network which will be used as the nonlinearity compensator.

For the inverse characteristic $v = NI(u_d,\alpha)$ such that

$$u = N(NI(u_d,\alpha),\alpha) = u_d, \tag{4}$$

where u_d is the desired input signal, we use another neural network $W_i^T\sigma_i(V_i^T x_2 + v_{0i})$ which acts as the compensator, to estimate the function u_{dNN} given by

$$\begin{aligned} u_{dNN} &\triangleq NI(u_d,\alpha) - u_d \\ &= W_i^T\sigma_i(V_i^T x_2 + v_{0i}) + \epsilon_2(u_d,\alpha), \end{aligned} \tag{5}$$

where $\sigma_i(\cdot) \in R^{L_i}$ is a hidden layer activation vector, V_i and v_{0i} are the basic NN elements (a matrix and a vector), $x_2 = [u_d,\alpha]^T$ and $\epsilon_2(u_d,\alpha)$ is the network approximation error. Then, the estimate of u_{dNN} is

$$\hat{u}_{dNN} = \hat{W}_i^T\sigma_i(V_i^T x_2 + v_{0i}), \tag{6}$$

where $\hat{W}_i$ is an estimate of the NN weighting vector W_i. The weights V and V_i are chosen based on the activation functions σ and σ_i, and they are kept fixed.

An estimation scheme for nonsmooth actuator nonlinearities with first order parametrization is presented in [8]. This paper extends that work to higher order parametrization and approximation in presence of signal dependent nonlinearities.

From (2) and (4) we get

$$u_d = W^T\sigma\left(V^T\left[u_{dNN} + u_d, \alpha\right]^T + v_0\right) + \epsilon_1(v,\alpha). \tag{7}$$

Using (5) and (6), we get

$$\begin{aligned} u_d \;=\; & W^T\sigma\Big(V^T\left[\tilde{W}_i^T\sigma_i(V_i^T x_2 + v_{0i}) + \epsilon_2(u_d,\alpha), 0\right]^T \\ & +V^T\left[x_2 + [\hat{u}_{dNN}, 0]^T\right] + v_0\Big) + \epsilon_1(v,\alpha), \end{aligned} \tag{8}$$

where $\tilde{W}_i = W_i - \hat{W}_i$ is the weight parameter error.

A. **New Inverse Parametrization**

Next, we describe the effectiveness of this coupled neural network nonlinearity approximation by developing a control error expression. This expression is critical in developing adaptive update laws for the parameter estimates. For developing control error, we assume that

(A1) the ideal weights W and W_i are bounded such that $\|W\|_F \le W_M$ and $\|W_i\|_F \le W_{iM}$ with W_M and W_{iM} known bounds.

The key to extending theory developed for linear-in parameter functions [9] to neural networks with one hidden layer involves a Taylor series expansion of the hidden-layer output. Using Taylor series expansion of u_d in (8) for W_i at $\hat{W}_i$ up to the second order (for ϵ_2 at 0 nominally), we get

$$\begin{aligned} u_d \;=\; & W^T\sigma\left(V^T\left(x_2 + [\hat{u}_{dNN},0]^T\right) + v_0\right) + \epsilon_1(v,\alpha) \\ & +W^T\sigma'\left(V^T\left(x_2 + [\hat{u}_{dNN},0]^T\right) + v_0\right)V^T \\ & \cdot\left[\tilde{W}_i^T\sigma_i\left(V_i^T x_2 + v_{0i}\right) + \epsilon_2(u_d,\alpha), 0\right]^T \\ & +\frac{1}{2}W^T\sigma''\left(V^T\left(x_2 + [\hat{u}_{dNN},0]^T\right) + v_0\right) \\ & \cdot\left[\tilde{W}_i^T\sigma_i\left(V_i^T x_2 + v_{0i}\right) + \epsilon_2(u_d,\alpha), 0\right]VV^T \\ & \cdot\left[\tilde{W}_i^T\sigma_i\left(V_i^T x_2 + v_{0i}\right) + \epsilon_2(u_d,\alpha), 0\right]^T \\ & +W^T R_2\left(\tilde{W}_i, u_d, \alpha\right), \end{aligned} \tag{9}$$

where $\tilde{W} = W - \hat{W}$ is a weight estimation error, and $W^T R_2\left(\tilde{W}_i, u_d, \alpha\right)$ is the remainder of the second Taylor polynomial and $\sigma'(z)$ and $\sigma''(z)$ is the first and second derivatives of $\sigma(z) = [\sigma_1(z_1), \ldots, \sigma_L(z_L)]^T$ with respect to $z = [z_1, \ldots, z_L]^T$ respectively.

Remark 2.1: The higher order approximation naturally provides a more accurate approximation compared to the first order approximation presented in [8]. In fact, it can be shown that for higher order approximation even beyond second order, the control error can be expressed as a linear parametrization of weighting errors $\tilde{W}$ and $\tilde{W}_i$, with the model mismatch term accounting for the higher order components. The norm of this model mismatch term, in principle, is lower for higher order approximations.

The estimate on the remainder of the second Taylor polynomial describes how far the approximation is from the true function. Since we use the second Taylor polynomial, the reminder term is only degree 3 and satisfy the inequality.

Since $x_1 = x_2 + [\hat{u}_{dNN}, 0]^T$, using (2), we get

$$u = W^T \sigma \left(V^T \left(x_2 + [\hat{u}_{dNN}, 0]^T\right) + v_0\right) + \epsilon_1(v, \alpha). \quad (10)$$

Substituting (10) into (9), we get

$$\begin{aligned} u - u_d \;=\; & -W^T \sigma' \left(V^T x_1 + v_0\right) V^T \left[\epsilon_2(u_d, \alpha), 0\right]^T \\ & -W^T \sigma' \left(V^T x_1 + v_0\right) V^T \Big[\tilde{W}_i^T \sigma_i \left(V_i^T x_2\right. \\ & \left.+ v_{0i}\right), 0\Big]^T - \frac{1}{2} W^T \sigma'' \left(V^T x_1 + v_0\right) \\ & \cdot \left[\tilde{W}_i^T \sigma_i \left(V_i^T x_2 + v_{0i}\right) + \epsilon_2(u_d, \alpha), 0\right] V V^T \\ & \cdot \left[\tilde{W}_i^T \sigma_i \left(V_i^T x_2 + v_{0i}\right) + \epsilon_2(u_d, \alpha), 0\right]^T \\ & -W^T R_2 \left(\tilde{W}_i, u_d, \alpha\right). \end{aligned} \quad (11)$$

Next, utilizing the weighting errors $\tilde{W} = W - \hat{W}$, $\tilde{W}_i = W_i - \hat{W}_i$, we express (11) such that it is composed of terms which are linear in $\tilde{W}$ and $\tilde{W}_i$, and the model mismatch term ϵ. The model mismatch term is composed of the nonlinear terms in $\tilde{W}$ and $\tilde{W}_i$, and terms with $\epsilon(u_d, \alpha)$ components. Finally, the control error $u - u_d$ is given by

$$\begin{aligned} u - u_d \;=\; & \tilde{W}^T \sigma' \left(V^T x_1 + v_0\right) V^T \left[\hat{u}_{dNN}, 0\right]^T - \hat{W}^T \\ & \cdot \sigma' (V^T x_1 + v_0) V^T \left[\tilde{W}_i^T \sigma_i \left(V_i^T x_2 + v_{0i}\right), 0\right]^T \\ & -\frac{1}{2} \tilde{W}^T \sigma'' (V^T x_1 + v_0) \left[\hat{u}_{dNN}, 0\right] V V^T \left[\hat{u}_{dNN}, 0\right]^T \\ & +\frac{1}{2} \hat{W}^T \sigma'' (V^T x_1 + v_0) V^T \left[\hat{u}_{dNN}, 0\right]^T \\ & \cdot \left[\tilde{W}_i^T \sigma_i \left(V_i^T x_2 + v_{0i}\right), 0\right] V + \epsilon \\ =\; & -\hat{W}^T \Psi \left[\tilde{W}_i^T \sigma_i \left(V_i^T x_2 + v_{0i}\right), 0\right]^T \\ & +\tilde{W}^T \Psi \left[\hat{u}_{dNN}, 0\right]^T + \epsilon, \end{aligned} \quad (12)$$

where Ψ is given by

$$\Psi = \left(\sigma' \left(V^T x_1 + v_0\right) - \frac{1}{2} \sigma'' (V^T x_1 + v_0) [\hat{u}_{dNN}, 0] \, V\right) V^T,$$

and ϵ defined as the modeling mismatch error with a desirable bounded norm is given by

$$\begin{aligned} \epsilon \;=\; & -W^T \sigma' \left(V^T x_1 + v_0\right) V^T \left[\epsilon_2(u_d, \alpha), 0\right]^T \\ & -\tilde{W}^T \sigma' \left(V^T x_1 + v_0\right) V^T \left[W_i^T \sigma_i \left(V_i^T x_2 + v_{0i}\right), 0\right]^T \\ & -\frac{1}{2} W^T \sigma'' \left(V^T x_1 + v_0\right) V^T \left[\epsilon_2^2(u_d, \alpha), 0\right]^T [1, 0] \, V \\ & -W^T \sigma'' \left(V^T x_1 + v_0\right) V^T [1, 0]^T \\ & \cdot \left[\tilde{W}_i^T \sigma_i \left(V_i^T x_2 + v_{0i}\right) \epsilon_2(u_d, \alpha), 0\right] V - W^T \\ & \cdot \sigma'' \left(V^T x_1 + v_0\right) \left[\tilde{W}_i^T \sigma_i \left(V_i^T x_2 + v_{0i}\right), 0\right] V V^T \\ & \cdot \left[W_i^T \sigma_i \left(V_i^T x_2 + v_{0i}\right), 0\right]^T + \frac{1}{2} \tilde{W}^T \sigma'' \left(V^T x_1 + v_0\right) \\ & \cdot V^T \left[\hat{u}_{dNN}, 0\right]^T \left[W_i^T \sigma_i \left(V_i^T x_2 + v_{0i}\right), 0\right] V \\ & -W^T R_2 \left(\tilde{W}_i, u_d, \alpha\right). \end{aligned} \quad (13)$$

Remark 2.2: The important features of (12) are: (i) it contains the second-order approximation terms related to σ'', and (ii) it has a parametrization linear in terms of the neural network weight errors $\tilde{W}_i^T$, $\tilde{W}^T$, which are desirable for the adaptive inversion design. Unlike that with a first-order approximation seen in the literature, the inclusion of the second-order terms related to σ'' has the potential to increase the approximation accuracy. Moreover, higher-order approximations can be similarly developed.

B. Approximation Error Analysis

It is important to note that the choice of W, V, v_0 and v_{0i} is based on the space $x = [v, u_d, \alpha]^T$. The term $R_2\left(\tilde{W}_i, u_d, \alpha\right)$ is bounded as

$$\begin{aligned} \|R_2(\cdot)\|_F \;\le\; & \frac{1}{6} \|\sigma'''(\cdot)\| \|V\|_F^3 \Big(\|\tilde{W}_i\|_F^3 \|\sigma_i(\cdot)\|^3 \\ + \; & \|\epsilon_2(u_d, \alpha)\|^3 + 3 \|\tilde{W}_i\|_F^2 \|\sigma_i(\cdot)\|^2 \|\epsilon_2(u_d, \alpha)\| \\ & + 3 \|\tilde{W}_i\|_F \|\sigma_i(\cdot)\| \|\epsilon_2(u_d, \alpha)\|^2 \Big), \end{aligned} \quad (14)$$

where $\sigma'''(x)$ is the third derivative of $\sigma(x)$.

Using (13) and (14), the model mismatch error $\epsilon(t)$ satisfies the following condition

$$\begin{aligned} |\epsilon| \;\le\; & \|W\|_F \|\sigma'(\cdot)\| \|V\|_F \|\epsilon_2(u_d, \alpha)\| \\ & + \|\tilde{W}\|_F \|\sigma'(\cdot)\| \|V\|_F \|W_i\|_F \|\sigma_i(\cdot)\| \\ & + \frac{1}{2} \|W\|_F \|\sigma''(\cdot)\| \|V\|_F^2 \|\epsilon_2(u_d, \alpha)\|^2 \\ & + \|W\|_F \|\sigma''(\cdot)\| \|V\|_F^2 \|\tilde{W}_i\|_F \|\sigma_i(\cdot)\| \|\epsilon_2(u_d, \alpha)\| \\ & + \|W\|_F \|\sigma''(\cdot)\| \|V\|_F^2 \|W_i\|_F \|\sigma_i(\cdot)\|^2 \|\tilde{W}_i\|_F \\ & + \frac{1}{2} \|\tilde{W}\|_F \|\sigma''(\cdot)\| \|V\|_F^2 \|\hat{W}_i\|_F \|\sigma_i(\cdot)\|^2 \|W_i\|_F \\ & + \frac{1}{6} \|W\|_F \|\sigma'''(\cdot)\| \|V\|_F^3 \Big(\|\tilde{W}_i\|_F^3 \|\sigma_i(\cdot)\|^3 \\ & + \|\epsilon_2(u_d, \alpha)\|^3 + 3 \|\tilde{W}_i\|_F^2 \|\sigma_i(\cdot)\|^2 \|\epsilon_2(u_d, \alpha)\| \\ & + 3 \|\tilde{W}_i\|_F \|\sigma_i(\cdot)\| \|\epsilon_2(u_d, \alpha)\|^2 \Big) \\ =\; & \bar{\delta}_1 \|\tilde{W}_i\|_F^3 + \bar{\delta}_2 \|\tilde{W}_i\|_F^2 + \bar{\delta}_3 \|\tilde{W}_i\|_F \\ & + \bar{\delta}_4 \|\tilde{W}_i\|_F \|\tilde{W}\|_F + \bar{\delta}_5 \|\tilde{W}\|_F + \bar{\delta}_6, \end{aligned} \quad (15)$$

where $\bar{\delta}_1, \ldots, \bar{\delta}_6$ are unknown bounded constants. Using assumption (A1) and $\|\tilde{W}\|_F \leq W_M + \|\hat{W}\|_F$, $\|\tilde{W}_i\|_F \leq W_{iM} + \|\hat{W}_i\|_F$, we get

$$\begin{aligned} |\epsilon| \ \leq\ & \delta_1 \|\hat{W}_i\|_F^3 + \delta_2 \|\hat{W}_i\|_F^2 + \delta_3 \|\hat{W}_i\|_F \\ & + \delta_4 \|\hat{W}_i\|_F \|\hat{W}\|_F + \delta_5 \|\hat{W}\|_F + \delta_6 = \beta^T \Omega, \end{aligned} \quad (16)$$

where $\beta = [\delta_1, \ldots, \delta_6]^T$ is a vector of bounded constants (not assumed to be known and are adaptively updated), and $\Omega = \left[\|\hat{W}_i\|_F^3, \|\hat{W}_i\|_F^2, \|\hat{W}_i\|_F, \|\hat{W}_i\|_F \|\hat{W}\|_F, \|\hat{W}\|_F, 1\right]^T$ is a known vector.

The vector β is bounded because $\|\epsilon_2(u_d, \alpha)\|$ is bounded for bounded α. We define an estimator for the bound of the model mismatch error $\beta^T \Omega$, as

$$\hat{\epsilon} = \hat{\beta}^T \Omega, \quad (17)$$

where $\hat{\beta} = [\hat{\delta}_1, \ldots, \hat{\delta}_6]^T$ is updated from parameter projection based adaptive law.

C. Adaptive Compensation based Feedback Control

As the estimates $\hat{W}$ and $\hat{W}_i$ approach the actual NN parameters W and W_i, the neural network precompensator effectively provides an inverse for the synthetic jet actuator nonlinearity. Section IV will demonstrate tuning methodology so that $\hat{W}$ and $\hat{W}_i$ are close to W and W_i. The adaptive control system error given by (12) is directly related to the approximation error and is crucial in guaranteeing closed-loop stability. Through the second-order Taylor expansion, the control error $u - u_d$ is expressed in a linearly parametrizable form with respect to the parameter errors $\tilde{W}$ and $\tilde{W}_i$.

Figure 1 describes a state feedback inverse control system where two approximators are used. The first approximator is used as an estimator of actuator nonlinearity, while the second is used as a compensator. The state feedback scheme and the aircraft pitch plane dynamics given by $\dot{x} = f_0(x) + g(x)u$ is presented in the next section. The actuator nonlinearity model $N(\cdot)$ and the approximators $\widehat{N}(\cdot)$ and $\widehat{NI}(\cdot)$ are given in (2), (3), and (6) respectively. The signal u_a is an additional input signal.

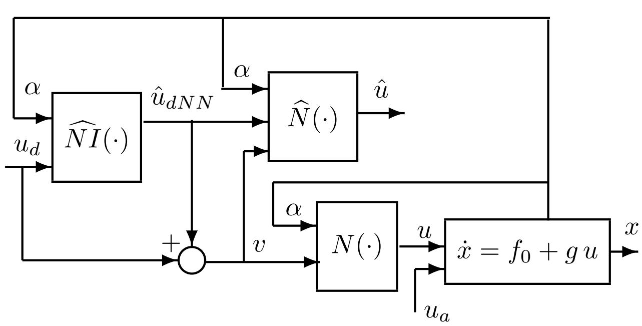

Fig. 1. Adaptive inverse compensation scheme.

III. A Benchmark Control System

In this section, synthetic jet application to state feedback adaptive control is studied for illustration of research motivation and demonstration of improved system performance. From a comparison of the low-angle-of-attack model[1] and the high-angle-of-attack model[2], we can conclude that the synthetic jet characteristic for a wide range of angles of attack inevitably depends on the angle of attack of the aircraft, and it is inherently nonlinear in nature. That is, in addition to the applied input voltage, the actuator nonlinearity characteristics change significantly with the angle of attack.

Our goal is to develop an adaptive inverse compensation scheme which is applicable at a wide range of angles of attack. We present a new state feedback adaptive inverse control scheme with two higher order approximators to cancel $N(\theta^*; v, \alpha)$. The most likely platforms for application of this technology are the new generation of stealth aircraft designed which lack vertical surfaces.

The states of the nonlinear system are

$$x = \left[\begin{array}{cccc} V_a & \gamma & \alpha & q \end{array}\right]^T, \quad (18)$$

where V_a is the magnitude of the aircraft velocity relative to the aircraft, γ is the flight-path angle which is assumed to be positive when the aircraft is climbing, q is the pitch angle, and α is the angle of attack.

Nonlinear Pitch-plane Dynamics

The nonlinear pitch-plane dynamics can be expressed as

$$\dot{V}_a = \frac{T}{m} \cos\alpha - g \sin\gamma, \quad (19a)$$

$$\dot{\gamma} = \frac{1}{mV_a} \left(u + T \sin\alpha - mg\cos\gamma\right), \quad (19b)$$

$$\dot{\alpha} = q - \frac{1}{mV_a} \left(u + T \sin\alpha - mg\cos\gamma\right), \quad (19c)$$

$$\dot{q} = \frac{I_{zz} - I_{xx}}{I_{yy}} \tau\, u, \quad (19d)$$

where u is the net lift force, m is the aircraft mass, $\tau\, u$ is the pitch moment, $T = u_a$ is the thrust, and inertias in the body axes are represented as $I_{ij},\ i = x, y, z,\ j = x, y, z$.

Control Inputs. The control inputs are the thrust signal T and the lift force $u(t)$. The objective is to develop an integrated control law $u_d(t)$ for all angles of attack until stall. We use the thrust T to control the velocity V_a, q to control α, and the desired lift u_d imparts a pitch moment to control q. The desired trajectory is specified by the reference states (V_c, α_c), which are differentiable and bounded.

For the feedback control design, we assume that

(A2) the velocity V_a does not approach 0, the angle γ does not approach $\pm 90°$, and $-24° \leq \alpha \leq 24°$,

so that the nonlinear aircraft dynamics avoid singularities.

A. Velocity Control

From the velocity dynamics in (19a), for velocity control, the desired thrust T as a control command is

$$T \ =\ \frac{m}{\cos\alpha} \left(g \sin\gamma + \dot{V}_c - K_V \tilde{V}\right), \quad (20)$$

where V_c is the reference signal, $\tilde{V} = V_a - V_c$, is the velocity error, $K_V > 0$, is a constant gain.

Using (19a), (20), the velocity error dynamics is given by

$$\dot{\tilde{V}} = -K_V \tilde{V}. \quad (21)$$

B. Flight path angle control

With thrust T given by (20), (19b) can be expressed as

$$\dot{\gamma} \;=\; f_\gamma + \frac{1}{mV_a}\left(u + T\sin\alpha\right), \tag{22}$$

where f_γ is a known function. From (22), we generate

$$\alpha_c \;=\; \arcsin\left(\frac{mV_a F_\gamma - u_d - \hat{\beta}^T\Omega}{T}\right), \tag{23}$$

where $F_\gamma = \dot{\gamma}_c - f_\gamma - K_\gamma\tilde{\gamma}$, and γ_c is the available flight path angle, $K_\gamma > 0$ is a constant gain, $\tilde{\gamma} = \gamma - \gamma_c$, is flight path error. Note that $\hat{\beta}^T\Omega$ term has been added for ensuring stability in the Lyapunov sense by cancelling the effect of $\beta^T\Omega$. The solution of α_c is valid in the region

$$\left|mV_a F_\gamma - u_d - \hat{\beta}^T\Omega\right| < \delta\left|T\right|, \tag{24}$$

where δ is a constant determined by the allowable values of α_c: $\alpha_c \in (-25°, 25°),\ \delta = 0.1736$.

Using (22)–(24), the dynamics of $\tilde{\gamma}$ is

$$\dot{\tilde{\gamma}} = -K_\gamma\tilde{\gamma} + \frac{1}{mV_a}\Big(u - u_d - \hat{\beta}^T\Omega + T\left(\sin\alpha - \sin\alpha_c\right)\Big).$$

Lyapunov stability analysis requires linearization of $\tilde{\gamma}$. For small errors $\tilde{\alpha} = \alpha - \alpha_c$, the flight path angle error is

$$\dot{\tilde{\gamma}} = -K_\gamma\tilde{\gamma} + \frac{1}{mV_a}\Big(u - u_d - \hat{\beta}^T\Omega + T(\tilde{\alpha}\cos\alpha_c + O(\tilde{\alpha}))\Big). \tag{25}$$

C. Angle of attack control

With thrust T given by (20), (19c) can be expressed as

$$\dot{\alpha} = f_\alpha - \frac{1}{mV_a}u + q, \tag{26}$$

where f_α is a known function. For angle of attack control, we generate the signals q_c, from the following equation

$$q_c = \dot{\alpha}_c - f_\alpha - K_\alpha\tilde{\alpha} + \frac{1}{mV_a}\left(u_d + \hat{\beta}^T\Omega + T\tilde{\gamma}\cos\alpha_c\right), \tag{27}$$

where α_c is a reference signal, and $K_\alpha > K_\gamma > 0$ are constant gains, $\tilde{\alpha} = \alpha - \alpha_c$, is angle of attack error, and $\hat{\beta}$ is an adaptive estimate of β. Note that $\hat{\beta}^T\Omega$ and $T\tilde{\gamma}\cos\alpha_c$ ensure stability in the Lyapunov sense.

Using (26), (27), and (12), the dynamics of $\tilde{\alpha}$ is

$$\dot{\tilde{\alpha}} \;=\; -K_\alpha\tilde{\alpha} + \tilde{q} - \frac{1}{mV_a}\left(u - u_d - \hat{\beta}^T\Omega\right), \tag{28}$$

where $\tilde{q}$ is the body axis rate error.

D. Pitch Rate Control

The state feedback control law can be expressed as

$$u_d = \frac{1}{\tau}\left(\frac{I_{yy}}{I_{zz} - I_{xx}}\right)\left(-K_q\tilde{q} - \tilde{\alpha} + \dot{q}_c\right) - \hat{\beta}^T\Omega, \tag{29}$$

where $K_q > K_\alpha > 0$, so that q_c given by (27), is achievable.

The first component in (29) represents a negative feedback of $\tilde{q}$. Using (19d), and (29), the dynamics of $\tilde{q}$ is

$$\dot{\tilde{q}} = -k_q\tilde{q} - \tilde{\alpha} + \frac{I_{zz} - I_{xx}}{I_{yy}}\tau\left(u - u_d - \hat{\beta}^T\Omega\right), \tag{30}$$

where $\tilde{q} = q - q_c$ and $u - u_d$ is available from (12).

The control objective now is to choose the adaptive laws for $\hat{W}$ and $\hat{W}_i$ for local stable system.

IV. Parameter Adaptation and Stability Analysis

It is possible to guarantee that $\hat{W}(t), \hat{W}_i(t)$, and $\hat{\beta}(t)$ remain within a convex region defined in a parameter space that contains the ideal target weights. In this section, we formulate update laws for the parameter estimates $\hat{W}(t), \hat{W}_i(t)$, and $\hat{\beta}(t)$ so that the control objective is achievable.

Adaptive update laws. We choose the adaptive laws as

$$\begin{aligned}\dot{\hat{W}}(t) &= g_a(t) + h_a(t),\\ \dot{\hat{W}}_i(t) &= g_c(t) + h_c(t),\\ \dot{\hat{\beta}}(t) &= \Gamma_3\Omega|e(t)| + h_\beta(t),\end{aligned} \tag{31}$$

where $g_a(t)$ and $g_c(t)$ are adaptation functions given by

$$\begin{aligned}g_c(t) &= -\Gamma_2 e(t)[\sigma_i\left(V_i^T x_2 + v_{0i}\right), 0]^T\Psi^T\hat{W},\\ g_a(t) &= \Gamma_1 e(t)\Psi[\hat{u}_{dNN}, 0]^T,\ \Gamma_i = \Gamma_i^T > 0,\end{aligned} \tag{32}$$

where $\Gamma_i, i = 1, 2, 3$, are adaptation gain matrices and

$$e(t) \;=\; \frac{\tilde{\gamma} - \tilde{\alpha}}{mV_a} + \frac{I_{zz} - I_{xx}}{I_{yy}}\tau\tilde{q}, \tag{33}$$

and $h_a(t), h_c(t)$ and $h_\beta(t)$ are projection functions which ensure that the parameter estimates stay in a convex region for certain desired physical properties.

For parameter projection we denote $W_j, \hat{W}_j(t), W_{i_j}$, $\hat{W}_{i_j}(t)$, $h_{a_j}(t)$ and $h_{c_j}(t)$ as the jth components of W, $\hat{W}(t), W_i, \hat{W}_i(t), h_a(t)$ and $h_c(t)$, respectively, for $j = 1, 2, \ldots, n_1$ and $j = 1, 2, \ldots, n_2$ as the case may be. The convex region with ideal target weights is

$$W_j \in \left[W_j^a, W_j^b\right],\ W_{i_j} \in \left[W_{i_j}^a, W_{i_j}^b\right]. \tag{34}$$

With $\hat{W}_j(0) \in [W_j^a, W_j^b], \hat{W}_{i_j}(0) \in [W_{i_j}^a, W_{i_j}^b]$, the projection functions $h_{a_j}(t), h_{c_j}(t)$ and $h_{\beta k}(t)$ are

$$h_{a_j}(t) = \begin{cases} 0 & \text{if } \ \hat{W}_j(t) \in (W_j^a, W_j^b),\\ & \text{if } \ \hat{W}_j(t) = W_j^a,\ g_{a_j}(t) \geq 0,\\ & \text{if } \ \hat{W}_j(t) = W_j^b,\ g_{a_j}(t) \leq 0,\\ -g_{a_j}(t) & \text{otherwise,}\end{cases} \tag{35}$$

$$h_{c_j}(t) = \begin{cases} 0 & \text{if } \ \hat{W}_{i_j}(t) \in (W_{i_j}^a, W_{i_j}^b),\\ & \text{if } \ \hat{W}_{i_j}(t) = W_{i_j}^a,\ g_{c_j}(t) \geq 0,\\ & \text{if } \ \hat{W}_{i_j}(t) = W_{i_j}^b,\ g_{c_j}(t) \leq 0,\\ -g_{c_j}(t) & \text{otherwise,}\end{cases} \tag{36}$$

$$h_{\beta_k}(t) = \begin{cases} 0 & \text{if } \ \hat{\delta}_k(t) \in \left(\delta_k^b, \delta_k^d\right),\\ & \text{if } \ \hat{\delta}_k(t) = \delta_k^b,\ \Gamma_3\Omega e(t) \leq 0,\\ & \text{if } \ \hat{\delta}_k(t) = \delta_k^d,\ \Gamma_3\Omega e(t) \geq 0,\\ \Gamma_3\Omega e(t) & \text{otherwise,}\end{cases} \tag{37}$$

where $k = 1, \ldots, 4,\ h_\beta = [h_{\beta 1}, \ldots, h_{\beta 2}]^T$.

Note the coupled nature of the adaptive laws $\hat{W}(t)$ and $\hat{W}_i(t)$ reinforcing the mutual dependence of the two neural networks. Next, we analyze the aircraft stability performance.

Theorem 1: Under assumptions (A1) and (A2), the error dynamics (21), (25), (28), and (30) from state feedback control law governed by (20), (23), (27) and (29), with parameters updated from the adaptive laws (31) guarantee that the overall closed-loop system is locally stable and $\tilde{x}$ converge to the compact set $S(\delta^2)$ given by

$$S(\delta^2) = \left\{ \tilde{x} \,\middle|\, \int_t^{t+T} \|\tilde{x}(\tau)\|_2^2 d\tau \leq \delta^2 T + c_0 \right\},$$

that is, $\tilde{x}$ is δ^2-small in the mean squared sense, and c_0 is a positive constant, for $\tilde{x} = [\tilde{V} \;\; \tilde{\gamma} \;\; \tilde{\alpha} \;\; \tilde{q}]^T$.

Proof: Consider the positive definite function

$$V_L = \frac{1}{2}\tilde{x}^T\tilde{x} + \frac{1}{2}\tilde{W}^T\Gamma_1^{-1}\tilde{W} + \frac{1}{2}\tilde{W}_i^T\Gamma_2^{-1}\tilde{W}_i + \frac{1}{2}\tilde{\beta}^T\Gamma_3^{-1}\tilde{\beta}, \tag{38}$$

where $\tilde{\beta}(t) = \beta - \hat{\beta}(t)$. Using (21), (25), (28), (30), and (33), and differentiating V_L, we have

$$\begin{aligned} \dot{V}_L \;=\; & -K_V\tilde{V}^2 - K_\gamma\tilde{\gamma}^2 - K_\alpha\tilde{\alpha}^2 - K_q\tilde{q}^2 - \tilde{W}^T\Gamma_1^{-1}\dot{\hat{W}} \\ & -\tilde{W}_i^T\Gamma_2^{-1}\dot{\hat{W}}_i + e(t)\left(u - u_d - \hat{\beta}^T\Omega\right) - \tilde{\beta}^T\Gamma_3^{-1}\dot{\hat{\beta}} \\ & +\Delta, \end{aligned} \tag{39}$$

where $\Delta = \frac{T\tilde{\gamma}}{mV_a}O(\tilde{\alpha}) \leq \eta\tilde{\gamma}$ such that $O(\tilde{\alpha})$ represent higher order error terms for computable bound η.

Substituting (12) and (31) in (39), we have

$$\begin{aligned} \dot{V}_L \;=\; & -K_V\tilde{V}^2 - K_\gamma\tilde{\gamma}^2 - K_\alpha\tilde{\alpha}^2 - K_q\tilde{q}^2 - \tilde{W}^T\Gamma_1^{-1}h_a(t) \\ & -\tilde{W}_i^T\Gamma_2^{-1}h_c(t) - \tilde{W}e(t)\Psi[\hat{u}_{dNN},0]^T - e(t)\tilde{\beta}^T\Omega \\ & +\tilde{W}_i^Te(t)[\sigma_i\left(V_i^Tx_2 + v_{0i}\right),0]^T\Psi^T\hat{W} - \tilde{\beta}^T\Omega|e(t)| \\ & -\tilde{\beta}^T\Gamma_3^{-1}h_\beta(t) + \Delta + e(t)\left(\tilde{W}^T\Psi\left[\hat{u}_{dNN},0\right]^T\right. \\ & \left. -\hat{W}^T\Psi\left[\tilde{W}_i^T\sigma_i\left(V_i^Tx_2 + v_{0i}\right),0\right]^T + \epsilon\right). \end{aligned} \tag{40}$$

Since $|\epsilon| \leq \beta^T\Omega$, on cancellation of terms, we have

$$\begin{aligned} \dot{V}_L \;=\; & -K_V\tilde{V}^2 - K_\gamma\tilde{\gamma}^2 - K_\alpha\tilde{\alpha}^2 - K_q\tilde{q}^2 - \tilde{\beta}^T\Gamma_3^{-1}h_\beta(t) \\ & -\tilde{W}^T\Gamma_1^{-1}h_a(t) - \tilde{W}_i^T\Gamma_2^{-1}h_c(t) + \eta\tilde{\gamma}. \end{aligned} \tag{41}$$

With bounds on the initial estimates and the projection functions (35)-(37), it follows that

$$\begin{aligned} \hat{W}_j \;&\in\; \left[W_j^a, W_j^b\right], j = 1, \ldots, n_1, \\ \hat{W}_{i_j} \;&\in\; \left[W_{i_j}^a, W_{i_j}^b\right], j = 1, \ldots, n_2, \end{aligned} \tag{42}$$

$$\tilde{W}^Th_a(t) \geq 0, \quad \tilde{W}_i^Th_c(t) \geq 0, \quad \tilde{\beta}^Th_\beta(t) \geq 0. \tag{43}$$

With this property, (41) reduces to

$$\dot{V}_L = -K_V\tilde{V}^2 - K_\gamma\tilde{\gamma}^2 - K_\alpha\tilde{\alpha}^2 - K_q\tilde{q}^2 + \eta\tilde{\gamma}. \tag{44}$$

Using the inequality $xy \leq \zeta^2x^2 + \frac{1}{4\zeta^2}y^2, \; \forall\zeta \neq 0,$ for $\zeta^2 = \frac{K_\gamma}{2}$, $|\eta| \leq \sigma$, we find that

$$\dot{V}_L \leq -K\tilde{x}^2 + \frac{1}{2K_\gamma}\sigma^2, \tag{45}$$

where $K = [K_V \;\; \frac{K_\gamma}{2} \;\; K_\alpha \;\; K_q]$. Therefore, we have

$$\int_t^{t+T} \|\tilde{x}(\tau)\|_2^2 d\tau \leq \delta^2 T + c_0, \tag{46}$$

where $\forall t \geq 0$ and any $T \leq 0$, c_0 and δ^2 are given as

$$c_0 = \sup_{t\geq 0} \frac{1}{\|K\|}\left(V_L(t) - V_L(t+T)\right), \; \delta^2 = \frac{\sigma^2}{2\|K\|K_\gamma}. \tag{47}$$

Hence, we have that $\tilde{x}(\tau) \in S(\delta^2)$, where δ can be adjusted by appropriately choosing the design parameters, and that $\tilde{W}$ and $\tilde{W}_i$ are bounded by parameter projection. ∇

Thus far, we have developed a higher-order approximation based adaptive inverse design for compensating signal-dependent actuator nonlinearities and applied it to a benchmark aircraft flight dynamic system with synthetic jet actuators. A detailed numerical evaluation of such an adaptive control system is crucial, in order to verify the desired system stability and tracking properties, but it is not available yet.

V. Concluding Remarks

Signal-dependent actuator nonlinearities are present in some important applications such as flight control of aircraft systems with synthetic jet actuators whose characteristics depend on the aircraft's angle of attack. Adaptive inversion of such actuator nonlinearities needs to be specifically designed to handle the actuator nonlinearity's uncertain structure and signal dependence, to increase the inversion accuracy. In this paper we have developed a framework for such a desired adaptive inverse, by using a higher-order approximation scheme for implementing the adaptive inverse. An important future task is to evaluate the performance of such an adaptive inverse compensation control system.

References

[1] D. Deb, G. Tao, J. Burkholder and D. Smith, "Adaptive compensation of nonlinearities in synthetic jet actuators for nonlinear aircraft dynamics," *Proceedings of the 2006 American Control Conference*, pp. 327–332, Minneapolis, MN, 2006.

[2] D. Deb, G. Tao, J. Burkholder and D. Smith, "Modeling and Compensation of Synthetic Jet Actuators for Airfoil Flow Separation Control," Proceedings of the *2006 AIAA Guidance, Navigation and Control Conference*, Keystone, CO, 2006.

[3] D. Deb, G. Tao and J. Burkholder, "Adaptive approximation based compensation of synthetic jet actuator nonlinearities for aircraft control," Proceedings of the *2007 AIAA Guidance, Navigation and Control Conference*, Hilton Head, SC, 2007.

[4] J. A. Farrell and M. M. Polycarpou, *Adaptive Approximation Based Control*, John Wiley & Sons, 2006.

[5] S. S. Ge and C. Wang, "Adaptive NN control of uncertain nonlinear pure-feedback systems," *Automatica*, vol. 38, pp. 671-682, 2002.

[6] F. L. Lewis, "Nonlinear structures for feedback control," *Asian Journal of Control*, vol. 1 (1999), pp. 205-228.

[7] F. L. Lewis, J. Campos and R. Selmic, *Neuro-Fuzzy Control of Industrial Systems with Actuator Nonlinearities*, SIAM: Frontiers in Applied Mathematics, Philadelphia, PA, 2002.

[8] R. Selmic and F. L. Lewis, "Dead-zone compensation in motion control systems using neural networks," *IEEE Transactions on Automatic Control*, vol. 45, no. 4, pp. 602–613, April 2000.

[9] G. Tao, *Adaptive Control Design and Analysis,*, John Wiley & Sons, 2003.