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Stability Analysis of Discrete Fault Tolerant Control Systems with Parameter Uncertainties

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***Abstract*— The stochastic behavior of DTFTCS with norm bounded parameter uncertainties in noisy environments is studied. The uncertainties are assumed to be unknown but bounded. Second moment stability for the uncertain DTFTCS driven by a state feedback control law is to be investigated. Sufficient conditions that guarantee the second moment stability and achieve a minimum of δ-level of disturbance rejection are to be derived. Conditions are to be formulated as a feasibility solution for a set of linear matrix inequalities (LMIs) which allow the utilization of linear optimization tools. The results are verified by means of a numerical example.**

I. Introduction

Active Fault Tolerant Control Systems (AFTCSs) are highly sophisticated control functions designed to maintain high levels of system survivability for safety-critical systems. An AFTCS utilizes a fault detection and isolation (FDI) scheme and a control reconfiguration mechanism to respond to a faulty condition. Fault induced changes as detected by the FDI scheme are used to reconfigure the control law in real time basis. Control reconfiguration mechanisms can be broadly classified as: projection-based methods, or automatic on-line controller calculation methods. In projection-based methods the control gains for possible fault scenarios are computed off-line and only gain selection is completed on-line based on the decisions of the FDI scheme. On the other hand, on-line controller calculation techniques never assume any faulty scenario a priori. The on-line controller computation is more involved and put heavy constraints on response time, yet they are able to deal with unanticipated faults. A bibliographical review on the definition of FTCS, classes and major components of FTCS, and preview of some design methodologies can be found in [14]. Generally speaking, the dynamical behavior of AFTCS is governed by stochastic differential equations and modeled as a hybrid system combining the Euclidean space for system dynamics and the discrete space for fault-induced changes.

The research of hybrid systems evolved into two major classes: Jump Linear Systems (JLS) and Fault Tolerant Control Systems with Markovian Parameters (FTCSMP). The modelling of JLS defines a single finite state Markov chain called plant regime mode [3], [7] to represent the random variations in the coefficients of the system to be controlled. This modelling lead to an inherit assumption of perfect regime knowledge which can not be enforced in practical environments corrupted by all types of disturbances and noises. This limitation motivated the introduction of FTCSMP [13]. In FTCSMP two separate random processes with different state spaces are defined: one represents system component failures and the second represents the non-deterministic decisions of the FDI process. This unique modeling allows the consideration of practical implementation issues and physical limitations for a particular system to be controlled. were studied in [8],[9] and [10]. A comprehensive review of the stochastic stability and stabilization of continuous FTCSMP using Lyapunov function approach can be found in [11]. The citation also studied the stochastic behavior of FTCSMP in the presence of noise, detection errors, detection delays, parameter uncertainties and actuator saturation. Just lately, the analysis of stochastic stability and H_∞ stabilization of continuous FTCSMP was revisited in [1] and [2] using convex programming framework. The results provided an LMI characterization of output feedback controllers that stochastically stabilize FTCSMP and ensures H_∞ constraints. Integral Quadratic Constraints were defined for FTCSMP and a stabilizing controller was synthesized in [15] and [16], optimal H_2 performance was investigated in [17] and [18]. In [19] FTCSMP were modeled and analyzed using randomized algorithms. The vital issue of detection delays has been revisited in a more rigorous form in [20].

Just recently the class of discrete time fault tolerant control systems (DTFTCS) has drawn the attention of several researchers. The reason for the delay in studying and characterizing the stochastic behavior of DTFTCS was due to the complexity of the model and tools needed to complete the studies. [6] studied the stochastic stability and controller design for the nominal DTFTCS, [12] extended the results to include norm bounded parameter uncertainties. In both citations, a control law was synthesized by solving a set of Riccati-Like matrix inequalities. It was concluded that this DTFTCS model yield results that are more complex than the case of continuous FTCSMP.

The problem of stabilizing an uncertain DTFTCS was tackled in [12]. However, it dealt with the case of DTFTCS operating in ideal noise-free environments and carried the analysis utilizing a less unified approach that lead to stability results in terms of nonlinear Riccati-like matrix equations that are difficult to solve and to test. It is the objective of this article to model and characterize the stochastic behavior of uncertain DTFTCS. Parameter uncertainties are assumed to be unknown yet bounded having norm bounded structure. Stability performance of uncertain DTFTCS actuated by a state feedback controller in noisy environments is to be studied. A unified approach of analysis is outlined and an

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easier to test results are stated in terms of LMI feasibility solution. The findings are to be demonstrated by a numerical example.

This paper is organized as follows: Section II describes the dynamical model of uncertain DTFTCS and outlines the structure of norm bound parameter uncertainties. Stochastic stability of uncertain DTFTCS is considered in Section III. Conditions for the existence of a stabilizing controller for uncertain DTFTCS in terms of feasibility solutions of some LMIs are derived in Section IV. A numerical example is given in Section V. A concluding summary is given in Section VI.

II. The Dynamical Model of DTFTCS

A class of uncertain DTFTCS in noisy environment and subject to random actuator failures can be described as

$$\begin{aligned} x_{k+1} &= [A(\eta_k) + \Delta A(\eta_k)]x_k + [B(\eta_k)+ \\ &\quad \Delta B(\eta_k)]u_k(\Psi_k) + \varphi_x(\eta_k)w_k \\ y_k &= C(\eta_k)x_k + \varphi_y(\eta_k)w_k \\ z_k &= D(\eta_k)x_k + E(\eta_k)\, u_k(\Psi_k) \end{aligned} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the system state, $u_k \in \mathbb{R}^m$ is the system input, $y_k \in \mathbb{R}^p$ is the system measured output, and $z_k \in \mathbb{R}^q$ is the system controlled output. $w_k \in \mathbb{R}^t$, $\{w_k\} \in l_2[0,\infty)$ is an exogenous disturbance input which belongs to the space of square summable infinite vector sequences on $[0,\infty)$, that is;

$$\|w\|_2^2 = \mathcal{E}\{\sum_{k\in\mathbb{N}} |w_k|^2\} < \infty \tag{2}$$

$\mathbb{N}$ is the set of natural numbers. $A(\eta_k)$, $B(\eta_k)$, $D(\eta_k)$, $E(\eta_k)$, $\varphi_{x_j}(\eta_k)$ and $\varphi_y(\eta_k)$ are properly dimensioned real-valued system matrices, and are random in nature with Markovian transition characteristics. $\Delta A(\eta_k)$ and $\Delta B(\eta_k)$ are unknown time-varying bounded matrices representing system uncertainties. η_k and Ψ_k are homogeneous discrete-time discrete-state Markov chains [5] with finite state spaces $S = \{1, 2, ..., s\}$ and $R = \{1, 2, ..., r\}$, respectively. The one-step transition probabilities from state (m) at time instant (k) to state (n) at time instant $(k+1)$ for the plant component failure process, $\{\eta_k\}$, is

$$\begin{aligned} \alpha_{mn} &= Pr\ \{\ \eta_{k+1} = n,\ \eta_k = m\ \} \\ \sum_{n=1}^{s} \alpha_{mn} &= 1.0 \quad \forall\, m \in\ S, \quad \alpha_{mn}\ \geq\ 0 \end{aligned} \tag{3}$$

The conditional transition probability for $\{\Psi_k\}$, is

$$\begin{aligned} q^i_{mn} &= Pr\ \{\ \Psi_{k+1} = m,\ \Psi_k = n,\ \eta_k = j\} \\ \sum_{n=1}^{r} q^i_{mn} &= 1.0\ \forall\, m \in\ R \text{ and } i \in S, \quad q^i_{mn}\ \geq\ 0 \end{aligned} \tag{4}$$

α_{mn} and q^i_{mn} are directly related to the component failure rates, and the FDI transition rates, respectively. These rates play a key-role in modelling different behaviors for the general class of AFTCSMP [11], [13]. For uncertain DTFTCS, the control law is only a function of the measurable FDI process with constant feedback controller gains. In this article, a state feedback controller is used and is described by

$$u_k(\Psi_k) = -K(\Psi_k)x_k \tag{5}$$

The DTFTCS (1) is assumed to satisfy both the growth and global uniform Lipschitz condition, the solution, x_k, determines a family of unique discrete stochastic processes, one for each choice of the random variable x_0. The joint process $\{x_k, \eta_k, \Psi_k; k \in \mathbb{N}\}$ is a discrete Markov process.

Notations The following notations are used in the paper, the notation $M > N (\geq, <, \leq)\ 0$ is used to denote that $M - N$ is positive definite (positive semi-definite, negative definite, negative semi-definite) matrix. $\lambda_{min}(\cdot)$, $\lambda_{max}(\cdot)$ denote the minimum and the maximum eigenvalue, respectively. $\mathcal{E}[\cdot]$ stands for the mathematical expectation. Also, $A(\eta_k) = A_i$, $\Delta A(\eta_k) = \Delta A_i$, $B(\eta_k) = B_i$, $\Delta B(\eta_k) = \Delta B_i$, $C(\eta_k) = C_i$, $D(\eta_k) = D_i$, $E(\eta_k) = E_i$, $\varphi_x(\eta_k) = \varphi_{x_i}$, and $\varphi_y(\eta_k) = \varphi_{y_i}$, when $\eta_k = i \in S$ and $K(\Psi_k) = K_j$ when $\Psi_k = j \in R$. To reduce repetition, a symmetric matrix A is written as

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{12}^T & a_{22} \end{bmatrix} \triangleq \begin{bmatrix} a_{11} & a_{12} \\ * & a_{22} \end{bmatrix}$$

Plant Uncertainty Models

The plant models commonly used in robust control system design and analysis are: state-variable models, transfer-function matrix models, and matrix-fraction models. Each plant model can have its own type of uncertainty representation. In general, system uncertainty is classified in a number of different ways. For example, uncertainty is characterized as parametric versus non-parametric, structured versus non-structured, etc. Norm bounded uncertainty is the most adopted form of structured parameter uncertainties in robust stability analysis [11]. In this form, the admissible parameter uncertainty is modelled as:

$$\begin{aligned} \Delta A(\eta_k) &= H^a(\eta_k)F^a(\eta_k, k)M^a(\eta_k) \\ \Delta B(\eta_k) &= H^b(\eta_k)F^b(\eta_k, k)M^b(\eta_k) \end{aligned} \tag{6}$$

where $H^a(\eta_k), H^b(\eta_k) \in \Re^{n\times n_h}$, and $M^a(\eta_k), M^b(\eta_k) \in \Re^{n_e\times n}$ are known constant matrices $\forall k \geq 0$. $F^a(\eta_k, k)$ and $F^b(\eta_k, k)$ are Lipschitz measurable unknown matrix functions satisfying the condition

$$\begin{aligned} &F^{aT}(\eta_k, k)F^a(\eta_k, k) \leq I_{ne},\ \forall k \geq 0 \\ &F^{bT}(\eta_k, k)F^b((\eta_k, k) \leq I_{ne},\ \forall k \geq 0,\ \eta_k = i \in S \end{aligned} \tag{7}$$

Other forms of admissible parameter uncertainties can be alternatively used in the modelling of uncertain DTFTCS, yet it was shown that these forms can be studied in unified systematic approach [11]. Therefore and without loss of generality, admissible parameter uncertainties in this article will be assumed to be time varying with norm bounded structure satisfying

$$\begin{aligned} &[\Delta A(\eta_k)\ \ \Delta B(\eta_k)] = H(\eta_k)F(\eta_k, k)[M^a(\eta_k)\ M^b(\eta_k)] \\ &\text{with} \\ &F^T(\eta_k, k)F^(\eta_k, k) \leq I_{ne},\ \forall k \geq 0,\ \eta_k = i \in S \end{aligned} \tag{8}$$

Before concluding this section, the following Lemmas will be used in the proof of the main results.

Lemma 1: (Shur Complement [4]) For appropriately dimensioned constant matrices $\phi = \phi^T$, $\omega = \omega^T$, and θ, the linear matrix inequality

$$\begin{bmatrix} \phi & \theta^T \\ \theta & -\omega \end{bmatrix} < 0$$

is equivalent to $\phi + \theta^T \omega^{-1} \theta < 0$ and $\omega > 0$.

Lemma 2: ([21]) For appropriately dimensioned real matrices ϕ, ψ, θ and *E*, such that $\theta^T \theta \leq I$, $\varepsilon \geq 0$, and $\mathcal{W} - \varepsilon \psi^T \psi > 0$, the following inequality is satisfied

$$(\phi+\psi\theta\omega)^T \mathcal{W}^{-1}(\phi+\psi\theta\omega) \leq \phi^T(\mathcal{W}-\varepsilon\psi\psi^T)^{-1}\phi+\varepsilon^{-1}\omega^T\omega$$

III. Stability Properties of Uncertain DTFTCS

Second moment stochastic stability for the uncertain DTFTCS will be studied. The uncertain DTFTCS without input and noise is first considered, then H_∞ performance of the uncertain DTFTCS is investigated. The investigation of H_∞ stabilization given constant control gains will be deferred to the next section.

A. Stability for Autonomous Uncertain DTFTCS

An autonomous uncertain DTFTCS is a noise-free input-free uncertain DTFTCS and is described as

$$\begin{aligned} x_{k+1} &= [A_i + \Delta A_i] x_k \\ y_k &= C_i x_k, \quad z_k = D_i x_k \end{aligned} \tag{9}$$

Select a quadratic Lyapunov function as

$$V(x_k, \eta_k, \Psi_k) = x_k^T P(\eta_k = i, \Psi_k = j) x_k \tag{10}$$

Given $Q_{ij} > 0$, the one-step forward increment for uncertain DTFTCS (9) gives

$$(A_i + \Delta A_i)^T \tilde{P}_{ij} (A_i + \Delta A_i) - P_{ij} + Q_{ij} < 0 \tag{11}$$

where

$$\tilde{P}_{ij} = \sum_{n=1}^{s} \alpha_{nj} \sum_{m=1}^{r} q_{im}^{j} P_{nm} \tag{12}$$

Define $\chi_{ij} = \chi_{ij}^T = P_{ij}^{-1}$, pre- and post- multiply (11) by χ_{ij}, we get

$$\chi_{ij}(A_i + \Delta A_i)^T \tilde{\chi}_{ij}^{-1} (A_i + \Delta A_i)\chi_{ij} - \chi_{ij} + \chi_{ij} Q_{ij} \chi_{ij} < 0 \tag{13}$$

Define

$$\begin{aligned} \mathbb{Z} &= diag\{[\chi_{i1},\ \chi_{i2},\ \ldots, \chi_{ir}]_{i=1,2,\ldots,s}\} \\ \mathbb{I}_{ij} &= \nabla_{ij}[\mathcal{I}] \end{aligned} \tag{14}$$

where

$$\begin{aligned} \nabla_{ij}[.] = \Big[&\sqrt{\alpha_{1i} q_{j1}^i}[.] \ldots \sqrt{\alpha_{1i} q_{jr}^i}[.] \ \sqrt{\alpha_{2i} q_{j1}^i}[.] \ldots \\ &\sqrt{\alpha_{2i} q_{jr}^i}[.] \ldots \sqrt{\alpha_{si} q_{j1}^i}[.] \ \ldots \ \sqrt{\alpha_{si} q_{jr}^i}[.] \ \Big] \end{aligned} \tag{15}$$

This will lead to the identification

$$\begin{aligned} &\chi_{ij}(A_i + \Delta A_i)^T \tilde{\chi}_{ij}^{-1}(A_i + \Delta A_i)\chi_{ij} \\ &= \chi_{ij}[\mathbb{I}_{ij}^T A_i + \mathbb{I}_{ij}^T H_i F_{i_k} M_i^a]^T \mathbb{Z}^{-1} [\mathbb{I}_{ij}^T A_i + \mathbb{I}_{ij}^T H_i F_{i_k} M_i^a]\chi_{ij} \end{aligned} \tag{16}$$

For the particular parameter uncertainties and the results of Lemma 2, Inequality (13) can be equivalently written as

$$\begin{aligned} &\chi_{ij} A_i^T \mathbb{I}_{ij} (\mathbb{Z} - \varepsilon_{ij} \mathbb{I}_{ij}^T H_i H_i^T \mathbb{I}_{ij})^{-1} \mathbb{I}_{ij}^T A_i \chi_{ij} + \\ &\quad \varepsilon_{ij}^{-1} \chi_{ij} M_i^{aT} M_i^a \chi_{ij} - \chi_{ij} + \chi_{ij} Q_{ij} \chi_{ij} < 0 \end{aligned} \tag{17}$$

Applying Shur complement to the term $\chi_{ij} A_i^T \mathbb{I}_{ij} (\mathbb{Z} - \varepsilon_{ij} \mathbb{I}_{ij}^T H_i H_i^T \mathbb{I}_{ij})^{-1} \mathbb{I}_{ij}^T A_i \chi_{ij}$, we get

$$\begin{bmatrix} \Omega_{11} & \chi_{ij} A_i^T \mathbb{I}_{ij} \\ * & (\varepsilon_{ij} \mathbb{I}_{ij}^T H_i H_i^T \mathbb{I}_{ij} - \mathbb{Z}) \end{bmatrix} \leq 0 \tag{18}$$

where $\Omega_{11} = -\chi_{ij} + \chi_{ij} Q_{ij} \chi_{ij} + \varepsilon_{ij}^{-1} \chi_{ij} M_i^{aT} M_i^a \chi_{ij}$. Applying Shur complement to the term $\chi_{ij} M_i^{aT} \varepsilon_{ij}^{-1} M_i^a \chi_{ij}$, we get

$$\begin{bmatrix} -\chi_{ij} + \chi_{ij} Q_{ij} \chi_{ij} & \chi_{ij} A_i^T \mathbb{I}_{ij} & \chi_{ij} M_i^{aT} \\ * & (\varepsilon_{ij} \mathbb{I}_{ij}^T H_i H_i^T \mathbb{I}_{ij} - \mathbb{Z}) & 0 \\ * & * & -\varepsilon_{ij} \mathcal{I} \end{bmatrix} < 0 \tag{19}$$

The above result is formally stated in Theorem 1.

Theorem 1: A sufficient condition for the second moment stability for the autonomous uncertain DTFTCS (9) is the existence of $\chi_{ij} = \chi_{ij}^T > 0$ and scalars $\varepsilon_{ij} > 0$, for some preselected $Q_{ij} > 0$, satisfying the matrix inequality (19).

B. H_∞ Performance of Uncertain DTFTCS

A noisy input-free uncertain DTFTCS is described as

$$\begin{aligned} x_{k+1} &= [A_i + \Delta A_i] x_k + \varphi_{x_i} w_k \\ y_k &= C_i x_k + \varphi_{y_i} w_k \\ z_k &= D_i x_k \end{aligned} \tag{20}$$

For a quadratic Lyapunov function, (10), the one-step increment for the noisy input-free uncertain DTFTCS (20) gives

$$\begin{aligned} &= x_k^T (A_i + \Delta A_i)^T \tilde{P}_{ij} (A_i + \Delta A_i) x_k + x_k^T [-P_{ij} + Q_{ij}] x_k \\ &+ x_k^T (A_i + \Delta A_i)^T \tilde{P}_{ij} \varphi_{x_i} w_k + w_k^T \varphi_{x_i}^T \tilde{P}_{ij} (A_i + \Delta A_i) x_k \\ &+ w_k^T \varphi_{x_i}^T \tilde{P}_{ij} \varphi_{x_i} w_k < 0 \end{aligned} \tag{21}$$

Define $\chi_{ij} = \chi_{ij}^T = P_{ij}^{-1}$, and using the identification (14) gives

$$\begin{aligned} &(A_i + \Delta A_i)^T \tilde{\chi}_{ij}^{-1} (A_i + \Delta A_i) = \\ &(A_i + \Delta A_i)^T \mathbb{I}_{ij} \mathbb{Z}^{-1} \mathbb{I}_{ij}^T (A_i + \Delta A_i) \\ &= [\mathbb{I}_{ij}^T A_i + \mathbb{I}_{ij}^T H_i F_{i_k} M_i^a]^T \mathbb{Z}^{-1} [\mathbb{I}_{ij}^T A_i + \mathbb{I}_{ij}^T H_i F_{i_k} M_i^a] \end{aligned} \tag{22}$$

The results of Lemma 2 give

$$\begin{aligned} &= x_k^T (A_i + \Delta A_i)^T \tilde{\chi}_{ij}^{-1} (A_i + \Delta A_i) x_k + x_k^T [-\chi_{ij}^{-1} + Q_{ij}] x_k \\ &+ x_k^T (A_i + \Delta A_i)^T \tilde{\chi}_{ij}^{-1} \varphi_{x_i} w_k + w_k^T \varphi_{x_i}^T \tilde{\chi}_{ij}^{-1} (A_i + \Delta A_i) x_k \\ &+ w_k^T \varphi_{x_i}^T \tilde{\chi}_{ij}^{-1} \varphi_{x_i} w_k \leq \\ &x_k^T [A_i^T \mathbb{I}_{ij} (\mathbb{Z} - \varepsilon_{ij} \mathbb{I}_{ij}^T H_i H_i^T \mathbb{I}_{ij})^{-1} \mathbb{I}_{ij}^T A_i + \varepsilon_{ij}^{-1} M_i^{aT} M_i^a] x_k \\ &+ x_k^T (A_i + \Delta A_i)^T \tilde{\chi}_{ij}^{-1} \varphi_{x_i} w_k + w_k^T \varphi_{x_i}^T \tilde{\chi}_{ij}^{-1} (A_i + \Delta A_i) x_k \\ &+ w_k^T \varphi_{x_i}^T \tilde{\chi}_{ij}^{-1} \varphi_{x_i} w_k + x_k^T [-\chi_{ij}^{-1} + Q_{ij}] x_k < 0 \end{aligned} \tag{23}$$

The noisy input-free uncertain DTFTCS (20) has a δ-level disturbance rejection, if the following is satisfied

$$z_k^T z_k - \delta^2 w_k^T w_k < 0$$

The one-step increment for the controlled output gives

$$x_k^T D_i^T D_i x_k - w_k^T (\delta^2 \mathcal{I}) w_k \leq 0 \tag{24}$$

Combining (23) and (24) by $y_k^T = [x_k^T \; w_k^T]$ gives

$$y_k^T \Theta y_k \leq 0 \tag{25}$$

where

$$\begin{aligned}
\Theta_{11} = &\; A_i^T \mathbb{I}_{ij}(\mathbb{Z} - \varepsilon_{ij}\mathbb{I}_{ij}^T H_i H_i^T \mathbb{I}_{ij})^{-1}\mathbb{I}_{ij}^T A_i + \varepsilon_{ij}^{-1} M_i^{aT} M_i^a \\
&\; D_i^T D_i - \chi_{ij}^{-1} + Q_{ij} \\
\Theta_{12} = &\; \varphi_{x_i}^T \tilde{\chi}_{ij}^{-1}(A_i + \Delta A_i) = \Theta_{21}^T \\
\Theta_{22} = &\; -\delta^2 \mathcal{I} + \varphi_{x_i}^T \tilde{\chi}_{ij}^{-1} \varphi_{x_i}
\end{aligned} \tag{26}$$

Pre- and post- multiply Θ as follows

$$\begin{bmatrix} \chi_{ij} & 0 \\ 0 & \mathcal{I} \end{bmatrix} \Theta \begin{bmatrix} \chi_{ij} & 0 \\ 0 & \mathcal{I} \end{bmatrix} = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} < 0 \tag{27}$$

where

$$\begin{aligned}
\Omega_{11} = &\; \chi_{ij} A_i^T \mathbb{I}_{ij}(\mathbb{Z} - \varepsilon_{ij}\mathbb{I}_{ij}^T H_i H_i^T \mathbb{I}_{ij})^{-1}\mathbb{I}_{ij}^T A_i \chi_{ij} \\
&\; \varepsilon_{ij}^{-1}\chi_{ij} M_i^{aT} M_i^a \chi_{ij} + \chi_{ij} D_i^T D_i \chi_{ij} - \chi_{ij} + \chi_{ij} Q_{ij} \chi_{ij} \\
\Omega_{12} = &\; \chi_{ij}\varphi_{x_i}^T \tilde{\chi}_{ij}^{-1}(A_i + \Delta A_i) = \Omega_{21}^T \\
\Omega_{22} = &\; -\delta^2 \mathcal{I} + \mathbb{W}_{ij}^T \mathbb{Z}^{-1} \mathbb{W}_{ij}
\end{aligned}$$

Applying Shur complement to the term $\chi_{ij} A_i^T \mathbb{I}_{ij}(\mathbb{Z} - \varepsilon_{ij}\mathbb{I}_{ij}^T H_i H_i^T \mathbb{I}_{ij})^{-1}\mathbb{I}_{ij}^T A_i \chi_{ij}$, Inequality (27) is feasible if the following inequality is feasible

$$\begin{bmatrix} \Lambda_{11} & 0 & \chi_{ij} A_i^T \mathbb{I}_{ij} \\ * & -\delta^2\mathcal{I} + \mathbb{W}_{ij}^T \mathbb{Z}^{-1}\mathbb{W}_{ij} & 0 \\ * & * & (\varepsilon_{ij}\mathbb{I}_{ij}^T H_i H_i^T \mathbb{I}_{ij} - \mathbb{Z}) \end{bmatrix} < 0 \tag{28}$$

where $\Lambda_{11} = \varepsilon_{ij}^{-1}\chi_{ij} M_i^{aT} M_i^a \chi_{ij} + \chi_{ij} D_i^T D_i \chi_{ij} + \chi_{ij} Q_{ij} \chi_{ij} - \chi_{ij}$. Applying Shur complement, in order, to the terms $\mathbb{W}_{ij}^T \mathbb{Z}^{-1}\mathbb{W}_{ij}$, $\chi_{ij} M_i^{aT} \varepsilon_{ij}^{-1} M_i^a \chi_{ij}$, and $\chi_{ij} D_i^T D_i \chi_{ij}$, we get

$$\begin{bmatrix} \triangle_{11} & 0 & \chi_{ij} A_i^T \mathbb{I}_{ij} & 0 & \chi_{ij} M_i^{aT} & \chi_{ij} D_i^T \\ * & -\delta^2 \mathcal{I} & 0 & \mathbb{W}_{ij}^T & 0 & 0 \\ * & * & (\varepsilon_{ij}\mathbb{I}_{ij}^T H_i H_i^T \mathbb{I}_{ij} - \mathbb{Z}) & 0 & 0 & 0 \\ * & * & * & -\mathbb{Z} & 0 & 0 \\ * & * & * & * & -\varepsilon_{ij}\mathcal{I} & 0 \\ * & * & * & * & * & -\mathcal{I} \end{bmatrix} < 0 \tag{29}$$

with $\triangle_{11} = -\chi_{ij} + \chi_{ij} Q_{ij} \chi_{ij}$. Define

$$J_N \triangleq \|z\|_2^2 - \delta^2 \|w_k\|_2^2 = \mathcal{E}\{\sum_{k=0}^{N-1} [z_k^T z_k - \delta^2 w_k^T w_k]\} \tag{30}$$

Dykin's formula leads to

$$\begin{aligned}
&\mathcal{E}\{V(x_N, \eta_N, \Psi_N)\} - V(x_0, \eta_0, \Psi_0) \\
&= \mathcal{E}\{\sum_{k=0}^{N-1} V(x_{k+1}, \eta_{k+1}, \Psi_{k+1}) - V(x_k, \eta_k, \Psi_k)\}
\end{aligned} \tag{31}$$

Assuming zero initial conditions, we get

$$\begin{aligned}
&\mathcal{E}\{V(x_N, \eta_N, \Psi_N)\} = \\
&\mathcal{E}\{\sum_{k=0}^{N-1} V(x_{k+1}, \eta_{k+1}, \Psi_{k+1}) - V(x_k, \eta_k, \Psi_k)\}
\end{aligned} \tag{32}$$

Equations (30) and (32) give

$$\begin{aligned}
J_N = &\; \mathcal{E}\{\sum_{k=0}^{N-1} z_k^T z_k + V(x_{k+1}, \eta_{k+1}, \Psi_{k+1}) - V(x_k, \eta_k, \Psi_k) \\
&- \delta^2 w_k^T w_k\} - \mathcal{E}\{V(x_N, \eta_N, \Psi_N)\} < \\
&\mathcal{E}\{\sum_{k=0}^{N-1} z_k^T z_k + V(x_{k+1}, \eta_{k+1}, \Psi_{k+1}) - V(x_k, \eta_k, \Psi_k) \\
&- \delta^2 w_k^T w_k\} = \sum_{k=0}^{N-1} y_k^T \Theta y_k < 0
\end{aligned} \tag{33}$$

Hence, the dissipativity condition

$$\|z_k^T z_k\| < \delta^2 \|w_k^T w_k\|^2$$

is satisfied and the uncertain DTFTCS (20) is second moment stable as stated in the following Theorem

Theorem 2: The noisy input-free uncertain DTFTCS (20) is second moment stable with δ-disturbance attenuation for any noise disturbance $w_k \in l_2$, if for some preselected $Q_{ij} > 0$, there exist symmetric $\chi_{ij} > 0$ and some scalars $\varepsilon_{ij} > 0$ that satisfy matrix inequality (29).

IV. Stochastic Stabilization of Uncertain DTFTCS

This section derives conditions that test the stability of the uncertain DTFTCS given a linear state feedback controller with constant gains $K_j \; \forall j \in R$. The study considers both noise-free and noisy environments.

A. Uncertain DTFTCS in Noise-Free Environments

An uncertain DTFTCS in noise-free environments driven by a state feedback controller $u_k = -K_j x_k$ can be described as

$$\begin{aligned}
x_{k+1} &= [\hat{A}_{ij} + \hat{\Delta} A_{ij}] x_k \\
y_k &= C_i x_k, \;\; z_k = \hat{D}_i x_k
\end{aligned} \tag{34}$$

where

$$\begin{aligned}
\hat{A}_{ij} &= A_i - B_i K_j \\
\hat{\Delta} A_{ij} &= \Delta A_i - \Delta B_i K_j \\
\hat{D}_{ij} &= D_i - E_i K_j
\end{aligned} \tag{35}$$

Equation (8) combines admissible parameter uncertainties for the closed-loop system together as follows

$$\hat{\Delta} A(\eta_k) = H(\eta_k) F(\eta_k, k)[M^a(\eta_k) - M^b(\eta_k) K(\Psi_k)] \tag{36}$$

Theorem 3: Given a state feedback controller with constant gains, K_j, the noise-free uncertain DTFTCS (34) is second moment stable, if for some preselected $Q_{ij} > 0$, there exist $\chi_{ij} = \chi_{ij}^T > 0$ and some scalars $\varepsilon_{ij} > 0$ satisfying the following matrix inequality

$$\begin{bmatrix} \chi_{ij} Q_{ij} \chi_{ij} - \chi_{ij} & \chi_{ij}(A_i - B_i K_j)^T \mathbb{I}_{ij} & \chi_{ij}(M_i^a - M_i^b K_j)^T \\ * & (\varepsilon_{ij}\mathbb{I}_{ij}^T H_i H_i^T \mathbb{I}_{ij} - \mathbb{Z}) & 0 \\ * & * & -\varepsilon_{ij}\mathcal{I} \end{bmatrix} < 0$$

Proof: A careful look reveals that the noise-free uncertain DTFTCS (34) has similar structure to the autonomous uncertain DTFTCS (9) with the augmented system matrices $\hat{A}$, $\hat{\Delta} A$, and $\hat{D}$ replace the system open loop system matrices A, ΔA, and D respectively. As a result, the proof is omitted.

B. Uncertain DTFTCS in Noisy Environments

An uncertain DTFTCS in noisy environment with state feedback controller $u_k = -K_j x_k$ is described as

$$\begin{aligned} x_{k+1} &= [\hat{A}_{ij} + \Delta\hat{A}_{ij}]x_k + \varphi_{x_i} w_k \\ y_k &= C_i x_k + \varphi_{y_i} w_k \\ z_k &= \hat{D}_{ij} x_k \end{aligned} \tag{37}$$

where $\hat{A}_{ij}$, $\hat{\Delta}A_{ij}$ and $\hat{D}_{ij}$ were defined in (35) and the admissible parameter uncertainties were combined in (36). The one-step increment for the uncertain DTFTCS (37) gives

$$\begin{aligned} &x_k^T(\hat{A}_{ij} + \Delta\hat{A}_{ij})^T \tilde{P}_{ij}(\hat{A}_{ij} + \Delta\hat{A}_{ij})x_k \\ &+x_k^T(\hat{A}_{ij} + \Delta\hat{A}_{ij})^T \tilde{P}_{ij}\varphi_{x_i} w_k + w_k^T \varphi_{x_i}^T \tilde{P}_{ij}(\hat{A}_{ij} + \Delta\hat{A}_{ij})x_k \\ &+w_k^T \varphi_{x_i}^T \tilde{P}_{ij}\varphi_{x_i} w_k - x_k^T P_{ij} x_k + x_k^T Q_{ij} x_k < 0 \end{aligned} \tag{38}$$

Equivalently,

$$\begin{aligned} &x_k^T(\hat{A}_{ij} + \Delta\hat{A}_{ij})^T \tilde{\chi}_{ij}^{-1}(\hat{A}_{ij} + \Delta\hat{A}_{ij})x_k \\ &+x_k^T(\hat{A}_{ij} + \Delta\hat{A}_{ij})^T \tilde{\chi}_{ij}^{-1}\varphi_{x_i} w_k + w_k^T \varphi_{x_i}^T \tilde{\chi}_{ij}^{-1}(\hat{A}_{ij} + \Delta\hat{A}_{ij})x_k \\ &+w_k^T \varphi_{x_i}^T \tilde{\chi}_{ij}^{-1}\varphi_{x_i} w_k - x_k^T \chi_{ij}^{-1} x_k + x_k^T Q_{ij} x_k < 0 \end{aligned} \tag{39}$$

Using the identification in (14), the results of Lemma 2 and the combined parameter uncertainties in (36) gives

$$\begin{aligned} &x_k^T(\hat{A}_{ij} + \Delta\hat{A}_{ij})^T \tilde{\chi}_{ij}^{-1}(\hat{A}_{ij} + \Delta\hat{A}_{ij})x_k \\ &+x_k^T(\hat{A}_{ij} + \Delta\hat{A}_{ij})^T \tilde{\chi}_{ij}^{-1}\varphi_{x_i} w_k + w_k^T \varphi_{x_i}^T \tilde{\chi}_{ij}^{-1}(\hat{A}_{ij} + \Delta\hat{A}_{ij})x_k \\ &+w_k^T \varphi_{x_i}^T \tilde{\chi}_{ij}^{-1}\varphi_{x_i} w_k - x_k^T \chi_{ij}^{-1} x_k + x_k^T Q_{ij} x_k \leq \\ &x_k^T[\hat{A}_{ij}^T \mathbb{I}_{ij}(\mathbb{Z} - \varepsilon_{ij}\mathbb{I}_{ij}^T H_i H_i^T \mathbb{I}_{ij})^{-1}\mathbb{I}_{ij}^T \hat{A}_{ij} \\ &+\varepsilon_{ij}^{-1}(M_i^a - M_i^b K_j)^T(M_i^a - M_i^b K_j)]x_k \\ &+x_k^T(\hat{A}_{ij} + \Delta\hat{A}_{ij})^T \tilde{\chi}_{ij}^{-1}\varphi_{x_i} w_k + w_k^T \varphi_{x_i}^T \tilde{\chi}_{ij}^{-1}(\hat{A}_{ij} + \Delta\hat{A}_{ij})x_k \\ &+w_k^T \varphi_{x_i}^T \tilde{\chi}_{ij}^{-1}\varphi_{x_i} w_k + x_k^T[-\chi_{ij}^{-1} + Q_{ij}]x_k < 0 \end{aligned} \tag{40}$$

The uncertain DTFTCS (37) have a δ-level disturbance rejection if

$$x_k^T \hat{D}_{ij}^T \hat{D}_{ij} x_k - w_k^T(\delta^2\mathcal{I})w_k < 0 \tag{41}$$

Combining (40) and (41) using $y_k^T = [x_k^T \ w_k^T]$, pre- and post-multiply by $diag\{\chi_{ij}, \mathcal{I}\}$ and applying Shur complement to the term $\chi_{ij}\hat{A}_{ij}^T\mathbb{I}_{ij}(\mathbb{Z} - \varepsilon_{ij}\mathbb{I}_{ij}^T H_i H_i^T \mathbb{I}_{ij})^{-1}\ \mathbb{I}_{ij}^T \hat{A}_{ij}\chi_{ij}$ and the term $\mathbb{W}_{ij}^T\mathbb{Z}^{-1}\mathbb{W}_{ij}$, we get

$$\begin{bmatrix} \Pi_{11} & 0 & \chi_{ij}\hat{A}_{ij}^T\mathbb{I}_{ij} & 0 \\ * & -\delta^2\mathcal{I} & 0 & \mathbb{W}_{ij}^T \\ * & * & (\varepsilon_{ij}\mathbb{I}_{ij}^T H_i H_i^T \mathbb{I}_{ij} - \mathbb{Z}) & 0 \\ * & * & * & -\mathbb{Z} \end{bmatrix} < 0 \tag{42}$$

where $\Pi_{11} = \varepsilon_{ij}^{-1}\chi_{ij}(M_i^a - M_i^b K_j)^T(M_i^a - M_i^b K_j)\chi_{ij} + \chi_{ij}\hat{D}_{ij}^T\hat{D}_{ij}\chi_{ij} - \chi_{ij} + \chi_{ij}Q_{ij}\chi_{ij}$. Further application of Shur complement to the term $\chi_{ij}(M_i^a - M_i^b K_j)^T\varepsilon_{ij}^{-1}(M_i^a - M_i^b K_j)\chi_{ij}$, we get

$$\begin{bmatrix} \nabla_{11} & 0 & \chi_{ij}\hat{A}_{ij}^T\mathbb{I}_{ij} & 0 & \chi_{ij}(M_i^a - M_i^b K_j)^T \\ * & -\delta^2\mathcal{I} & 0 & \mathbb{W}_{ij}^T & 0 \\ * & * & (\varepsilon_{ij}\mathbb{I}_{ij}^T H_i H_i^T \mathbb{I}_{ij} - \mathbb{Z}) & 0 & 0 \\ * & * & * & -\mathbb{Z} & 0 \\ * & * & * & * & -\varepsilon_{ij}\mathcal{I} \end{bmatrix} < 0 \tag{43}$$

where $\nabla_{11} = \chi_{ij}Q_{ij}\chi_{ij} - \chi_{ij} + \chi_{ij}\hat{D}_{ij}^T\hat{D}_{ij}\chi_{ij}$. Finally, Shur complement to the term $\chi_{ij}\hat{D}_{ij}^T\hat{D}_{ij}\chi_{ij}$, lead to

$$\begin{bmatrix} \chi_{ij}Q_{ij}\chi_{ij} - \chi_{ij} & 0 & \chi_{ij}\hat{A}_{ij}^T\mathbb{I}_{ij} & 0 & \chi_{ij}(M_i^a - M_i^b K_j)^T & \chi_{ij}\hat{D}_{ij}^T \\ * & -\delta^2\mathcal{I} & 0 & \mathbb{W}_{ij}^T & 0 & 0 \\ * & * & (\varepsilon_{ij}\mathbb{I}_{ij}^T H_i H_i^T \mathbb{I}_{ij} - \mathbb{Z}) & 0 & 0 & 0 \\ * & * & * & -\mathbb{Z} & 0 & 0 \\ * & * & * & * & -\varepsilon_{ij}\mathcal{I} & 0 \\ * & * & * & * & * & -\mathcal{I} \end{bmatrix} < 0 \tag{44}$$

Theorem 4: Given a state feedback controller with constant gains, K_j, the uncertain DTFTCS (37) is second moment stable with δ-disturbance attenuation for any noise disturbance $w_k \in l_2$, if for some preselected $R_{ij} > 0$, there exist $\chi_{ij} = \chi_{ij}^T > 0$ and scalars $\varepsilon_{ij} > 0$ satisfying the following linear matrix inequality

$$\begin{bmatrix} -\chi_{ij} & 0 & \chi_{ij}\hat{A}_{ij}^T\mathbb{I}_{ij} & 0 & \chi_{ij}(M_i^a - M_i^b K_j)^T & \chi_{ij}\hat{D}_{ij}^T & \chi_{ij} \\ * & -\delta^2\mathcal{I} & 0 & \mathbb{W}_{ij}^T & 0 & 0 & 0 \\ * & * & (\varepsilon_{ij}\mathbb{I}_{ij}^T H_i H_i^T \mathbb{I}_{ij} - \mathbb{Z}) & 0 & 0 & 0 & 0 \\ * & * & * & -\mathbb{Z} & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_{ij}\mathcal{I} & 0 & 0 \\ * & * & * & * & * & -\mathcal{I} & 0 \\ * & * & * & * & * & * & -R_{ij} \end{bmatrix} < 0$$

Proof: The same argument as those to get the matrix inequality (44) with the parametrization, $Q_{ij} = R_{ij}^{-1}$, is used to avoid the nonlinearity introduced by the slack term $\chi_{ij}Q_{ij}\chi_{ij}$. The proof is omitted.

V. Numerical Example

A second-order DTFTCS subject to single actuator failure has the following system parameters

$$A_1 = \begin{bmatrix} -1.0 & 0.0 \\ 0.2 & 1.2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1.0 & -2.0 \\ 1.0 & 1.0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 2.0 & 0.5 \\ 0.0 & 1.0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix},$$

$$H_1^T = \begin{bmatrix} 0.0 & 0.2 \end{bmatrix}, \quad M_1^a = \begin{bmatrix} 0.1 & 0.0 \end{bmatrix}, \quad M_1^b = \begin{bmatrix} 0.0 & -0.2 \end{bmatrix},$$

$$H_2^T = \begin{bmatrix} 0.1 & -0.1 \end{bmatrix}, \quad M_2^a = \begin{bmatrix} 0.3 & 0.0 \end{bmatrix}, \quad M_2^b = \begin{bmatrix} -0.1 & 0.1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.0 & 1.0 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0.0 & 0.5 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.6 & 0.2 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.2 & 0.6 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.5 & 0.1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.0 & -0.2 \end{bmatrix},$$

$$\varphi_{y_1} = \varphi_{y_2} = \begin{bmatrix} 0.1 \end{bmatrix}, \quad \varphi_{x_1} = \begin{bmatrix} 0.5 \\ 0.3 \end{bmatrix}, \quad \varphi_{x_2} = \begin{bmatrix} 0.0 \\ 0.2 \end{bmatrix},$$

For such system, the actuator failure, η_k, has a state space $S = \{1, 2\}$ and the FDI process, Ψ_k, has $R = \{1, 2\}$. The failure rates representing these two modes of operation are

$$\alpha_{mn} = \begin{bmatrix} 0.70 & 0.30 \\ 0.60 & 0.40 \end{bmatrix},$$

$$q_{mn}^1 = \begin{bmatrix} 0.20 & 0.80 \\ 0.2 & 0.80 \end{bmatrix}, \quad q_{mn}^2 = \begin{bmatrix} 0.10 & 0.90 \\ 0.15 & 0.85 \end{bmatrix}.$$

The following gains were selected and the exitance of feasible solutions, $\chi_{ij},\ \forall i, j = 1, 2$, of the LMI in Theorem 4 is a sufficient condition for the stability of the system under study when driven by this preselected state feedback gains.

$$K_1 = \begin{bmatrix} -0.20 & 1.00 \\ 1.00 & 1.20 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -2.20 & -2.20 \\ -0.20 & -0.23 \end{bmatrix},$$

A sample path simulation for the state trajectories and the controlled output are shown in Figure 1 and 2, respectively.

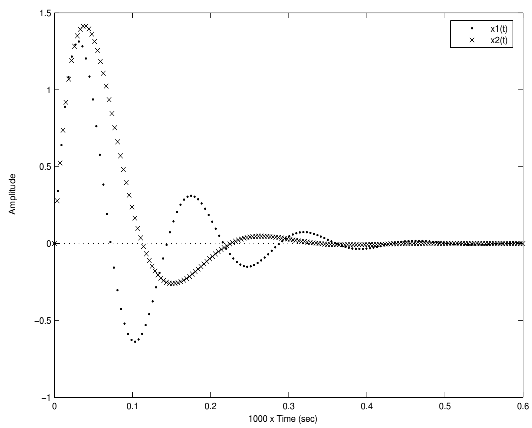

Fig. 1: Single sample path simulation: State Trajectory

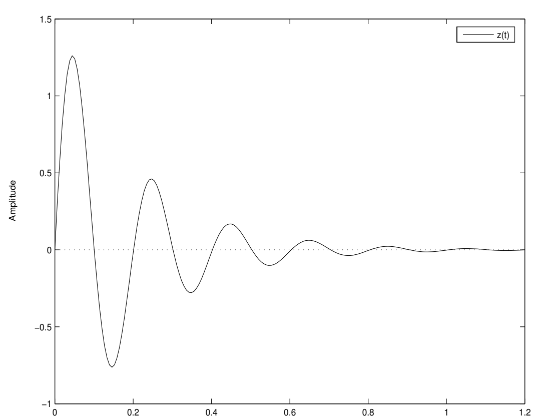

Fig. 2: Single sample path simulation: Controlled Output

VI. Conclusions

The stochastic behavior of DTFTCS with parameter uncertainties in noisy environments has been studied. The uncertainties are assumed to be unknown time varying with norm bounded structure. Second moment stability of the proposed uncertain DTFTCS has been investigated. Sufficient conditions that guarantee the second moment stability and achieve a minimum of δ-level of disturbance rejection were derived and formulated as feasibility solutions for some LMIs. This framework allows the utilization of linear optimization tools. The major results were illustrated by a numerical example. The current results provide a test criteria given a state feedback controller, future work is to develop a methodology to synthesis a stabilizing reconfigurable state feedback fault tolerant controller.

References

[1] S. Aberkane, D. Sauter and J.C. Ponsart, "H_∞ Stochastic Stabilization of Active Fault Tolerant Control Systems: Convex Approach," *Proc. of the 44th Conference on Decision and Control and European Control Conference*, Seville, Spain, December 2005, pp 3783-3788.

[2] S. Aberkane, J. Christophe, J.C. Ponsar and D. Sautert, Output Feedback H_∞ Control of A Class of Stochastic Hybrid Systems with Winner Process via Convex Analysis, *International Journal of Innovative Computing, Information and Control*,Vol. 2, 2006, pp.1179-1196.

[3] E.K. Boukas, Static Output Feedback Control for Stochastic Hybrid Systems: LMI Approach, *Automatica*, Vol. 42, 2006, pp. 183-188.

[4] S. Boyd, L. El Ghaoui, E. Feron, V. Balakrishnan,*Linear Matrix Inequalities in System and Control Theory*, SIAM, Philadelphia, 1994.

[5] J.L. Doob, *Stochastic Processes*, Willey, New York, 1990.

[6] J.H. GE and C.F. Lin, "H_∞ Control for Discrete Time Active Fault Tolerant Control Systems," *Proc. of the 13th IFAC World Congress*, San Franciso, USA, 1996, pp. 439-444.

[7] M. Mariton, *Jump Linear Systems in Automatic Control*, Marcel Dekker, New York, 1990.

[8] M.M. Mahmoud, J. Jiang and Y. Zhang, Stochastic Stability Analysis of Fault Tolerant Control Systems in the Presence of Noise, *IEEE Transaction on Automatic Control*, vol. 46, no. 11, 2001, pp. 1810-1815.

[9] M.M. Mahmoud, J. Jiang and Y. Zhang, Stochastic Stability Analysis for Fault Tolerant Control Systems with Multiple Failure Processes, *International Journal of Systems Science*, vol. 33, no 1, 2002, pp. 55-65.

[10] M.M. Mahmoud, J. Jiang and Y. Zhang, Stabilization Algorithm for Uncertain Fault Tolerant Control Systems, *Journal of Stochastic Analysis and Applications*, vol. 21, 2003, pp. 673-701.

[11] M.M. Mahmoud, J. Jiang and Y. Zhang, *Active Fault Tolerant Control Systems: Analysis and Development*, Springer, Germany, 2003.

[12] P. Shi, E.K. Boukas, S.K. Nguang and X. Guo, Robust Disturbance Attenuation for Discrete-Time Active Fault Tolerant Control Systems with Uncertainties, *Optimal Control Applications and Methods*, Vol. 24, 2003, pp. 85-101.

[13] R. Srichander and B. K. Walker, Stochastic Stability Analysis for Continuous-Time Fault Tolerant Control Systems, *Int. Journal of Control*, Vol. 57, 1993, pp. 433-452.

[14] Y. Zhang and J Jiang, "Bibliographical Review On Reconfigurable Fault Tolerant Control Systems," *Proceedings of IFAC Symposium SAFEPROCESS*, 2003, pp. 265-276.

[15] C. Cheng, Q. Zhao and F. Tao, Stability And Performance of The Stochastic Fault Tolerant Control Systems, *Proceedings of the 42nd IEEE Conference on Decision and Control*, Vol. 3, pp. 2484-2489, 2003.

[16] C. Cheng, Q. Zhao and F. Tao, Fault Tolerant Control Systems With Stochastic Integral Quadratic Constraints, *Journal of Dynamics of Discrete, Continuous and Impulsive Systems - series B*, Vol. 11 , No. 1-2, 2004, pp. 123-141.

[17] F. Tao and Q. Zhao, Design Of Stochastic Fault Tolerant Control For H_2 Performance, *Proceedings of the 44th IEEE Conference on Decision and Control and 2005 European Control Conference*, pp. 6274-6279, 2005.

[18] F. Tao and Q. Zhao, Stochastic fault tolerant control for optimal H_2 performance, *International Journal of Robust and Nonlinear Control*, Vol. 17, No. 1, 2007, pp. 1-24.

[19] H. Li and Q. Zhao, Analysis of Fault Tolerant Control by Using Randomized Algorithms, *Proceedings of the 2005 American Control Conference*, Vol. 3, 2005, pp. 2218-2223.

[20] F. Tao and Q. Zhao, Synthesis of Fault Tolerant Control in The Presence Of Random FDI Delay, *International Journal of Control*, Vol. 80, No. 5, 2007, pp. 684-694.

[21] Y. Wang, L. Xie and CE de Souza, Robust Control of a Class of Uncertian Nonlinear Systems , *Systems and Control Letters*, Vol. 19, 1992, pp. 139-149.