2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

Spatially Periodic Disturbance Rejection Using Spatial-based Output Feedback Adaptive Backstepping Repetitive Control

Y. –H. Yang, *Student Member, IEEE* and C. –L. Chen, *Member, IEEE*

Abstract—**In this paper, we propose a new design of spatial-based repetitive control for rotational motion systems required to operate at varying speeds and subject to spatially periodic disturbances. The system has known model structure with uncertain parameters. To synthesize a repetitive controller in spatial domain, a linear time-invariant system is reformulated with respect to a spatial coordinate (e.g., angular displacement), which results in a nonlinear system. A nonlinear state observer is then established for the system. Adaptive backstepping is applied to the system with the state observer so as to stabilize the system and reduce the tacking error. Moreover, a spatial-based repetitive controller is added and operates in parallel with the adaptively backstepped system, which further reduces the tracking error. The overall output feedback adaptive backstepping repetitive control system is robust to structured parameter uncertainty, capable of rejecting spatially periodic disturbances under varying process speeds, and can be shown to be stable and produce bounded state estimated error and bounded tracking error under sensible assumptions. Finally, feasibility and effectiveness of the proposed scheme is verified by simulation.**

I. Introduction

ROTATIONAL motion systems have found their application in many industry products. For most application, the systems are required to operate at varying speeds while following repetitive trajectories and/or rejecting disturbances with sinusoidal/periodic components. For example, the brushless dc motor in a typical laser printer may need to operate at different speed when driving the photosensitive drum for printing tasks of different media or resolution. Also, laser printing systems often suffer from a type of print artifacts (known as banding), which is mostly due to periodic disturbances affecting the angular velocity of the photosensitive drum (see [3]). Repetitive control systems have been shown to work well for tracking periodic reference commands or for rejecting periodic disturbances in regulation applications. Typical repetitive controllers are time-based controllers since they are synthesized and operate in temporal or time domain. For example, to synthesize the repetitive controller proposed by [15], one of the key steps is to determine the temporal period or frequency of the tracking or disturbance signal. To ensure effectiveness of the design, an underlying assumption is that the tracking or disturbance signal is stationary, i.e., the frequency/period of the tracking or disturbance signal does not vary with time. This assumption can easily be satisfied for many cases where the objective of the design is to track pre-specified repetitive trajectory. However, it might be violated for disturbance rejection problems where periods or frequencies of the disturbances are mostly time-varying.

Recent researches started studying control problems of rejecting/tracking spatially periodic disturbances/references for rotational motion systems using spatial-based repetitive controllers. A spatial-based repetitive controller has its repetitive kernel (i.e. e^{-Ls} with positive feedback) synthesized and operate with respect to a spatial coordinate, e.g., angular position or displacement. Hence its capability for rejecting or tracking spatially periodic disturbances or references will not degrade when the controlled system operates at varying speed. Note that a typical repetitive controller consists of repetitive (i.e., a repetitive kernel) and non-repetitive (e.g., a stabilizing controller) portions. With the repetitive kernel synthesized in spatial domain and given a time-domain open-loop system, design of the non-repetitive portion that properly interfaces the repetitive kernel and the open-loop system actually poses a challenge. For rejection of spatially periodic disturbances, Nakano *et al.* [10] reformulated a given open-loop linear time-invariant (LTI) system with respect to angular position, and linearized the resulting nonlinear system with respect to a constant operating speed. A stabilizing controller with built-in repetitive kernel was then synthesized for the obtained linear model using coprime factorization. A more recent and advanced design based on linearization using robust control was proposed by Chen *et al.* [4]. Although design methods for the linearized system are simple and straightforward, it is unclear whether the overall control system (which is nonlinear) will operate at varying speed or could sustain large velocity fluctuation without stability concern. For tracking of spatially periodic references, Mahawan and Luo [1] proposed and proved the feasibility of operating the repetitive kernel in spatial domain and the stabilizing controller in time domain. Thus, no reformulation of the open-loop system is required. For practical implementation, however, the proposed method requires solving an optimization problem in real-time to synchronize the hardware and software interrupts

C.-L. Chen obtained his PhD (2003) in Mechanical Engineering from Purdue University, WL (USA). He was with Lexmark International, Inc. (2003-2004), Lexington, KY 40517, USA. He is now with Department of Electrical Engineering (2004-now), National Chung Hsing University, Taichung 40225, Taiwan (phone: 886-4-22851549 ext 704; fax: 886-4-22851410; e-mail: chenc@dragon.nchu.edu.tw).

Y.-H. Yang is currently a graduate student of Department of Electrical Engineering, National Chung Hsing University, Taichung 40225, Taiwan (e-mail: g9664213@mail.nchu.edu.tw).

corresponding to time and angular position, respectively. Also, the function between time and angular position needs to be known *a priori*, which further limits the applicability of the proposed method. Both Nakano [10] and Mahawan [1] assumed the simplest scenario when making problem formulation. Namely, the open-loop system was assumed to be free of modeling uncertainty and nonlinearity. Chen and Chiu [5] showed that the nonlinear plant model can be formulated into a quasi-linear parameter varying (quasi-LPV) system. Then, an LPV gain-scheduling controller was obtained which addresses unstructured/bounded modeling uncertainties, actuator saturation and spatially periodic disturbances. The proposed approach, however, could lead to conservative design if the number of varying parameters increases, the varying parameter space is nonconvex, or the size of the modeling uncertainties becomes significant. To relieve the constraint and conservatism of modeling uncertainties imposed on controller design and control performance, Chen and Yang [6] formulated a spatial-based repetitive control system which combines adaptive backstepping [8] and repetitive control. However, this method requires full-state feedback and is thus not applicable to systems of which measurements of states are not available in real-time.

In this paper, we propose a new design of spatial-based repetitive control system which evolves from our previous work [6]. The proposed design resolves the major shortcoming in our former design, i.e., which requires full-state feedback, by incorporation of a nonlinear state observer known as the K-filters [2][7][9][12][13][17]. The proposed output feedback adaptive backstepping repetitive control (ABRC) system is robust to structured uncertainty of system parameters and capable of rejecting spatially periodic disturbances under variable process speed. Also, the overall system can be shown to be stable under bounded disturbance and parameter uncertainty. Furthermore, addition of the repetitive controller further improves the tracking error. A brushless dc motor of second-order is used for demonstration and derivation of the control algorithm. Simulation is performed to verify the feasibility and effectiveness of the proposed scheme.

This paper is organized as follows: Reformulation of an LTI rotational motion system with respect to angular displacement will be presented in Section II. Design of the state estimator is described in Section III. Section IV will cover derivation and stability analysis of the proposed output feedback ABRC scheme. Simulation verification for the proposed scheme will be presented in Section V. Conclusion and future work are given in Section VI.

II. Problem Statement

Suppose that a 2nd order LTI model for a rotational motion system is expressed as

$$Y(s) = (b_1 s + b_0)U(s) / (s^2 + a_1 s + a_0) + d(s)\,, \qquad (1)$$

where a_1, a_0, b_1 and b_0 are coefficients whose values depend on system parameters and are unknown (but might have known upper/lower bounds). $U(s)$ and $Y(s)$ correspond to control input and measured output angular velocity of the system, respectively. $d(s)$ represents a class of bounded output disturbances which are spatially periodic. The only available information of the disturbances is the number of distinctive spatial frequencies which need to be rejected. If no pole/zero cancellation occurs, a possible state space realization of (1) is

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t), \quad y(t) = \Psi x(t) + d(t), \qquad (2)$$

where

$$x(t) = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T, \begin{bmatrix} A \mid B \end{bmatrix} = \left[\begin{array}{cc|c} -a_1 & 1 & b_1 \\ -a_0 & 0 & b_0 \end{array}\right], \ \Psi = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

Following the same procedure as described in our previous work [6], we may rewrite (2) as

$$\hat{\omega}(\theta)\frac{d\hat{x}(\theta)}{d\theta} = A\hat{x}(\theta) + B\hat{u}(\theta), \quad \hat{y}(\theta) = \Psi\hat{x}(\theta) + \hat{d}(\theta). \qquad (3)$$

Equation (3) can be viewed as a nonlinear position-invariant (as opposed to the definition of time-invariant) system with the angular displacement θ as the independent variable. Note that the concept of transfer function is still valid for linear position-invariant systems if we define the Laplace transform of a signal $\hat{g}(\theta)$ in the angular displacement domain as

$$\hat{G}(\tilde{s}) = \int_0^\infty \hat{g}(\theta) e^{-\tilde{s}\theta} d\theta\,.$$

In addition, we will make the following assumptions for subsequent derivations: (1) The model structure (order, relative degree, and the sign of the high frequency gain) of the system (1) is known. Also, the system is assumed to be of minimum phase. (2) The disturbance $\hat{d}$ and the reference command $\hat{y}_r$ are smooth enough. (3) The uncertain parameters of the system have known bounds.

III. State Estimator

In this section, we will establish a state estimator for (3). To allow us to present the proposed design in a simpler context, we will focus on the case in which (2) has relative degree equal to two, i.e., $b_1 = 0$.

As first step, drop the θ notation and rewrite (3) in the form

$$\dot{\hat{x}}_1 = -a_1 + \hat{x}_2/\hat{x}_1\,,\ \dot{\hat{x}}_2 = -a_0 + b_0\hat{u}/\hat{x}_1\,,\ \hat{y} = \hat{\omega} + \hat{d} = \hat{x}_1 + \hat{d}, \quad (4)$$

where the state variables have been specified such that the angular velocity $\hat{\omega}$ is equal to $\hat{x}_1$, i.e., the undisturbed output. Suppose that both states in (4) cannot be measured in real time. To design a state estimator or the K-filters [13], we proceed as follows. First, rewrite the state equations in (4) as

$$\dot{\hat{x}} = A_0\hat{x} + \bar{k}\hat{x}_1 + \eta(\hat{x}_1)a + \varphi(\hat{x}_1) + \begin{bmatrix} 0 & b_0 \end{bmatrix}^T \sigma(\hat{x}_1)\hat{u}\,, \quad (5)$$

where

$$\hat{x}=\begin{bmatrix}\hat{x}_1\\ \hat{x}_2\end{bmatrix}, A_0=\begin{bmatrix}-k_1 & 1\\ -k_2 & 0\end{bmatrix}, \bar{k}=\begin{bmatrix}k_1\\ k_2\end{bmatrix}, \eta(\hat{x}_1)=\begin{bmatrix}-\hat{x}_1 & 0\\ 0 & -1\end{bmatrix},$$

$$a=\begin{bmatrix}a_1\\ a_0\end{bmatrix}, \varphi(\hat{x}_1)=\begin{bmatrix}\dot{\hat{x}}_1(-\hat{x}_1+1)\\ 0\end{bmatrix}, \sigma(\hat{x}_1)=\frac{1}{\hat{x}_1}.$$

By properly choosing k_1 and k_2, the matrix A_0, which determines the properties of the K-filters, can be made Hurwitz. Thus, a symmetric positive definite matrix P exists, i.e., $P>0$, $P=P^T$, such that

$$PA_0+A_0^TP=-I, \tag{6}$$

where I is an identity matrix. Next, we decide on the following observer structure:

$$\dot{\bar{x}}=A_0\bar{x}+\bar{k}\hat{y}+\eta(\hat{y})a+\varphi(\hat{y})+\begin{bmatrix}0 & b_0\end{bmatrix}^T\sigma(\hat{y})\hat{u}, \tag{7}$$

where $\bar{x}=\begin{bmatrix}\bar{x}_1 & \bar{x}_2\end{bmatrix}^T$ is the state estimates of $\hat{x}$,

$$\eta(\hat{y})=\begin{bmatrix}-\hat{y} & 0\\ 0 & -1\end{bmatrix}, \varphi(\hat{y})=\begin{bmatrix}\dot{\hat{y}}(-\hat{y}+1)\\ 0\end{bmatrix}, \sigma(\hat{y})=\frac{1}{\hat{y}}.$$

Equation (7) can be further expressed as

$$\dot{\bar{x}}=A_0\bar{x}+\bar{k}\hat{y}+\varphi(\hat{y})+F(\hat{y},\hat{u})^T\Theta, \tag{8}$$

where $\Theta=\begin{bmatrix}b_0 & a^T\end{bmatrix}^T\in\mathbb{R}^3$ is a parameter vector and

$$F(\hat{y},\hat{u})^T=\begin{bmatrix}0 & \\ \sigma(\hat{y})\hat{u} & \eta(\hat{y})\end{bmatrix}\in\mathbb{R}^{2\times 3}.$$

Define the state estimated error as $\varepsilon\triangleq\begin{bmatrix}\varepsilon_{\hat{x}_1} & \varepsilon_{\hat{x}_2}\end{bmatrix}^T\triangleq\hat{x}-\bar{x}$. Then the state space description of the estimated error can be obtained by subtracting (8) from (5), i.e.,

$$\dot{\varepsilon}=A_0\varepsilon+\Delta, \tag{9}$$

where

$$\Delta=-\bar{k}\hat{d}+\begin{bmatrix}\hat{d} & 0\\ 0 & 0\end{bmatrix}a+\begin{bmatrix}-\hat{d}-\hat{d}\dot{\hat{d}}+\frac{d}{d\theta}(\hat{d}\hat{y})\\ 0\end{bmatrix}+\begin{bmatrix}0\\ b_0\end{bmatrix}\frac{\hat{d}}{\hat{y}(\hat{y}-\hat{d})}\hat{u}$$

Define the state estimate $\bar{x}\triangleq\xi+\Omega^T\Theta$ such that $\xi=\begin{bmatrix}\xi_{11} & \xi_{12}\end{bmatrix}^T\in\mathbb{R}^2$ and $\Omega\in\mathbb{R}^{2\times 3}$. Substituting this definition into (8) gives

$$\dot{\xi}+\dot{\Omega}^T\Theta=A_0\xi+\bar{k}\hat{y}+\varphi(\hat{y})+\left(A_0\Omega^T+F(\hat{y},\hat{u})^T\right)\Theta.$$

Thus the following two filters may be employed

$$\dot{\xi}=A_0\xi+\bar{k}\hat{y}+\varphi(\hat{y}),\ \dot{\Omega}^T=A_0\Omega^T+F(\hat{y},\hat{u})^T. \tag{10}$$

Define $\Omega^T=\begin{bmatrix}v_0 & \Xi\end{bmatrix}$, i.e., the first column $v_0\triangleq\begin{bmatrix}v_{01} & v_{02}\end{bmatrix}^T\in\mathbb{R}^2$ and the rest as $\Xi\in\mathbb{R}^{2\times 2}$. The second equation in (10) can be split into two filters:

$$\dot{v}_0=A_0v_0+e_2\sigma(\hat{y})\hat{u},\ \dot{\Xi}=A_0\Xi+\eta(\hat{y}) \tag{11}$$

where $e_2=\begin{bmatrix}0 & 1\end{bmatrix}^T$ denotes one of the basis vector for $\mathbb{R}^2$. Expressing $\Xi=\begin{bmatrix}\Xi_1 & \Xi_2\end{bmatrix}^T$, where $\Xi_1\in\mathbb{R}^2$ and $\Xi_2\in\mathbb{R}^2$, and with the definition of the state estimates, we obtain

$$\bar{x}_1=\xi_{11}+v_{01}b_0+\Xi_1^Ta,\ \bar{x}_2=\xi_{12}+v_{02}b_0+\Xi_2^Ta. \tag{12}$$

Equation (11) and (12) will be used in the subsequent design.

IV. Output Feedback Adaptive Backstepping Repetitive Control System

With the definition of the state estimated error ε, the output equation in (4) can be expressed as

$$\hat{y}=\hat{x}_1+\hat{d}=\bar{x}_1+\varepsilon_{\hat{x}_1}+\hat{d}. \tag{13}$$

Substituting the first equation of (12) into (13), we have

$$\hat{y}=\xi_{11}+v_{01}b_0+\Xi_1^Ta+\varepsilon_{\hat{x}_1}+\hat{d}. \tag{14}$$

In accordance with the controller design procedure with K-filters [13], adaptive backstepping is applied to the following system

$$\dot{\hat{y}}=\dot{\xi}_{11}+(-k_1v_{01}+v_{02})b_0+\dot{\Xi}_1^Ta+\dot{\varepsilon}_{\hat{x}_1}+\dot{\hat{d}},\ \dot{v}_{02}=-k_2v_{01}+\sigma\hat{u}. \tag{15}$$

Define $\tilde{a}=\begin{bmatrix}\tilde{a}_1 & \tilde{a}_0\end{bmatrix}^T$, where $\tilde{a}_1$ and $\tilde{a}_0$ are the estimates of a_1 and a_0, and $\tilde{b}_0$ is the estimate of b_0. Introduce a new pair of coordinates

$$z_1=\hat{y}-\hat{y}_r,\ z_2=v_{02}-\alpha_1, \tag{16}$$

where $\hat{y}_r$ is the reference command and α_1 is a virtual input. With respect to the new coordinates, (15) can be expressed as

$$\begin{aligned}\dot{z}_1&=\dot{\xi}_{11}+(-k_1v_{01}+v_{02})b_0+\dot{\Xi}_1^Ta+\dot{\varepsilon}_{\hat{x}_1}+\dot{\hat{d}}-\dot{\hat{y}}_r,\\ \dot{z}_2&=-k_2v_{01}+\sigma\hat{u}-\dot{\alpha}_1.\end{aligned} \tag{17}$$

Regard v_{02} as an input to stabilize the first state equation in (17). To design v_{02}, consider a Lyapunov function $V_1=(1/2)z_1^2$ and calculate its derivative

$$\dot{V}_1=z_1\left(\dot{\xi}_{11}+(-k_1v_{01}+v_{02})b_0+\dot{\Xi}_1^Ta+\dot{\varepsilon}_{\hat{x}_1}+\dot{\hat{d}}-\dot{\hat{y}}_r\right).$$

Specify $v_{02}=\alpha_1=\bar{\alpha}_1/\tilde{b}_0$, where

$$\bar{\alpha}_1=-\dot{\xi}_{11}+k_1v_{01}\tilde{b}_0-\dot{\Xi}_1^T\tilde{a}+\dot{\hat{y}}_r-c_1z_1-d_1z_1-d_1k_1^2z_1.$$

Define the parameter error vector as

$$\Phi\triangleq\Theta-\tilde{\Theta}=\begin{bmatrix}\Phi_{b_0} & \Phi_a\end{bmatrix}^T,$$

where $\Phi_{b_0}=b_0-\tilde{b}_0$ and $\Phi_a=\begin{bmatrix}a_1-\tilde{a}_1 & a_0-\tilde{a}_0\end{bmatrix}^T$. The first state equation in (17) becomes

$$\begin{aligned}\dot{z}_1=&-k_1v_{01}\Phi_{b_0}+z_2\left(\tilde{b}_0+\Phi_{b_0}\right)+\alpha_1\Phi_{b_0}+\dot{\Xi}_1^T\Phi_a+\dot{\varepsilon}_{\hat{x}_1}+\dot{\hat{d}}\\ &-c_1z_1-d_1z_1-d_1k_1^2z_1,\end{aligned}$$

where c_1 and d_1 are two positive adjustable parameter. The derivative of V_1 now becomes

$$\begin{aligned}\dot{V}_1=&-z_1k_1v_{01}\Phi_{b_0}+z_1z_2\tilde{b}_0+z_1z_2\Phi_{b_0}+z_1\alpha_1\Phi_{b_0}+z_1\dot{\Xi}_1^T\Phi_a\\ &-c_1z_1^2-d_1z_1^2-d_1k_1^2z_1^2+z_1\left(\dot{\varepsilon}_{\hat{x}_1}+\dot{\hat{d}}\right).\end{aligned} \tag{18}$$

Before we proceed, calculation of $\dot{\alpha}_1$ needs to be performed, i.e.,

$$\dot{\alpha}_1=F_1+F_2\left(\dot{\xi}_{11}+(-k_1v_{01}+v_{02})b_0+\dot{\Xi}_1^Ta+\dot{\varepsilon}_{\hat{x}_1}+\dot{\hat{d}}\right)-F_3, \tag{19}$$

where

$$F_1 = \frac{1}{\tilde{b}_0}\Big(-\ddot{\xi}_{11} + k_1\dot{v}_{01}\tilde{b}_0 + k_1 v_{01}\dot{\tilde{b}}_0 - \ddot{\Xi}_1^T\tilde{a} - \dot{\Xi}_1^T\dot{\tilde{a}} + \ddot{y}_r$$

$$+\left(c_1 + d_1 + d_1 k_1^2\right)\dot{y}_r\Big),\ F_2 = -\frac{1}{\tilde{b}_0}\left(c_1 + d_1 + d_1 k_1^2\right),\ F_3 = \frac{1}{\tilde{b}_0}\alpha_1\dot{\tilde{b}}_0.$$

Consider another Lyapunov function

$$V_2 = V_1 + \frac{1}{2}z_2^2 + \sum_{i=1}^{2}\frac{1}{2d_i}\varepsilon^T P\varepsilon + \frac{1}{2}\Phi^T\Gamma^{-1}\Phi\ ,$$

where Γ is a symmetric positive definite matrix, i.e., $\Gamma = \Gamma^T > 0$ and d_i is an adjustable positive constant. With (6), (9), (18) and (19), we obtain the derivative of V_2, i.e.,

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + z_2\left(-k_2 v_{01} + \sigma\hat{u}\right)\\ &-z_2\left[F_1 + F_2\left(\dot{\xi}_{11} + \left(-k_1 v_{01} + v_{02}\right)b_0 + \dot{\Xi}_1^T a + \dot{\varepsilon}_{\hat{x}_1} + \dot{\hat{d}}\right) - F_3\right]\\ &-\sum_{i=1}^{2}\varepsilon^T\varepsilon\big/\left(2d_i\right) + \sum_{i=1}^{2}\left(\Delta^T P\varepsilon + \varepsilon^T P\Delta\right)\big/\left(2d_i\right) + \Phi^T\Gamma^{-1}\dot{\Phi}.\end{aligned}$$

Now we can specify

$$\begin{aligned}\hat{u} = \Big[&-z_1\tilde{b}_0 + k_2 v_{01} + F_1 + F_2\left(\dot{\xi}_{11} + \left(-k_1 v_{01} + v_{02}\right)\tilde{b}_0 + \dot{\Xi}_1^T\tilde{a}\right)\\ &-F_3 - c_2 z_2 - d_2 F_2^2 z_2 - d_2 k_1^2 F_2^2 z_2 + \hat{u}_{\hat{R}} - c_3 z_2 - \hat{l}^2(\theta)z_2 + \ddot{y}_r\big/\tilde{b}_0\Big]\Big/\sigma\end{aligned} \tag{20}$$

where c_2 and c_3 are adjustable positive constants, $\hat{u}_{\hat{R}}$ is a designable input used to focus on reduction of periodic output disturbance and the estimated error $\varepsilon_{\hat{x}_1}$, and $\hat{l}(\theta)$ is a designable function to be used to ensure stability of the overall system. Substituting (20) back into $\dot{V}_2$ and defining $e_1 \triangleq [1\quad 0]^T$, i.e., the other basis vector for $\mathbb{R}^2$, we obtain

$$\begin{aligned}\dot{V}_2 &= -c_1 z_1^2 - c_2 z_2^2 - d_1\left(z_1 - \varepsilon_{\hat{x}_2}\big/\left(2d_1\right)\right)^2 - d_1\left(k_1 z_1 + \varepsilon_{\hat{x}_1}\big/\left(2d_1\right)\right)^2\\ &\quad - d_2\left(F_2 z_2 + \varepsilon_{\hat{x}_2}\big/\left(2d_2\right)\right)^2 - d_2\left(k_1 F_2 z_2 - \varepsilon_{\hat{x}_1}\big/\left(2d_2\right)\right)^2\\ &+\sum_{i=1}^{2}\left(\Delta^T P\varepsilon + \varepsilon^T P\Delta\right)\big/\left(2d_i\right) + \left(z_1 - z_2 F_2\right)\left(e_1^T\Delta + \dot{\hat{d}}\right)\\ &+\left(-\dot{\Theta}^T\Gamma^{-1} + W^T\right)\Phi\ + z_2\ddot{y}_r\big/\tilde{b}_0 + \hat{u}_{\hat{R}} - \hat{l}^2(\theta)z_2^2 - c_3 z_2^2,\end{aligned}$$

where

$$W = \left[-z_1 k_1 v_{01} + z_1 z_2 + z_1\alpha_1 - z_2 F_2\dot{v}_{01}\quad z_1\dot{\Xi}_1^T - z_2 F_2\dot{\Xi}_1^T\right]^T$$

and Γ^{-1} can be usually specified as

$$\Gamma^{-1} = \begin{bmatrix}\gamma_{b_0}^{-1} & 0 & 0\\ 0 & \gamma_{a_1}^{-1} & 0\\ 0 & 0 & \gamma_{a_0}^{-1}\end{bmatrix},$$

in which $\gamma_{b_0}, \gamma_{a_1}, \gamma_{a_0}$ are three adjustable positive constants. The parameter update law is specified in order to cancel Φ related terms in $\dot{V}_2$, i.e.,

$$\begin{cases}\dot{\hat{b}} = P_R\left(\bullet\right),\\ \dot{\tilde{a}} = \begin{bmatrix}\gamma_{a_1} & 0\\ 0 & \gamma_{a_0}\end{bmatrix}\left(z_1\dot{\Xi}_1^T - z_2 F_2\dot{\Xi}_1^T\right)^T,\end{cases} \tag{21}$$

where $\bullet = \gamma_{b_0}\left(-z_1 k_1 v_{01} + z_1 z_2 + z_1\alpha_1 - z_2 F_2\dot{v}_{01}\right)$ and

$$P_R\left(\bullet\right) = \begin{cases}0 & \text{if } \tilde{b}_0 = \tilde{b}_{0\_\min} \text{ and } \dot{\tilde{b}}_0 < 0,\\ 0 & \text{if } \tilde{b}_0 = \tilde{b}_{0\_\max} \text{ and } \dot{\tilde{b}}_0 > 0,\\ \bullet & \text{otherwise.}\end{cases}$$

The projection function $P_R(\bullet)$ is imposed in order to prevent the estimated parameters from leaving the allowable variation set [11][16]. With the control input and the parameter update law specified as (20) and (21), $\dot{V}_2$ is reduced to

$$\begin{aligned}\dot{V}_2 &= -c_1 z_1^2 - c_2 z_2^2 - d_1\left(z_1 - \varepsilon_{\hat{x}_2}\big/\left(2d_1\right)\right)^2\\ &-d_1\left(k_1 z_1 + \varepsilon_{\hat{x}_1}\big/\left(2d_1\right)\right)^2 - d_2\left(F_2 z_2 + \varepsilon_{\hat{x}_2}\big/\left(2d_2\right)\right)^2\\ &-d_2\left(k_1 F_2 z_2 - \varepsilon_{\hat{x}_1}\big/\left(2d_2\right)\right)^2 + \sum_{i=1}^{2}\left(\Delta^T P\varepsilon + \varepsilon^T P\Delta\right)\big/\left(2d_i\right)\\ &+\left(z_1 - z_2 F_2\right)\left(e_1^T\Delta + \dot{\hat{d}}\right) + z_2\ddot{y}_r\big/\tilde{b}_0 + \hat{u}_{\hat{R}} - \hat{l}^2(\theta)z_2^2 - c_3 z_2^2.\end{aligned} \tag{22}$$

Note that all terms in (22) are minus complete squares except those in the last two lines.

In this paper, we consider a reduced-order and attenuated repetitive controller [14] whose continuous-position (as opposed to the definition of continuous-time) version can be expressed as

$$\hat{R}(\tilde{s}) = \prod_{i=1}^{k}\frac{\tilde{s}^2 + 2\zeta_i\omega_{ni}\tilde{s} + \omega_{ni}^2}{\tilde{s}^2 + 2\xi_i\omega_{ni}\tilde{s} + \omega_{ni}^2},$$

where k is the number of periodic frequencies to be rejected, ω_{ni} is determined based on the i^{th} disturbance frequency in rad/rev, and ξ_i and ζ_i are two damping ratios that satisfy $0 < \xi_i < \zeta_i < 1$. As shown in Figure 1, the tracking error z_1 and the control input $\hat{u}_{\hat{R}}$ is related by

$$\hat{u}_{\hat{R}} = -\hat{R}\left(\tilde{s}\right)\hat{C}\left(\tilde{s}\right)z_1, \tag{23}$$

where $\hat{C}(\tilde{s})$ is another controller which can be designed to reduce the effect of $\varepsilon_{\hat{x}_1}$ on the output. Differentiate (14) until the term involving control input $\hat{u}$ appears, namely,

$$\ddot{y} = \ddot{\xi}_{11} - \left(k_1\dot{v}_{01} + k_2 v_{01}\right)b_0 + b_0\sigma\left(\dot{y}\right)\hat{u} + \ddot{\Xi}_1^T a + \ddot{\varepsilon}_{\hat{x}_1} + \ddot{\hat{d}}. \tag{24}$$

Substituting (20) into (24), we have

$$\ddot{y} = P(.) + \hat{u}_{\hat{R}}b_0 + \ddot{y}_r + \ddot{\varepsilon}_{\hat{x}_1} + \ddot{\hat{d}}\ , \tag{25}$$

where

$$\begin{aligned}P(.) &= \ddot{\xi}_{11} - \left(k_1\dot{v}_{01} + k_2 v_{01}\right)b_0 + b_0\Big[-z_1\tilde{b}_0 + k_2 v_{01} + F_1\\ &+F_2\left(\dot{\xi}_{11} + \left(-k_1 v_{01} + v_{02}\right)\tilde{b}_0 + \dot{\Xi}_1^T\tilde{a}\right) - F_3 - c_2 z_2\\ &-d_2 F_2^2 z_2 - d_2 k_1^2 F_2^2 z_2 - c_3 z_2 - \hat{l}(\theta)z_2\Big] + \ddot{\Xi}_1^T a + \Phi_{b_0}\ddot{y}_r\big/\tilde{b}_0.\end{aligned}$$

Substituting (23) into (25) and taking Laplace transform, we arrive at

$$z_1 = M(\tilde{s})P(.) + M(\tilde{s})\tilde{s}^2\left(\varepsilon_{\hat{x}_1} + \hat{d}\right),$$

where

$$M(\tilde{s}) \triangleq 1\Big/\left[\tilde{s}^2 + b_0\hat{R}(\tilde{s})\hat{C}(\tilde{s})\right].$$

We see that $\hat{R}(\tilde{s})$ and $\hat{C}(\tilde{s})$ in $M(\tilde{s})$ can be suitably designed to reduce the effect of the disturbance $\hat{d}$ and the state estimated error $\varepsilon_{\hat{x}_1}$ on the output. Substituting (23) into (22), we obtain

$$\dot{V}_2 = \Gamma(\bullet) - \hat{R}(\tilde{s})\hat{C}(\tilde{s})z_1 - \hat{l}^2(\theta)z_2^2 - c_3 z_2^2,$$

where

$$\Gamma(\bullet) = -c_1 z_1^2 - c_2 z_2^2 - d_1\left(z_1 - \varepsilon_{\hat{x}_2}\big/(2d_1)\right)^2 - d_1\left(k_1 z_1 + \varepsilon_{\hat{x}_1}\big/(2d_1)\right)^2$$
$$- d_2\left(F_2 z_2 + \varepsilon_{\hat{x}_2}\big/(2d_2)\right)^2 - d_2\left(k_1 F_2 z_2 - \varepsilon_{\hat{x}_1}\big/(2d_2)\right)^2$$
$$+ \sum_{i=1}^{2}\left(\Delta^T P\varepsilon + \varepsilon^T P\Delta\right)\big/(2d_i) + (z_1 - z_2F_2)\left(e_1^T\Delta + \dot{\hat{d}}\right) + z_2\ddot{y}_r\big/\tilde{b}_0.$$

$\Gamma(\bullet)$ can be shown to be negative semidefinite, i.e., $\Gamma(\bullet) \le 0$ by following the steps proposed in [2]. The remaining three terms in $\dot{V}_2$ can be made negative semidefinite by properly choosing the constant c_3 and the function $\hat{l}(\theta)$.

V. Simulation Results

The proposed output feedback ABRC scheme is applied to a brushless dc motor system. The actual system is assumed to be a 2nd order system as described in (1) with $a_0 = 5155$, $a_1 = 1138$, $b_0 = 140368$, and $b_1 = 0$. The parameters are specified in accordance with the system identification results for an actual motor system from Shinano Kenshi Corp. For verification purpose, the output disturbance is assumed to be a rectangular periodic signal (with amplitude switching between -1 and 1), i.e.,

$$\hat{d} = 0.1\big/(\tilde{s}/20+1)^3\left[\sum_{l=-\infty}^{\infty}(-1)^l\,\Pi(\theta-1-l)\right],$$

where

$$\Pi(\theta) = \begin{cases} 1 & |\theta| < 1, \\ 0.5 & |\theta| = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Note that the disturbance has been low-pass filtered so that it is continuously differentiable. Parameters of repetitive controller are specified to target the fundamental frequency and the first three harmonic frequencies of the periodic disturbance, i.e.,

$$\hat{R}(\tilde{s}) = \prod_{i=1}^{4}\frac{\tilde{s}^2 + 2\zeta_i\omega_{ni}\tilde{s} + \omega_{ni}^2}{\tilde{s}^2 + 2\xi_i\omega_{ni}\tilde{s} + \omega_{ni}^2},$$

where $\varsigma_i = 0.9,\ \xi_i = 0.00009,\ \omega_{n1} = \pi,\ \omega_{n2} = 3\times\pi,$ $\omega_{n3} = 5\times\pi,\ \omega_{n4} = 7\times\pi$. Suppose that a motion control task demands the system to initially run at 40 rev/s and then speed up to 45 rev/s and finally speed down to 35 rev/s. To avoid getting infinite value when taking derivative, the reference command is specified to have smooth (instead of instant) change. Figure 2 compares the tracking performance of two scenarios. The figures on the left are for the pure output feedback adaptive backstepping design. The ones on the right are for the proposed output feedback ABRC design. Without repetitive control, the adaptive backstepping design has already shown superb tracking performance. We see that adding the repetitive control further reduces the magnitude of the tracking error (from 10^{-5} to 10^{-7}) without noticeable increase in the control input.

VI. Conclusion and Future Work

This paper presents the design of a new spatial-based repetitive control system, which can be applied to rotational motion systems with uncertain parameters operating at varying speeds and subject to spatially periodic disturbances. The proposed design combines two control paradigms, i.e., adaptive backstepping and repetitive control. The overall output feedback ABRC system can be shown to be stable and have bounded state estimated error and output tracking error. Feasibility and effectiveness of the proposed design are further justified by simulation. Although this paper only presents the design method for a 2nd order system, the proposed design may be extended to higher order systems, which is currently under our investigation.

References

[1] B. Mahawan and Z.-H. Luo, "Repetitive control of tracking systems with time-varying periodic references," International *Journal of Control,* 73(1), 1-10, 2000.

[2] B. Yao and L. Xu , "Output feedback adaptive robust control of uncertain linear systems with disturbances," *ASME Journal of Dynamic Systems, Measurement, and Control,* 128, 938-945, 2006.

[3] C.-L. Chen, G. T.-C. Chiu and Jan P. Allebach , "Banding reduction in EP processes using human contrast sensitivity function shaped photoconductor velocity control," *Journal of Imaging Science and Technology,* 47(3), 209-223, 2003.

[4] C.-L. Chen, G. T.-C. Chiu and Jan P. Allebach, "Robust spatial-sampling controller design for banding reduction in electrophotographic process," *Journal of Imaging Science and Technology,* 50(6), 1-7, 2006.

[5] C.-L. Chen and G. T.-C. Chiu, "Spatially periodic disturbance rejection with spatially sampled robust repetitive control," *To appear in ASME Journal of Dynamic Systems, Measurement, and Control.*

[6] C.-L. Chen and Y.-H. Yang, "Spatially Periodic Disturbance Rejection for Uncertain Rotational Motion Systems Using Spatial Domain Adaptive Backstepping Repetitive Control," accepted for presentation in 33rd Annual Conference of the IEEE Industrial Electronics Society (IECON; oral), Taipei, Taiwan, 2007.

[7] G. Kreisselmeier, "Adaptive Observers with Exponential Rate of Convergence," *IEEE Transactions on Automatic Control,* AC-22(1), 2-8, 1977.

[8] H. K. Khalil, *Nonlinear Systems (3rd edition)*. Upper Saddle River, NJ: Prentice Hall, 2002.

[9] Kanellakopoulos, P. V. Kokotovic and A. S. Morse, "Adaptive Output-Feedback Control of a Class of Nonlinear Systems," *In: Proc. IEEE Conference on Decision and Control,* 2, 1082-1087, 1991.

[10] M. Nakano, J.-H. She, Y. Mastuo, and T. Hino, "Elimination of position-dependent disturbances in constant-speed-rotation control systems," *Control Engineering Practice,* 4, 1241-1248, 1996.
[11] M. French and E. Rogers, "Approximate models for adaptive feedback linearization," *International Journal of Control,* 68(5), 1305-1321, 2000.
[12] M. Krstic, I. Kanellakopoulos and P. V. Kokotovic , "Nonliear Design of Adaptive Controllers for Linear Systems," *IEEE Transactions on Automatic Control,* 39 (4), 738-752, 1994.
[13] M. Krstic, I. Kanellakopoulos and P. Kokotovic, *Nonlinear and Adaptive Control Design.* New York, NY: John Wiley&Sons, 1995.
[14] R. C.-H. Lee and M.-C. Smith, "Repetitive control experiments for a CD player," *In: Proc. American Control Conference,* pp. 2682-2684, 1998.
[15] S. Hara, Y. Yamamoto, T. Omata, and M. Nakano, "Repetitive control system: A new type servo system for periodic exogenous signals," *IEEE Transactions on Automatic Control,* 33(7), 659-668, 1988.
[16] S. Sastry and A. Isidori, "Adaptive control of linearizable systems," *IEEE Transactions on Automatic Control,* 34(11), 1123-1131, 1989.
[17] Z.-J. Yang, K. Kunitoshi and S. Kanae, "Adaptive Robust Output Feedback Control of a Magnetic Levitation System by K-Filter Approach," *In: IEEE International Symposium on Intelligent Control,* 2004.

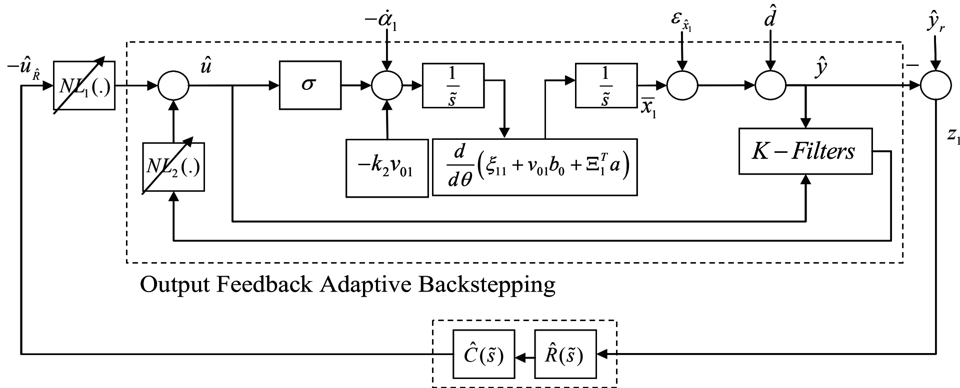

Figure 1: The control structure for the proposed adaptive robust repetitive control system.

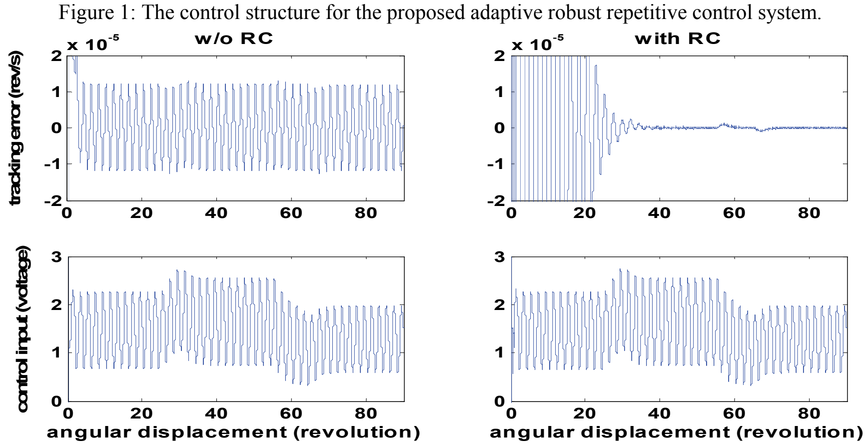

Figure 2: Comparison of tracking performance.