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Delay Independent Synchronization of Complex Network via Hybrid Control

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Abstract—**This paper studies synchronization properties of two classes of complex networks: directed time-varying network and undirected time-invariant network with constant edge weights. Based on Lyapunov stability theory, sufficient conditions for globally exponential and asymptotical synchronization are proposed for both of the two networks under control, which consists of an impulsive controller and a switching controller with time-invariant delayed. A numerical example is provided for illustration.**

I. INTRODUCTION

IN recent years, complex networks have attracted increasing attention in the scientific community because of the ubiquity of complex networks in sciences and societies naturally. Undoubtedly, many complex systems in nature can be modeled as networks [1, 2]. Furthermore, the real-world complex networks have the following features: (i) the dynamical system of coupled nodes may be normal systems, chaotic systems, or more complex systems; (ii) the topology could be time-varying; (iii) the states of the nodes may change because of the occurrence of impulses or switches, and the topology may change instantaneously, e.g., a star network may switch to a nearest-neighbor network; (iv) there exists time delays and some of them cannot be ignored.

One of the interesting phenomena is the synchronization of all dynamical nodes in complex network [3-9]. Nijmeijer et al studied the existence and stability of the linear invariant synchronous state in a simple coupled identical dynamical network through Lyapunov's direct method in Ref. [3]. A previously common method is to make the nonlinear nodes linear around the synchronous state and the coupling configuration matrix diagonal thereby [4-6]. For the networks which can not achieve synchronization themselves, the introduced controller can control them to synchronize [7-8]. Criteria for locally and globally adaptive synchronization of an uncertain network are deduced in Ref. [7]. Chen et al proved that a single controller can pin a complex network to a homogenous solution [8].

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In particular, some practical complex networks may change suddenly and sharply and thus the modes switch simultaneously. This kind of networks can be found in many evolutionary processes, such as bursting rhythm models in pathology, optimal control models in economics and so on. For these networks, we need a different controller, and [9] presents a hybrid control strategy.

However, in reality, time-delay in signal transmission and response of controller may cause time-delay in the controller (see more details in [10]). Unfortunately, the control with delay in complex network has not been studied yet.

In this paper, synchronization problems of two classes of complex networks, namely, directed time-varying network and undirected time-invariant network with constant edge weight are studied respectively. The control strategy introduced in this paper is a hybrid control method, which is a combination of an impulsive controller and a switching controller with constant time delay. This control strategy shows the advantages as follows: (i) it only uses the small control impulses in different modes [11]; (ii) it can reduce the information redundancy and simple to implement; (iii) it also acts on the complex system, of which behaviors are unpredictable; (iv) time delay in controller represents the response lag of each node, which can represent networks more realistically. Each isolated node of complex network under this hybrid control may work on several different modes according to different switching intervals. Based on the Lyapunov stability theory, sufficient conditions for globally exponential and asymptotical synchronization of both networks are obtained.

The organization of this paper is as follows. In section 2, synchronization of a class of directed time-varying network is discussed. A class of undirected time-invariant network is then discussed in section 3. A numerical example is given to verify the effectiveness of the proposed control methods in section 4.

II. SYNCHRONIZATION ANALYSIS OF HYBRID CONTROL COMPLEX NETWORK WITH TIME-VARYING TOPOLOGY

Consider a complex dynamical network with coupling time-delay. The network consists of N identical coupled nodes, of which each node is an n-dimensional dynamical subsystem with linear and nonlinear parts, which is described by

$$\dot{x}_i(t) = Ax_i(t) + g(t, x_i(t)) + \sum_{j=1}^{N} c_{ij}(t)\Gamma(t)x_j(t),$$
$$i = 1,2,\cdots,N, \quad (1)$$

where $t \in R_+$, $x_i(t) = (x_{i1}(t),\cdots,x_{in}(t))^T \in R^n$ is the state variable of node i. $g : R_+ \times R^n \to R^n$ is a nonlinear vector-valued function with $g(t,0) \equiv 0$. A is a known $n \times n$ matrix. $\Gamma(t) = (r_{ij}(t))_{n\times n} : R^+ \to \in R^{n\times n}$ is the inner-coupling link matrix at time t. If two coupled nodes are linked through their i th and j th state variables respectively, $r_{ij}(t) \neq 0(1 \le i, j \le n)$, otherwise, $r_{ij}(t) = 0(1 \le i, j \le n)$. $C(t) = (c_{ij}(t))_{n\times n}$ is the coupling configuration matrix. If there is a connection between node i and node j ($i \neq j$), $c_{ij}(t) \neq 0$; else, $c_{ij}(t) = 0(i \neq j)$, and the diagonal elements of matrix $C(t)$ are defined by

$$c_{ii}(t) = -\sum_{j=1, j\neq i}^{N} c_{ij}(t), i = 1,\cdots,N .$$

Remark 1. The varying asymmetric coupling matrix $C(t) = (c_{ij}(t))_{n\times n}$ shows two characters of the network: (i) the network is directed, i.e., $c_{ij}(t) \neq c_{ji}(t)$; (ii) the topology of the network is time-varying, i.e., the link from two corresponding nodes i to j may cut when $c_{ij}(t)$ changes from nonzero to zero, vice versa.

The network (1) is said to achieve synchronization when $x_i(t) = s(t)$, $(i = 1,2,\cdots,N)$ as $t \to \infty$, where $s(t)$ be a solution of an isolate node of the network (1) without coupling, i.e.,

$$\dot{s}(t) = As + g(t,s) . \quad (2)$$

As an isolate node of the network, $s(t)$ can be an equilibrium point, a nontrivial periodic orbit, or even a chaotic orbit.

Let the synchronization error as

$$e_i(t) = x_i(t) - s(t) . \quad (3)$$

For system (1), we design a hybrid impulsive and switching controller $u = u_1 + u_2$ as follows:

$$u_1 = \sum_{k=1}^{\infty} B_\sigma e_i(t-\tau) l_k(t),$$
$$u_2 = \sum_{k=1}^{\infty} E_k e_i(t)\delta(t-t_k), \quad (4)$$

where B_σ and E_k are $n \times n$ constant matrices, with switching signal $\sigma := \sigma(i,k)$. Let $L = \{1,2,\cdots N\}$, then $\sigma : L \times Z^+ \to \{1,2,\cdots,m\}$, $\delta(\bullet)$ is the Dirac impulse function, and $l_k(t)$ is the ladder function, $l_k(t) = 1$ for $t_k < t \le t_{k+1}$, otherwise, $l_k(t) = 0$, with discontinuity points $t_1 < t_2 < \cdots < t_k < \cdots$, $\lim_{k\to\infty} t_k = \infty$ where $t_1 > t_0$. Then from (1), (2), (3) and (4), the synchronization error system can be described as:

$$\begin{cases} \dot{e}_i(t) = Ae_i(t) + B_\sigma e_i(t-\tau) + \tilde{g}(t, x_i, s) \\ \qquad + \sum_{j=1}^{N} c_{ij}(t)\Gamma(t)e_j(t), t \in (t_{k-1}, t_k], \\ \Delta e_i(t) = e_i(t_k^+) - e(t_k) = E_k e_i(t), t = t_k, \\ e_i(t_0^+) = e_{i0}, k = 1,2,\cdots, i = 1,2,\cdots,N, \end{cases} \quad (5)$$

where $\tilde{g}(t, x_i, s) = g(t, x_i(t)) - g(t, s(t))$.

Lemma 1[12]. If X, Y are real matrices with appropriate dimensions, there exist a constant $\varepsilon > 0$, that

$$X^T Y + Y^T X \le \varepsilon X^T X + \varepsilon^{-1} Y^T Y .$$

Lemma 2[13]. Given a constant $\xi > 0$, then

$2x^T y \le \xi^{-1} x^T x + \xi y^T y$, for all $x \in R^n$ and $y \in R^n$.

Lemma 3[14]. If $P \in R^{n\times n}$ is a positive definite matrix, $Q \in R^{n\times n}$ is an symmetric matrix, then

$$\lambda_{\min}(P^{-1}Q)x^T Px \le x^T Qx$$
$$\le \lambda_{\max}(P^{-1}Q)x^T Px, x \in R^n ,$$

where $\lambda_{\min}(*)$ and $\lambda_{\max}(*)$ are the minimum and maximum eigenvalues of $*$, respectively.

The notations P_σ used throughout the Letter stands for some positive-definite matrices. Similar to Refs.[9, 13], the following assumptions are made for our results.

Assumption 1. For $t \in R_+$, there exists continuous function $\varphi(t) \ge 0$, which satisfy

$$\tilde{g}^T(t, x_i, s)P_\sigma e_i(t) \le \varphi(t)e_i^T(t)P_\sigma e_i(t) .$$

Assumption 2.

$$\|e_i(t-\tau)\|^2 \le \rho_k \|e_i(t)\|^2 , \; i = 1,\cdots,N ,$$

where $\rho_k = \dfrac{\lambda_{\min}(P_\sigma)}{\mu\lambda_{\max}(P_{\sigma-\mathrm{mode}(\tau)})\rho\eta_k}$, $\mu > 0$,

$$\rho = \frac{\max_{1\le\sigma\le m} \lambda_{\max}(P_\sigma)}{\min_{1\le\sigma\le m} \lambda_{\min}(P_\sigma)}, \quad (6)$$

$$\eta_k = \lambda_{\max}[(I + E_k)^T (I + E_k)] \ge 0, \; k = 1,2,\cdots, \quad (7)$$

and $[\sigma-\mathrm{mode}(\tau)]:=\sigma(i,\chi)$, where $\chi\in Z^{+}$ and $t_{\chi}<t-\tau<t_{\chi+1}$.

Assumption 3. If the state $e_i(t)$ of system (5) is in the σ mode, the delayed state $e_i(t-\tau)$ is in the $\sigma-\mathrm{mode}(\tau)$ mode of node i.

For convenience, define

$$\gamma(t)=\max_{1\le\sigma\le m}\left\{\lambda_{\max}P_{\sigma}^{-1}\left[\left(A^{T}P_{\sigma}+P_{\sigma}A+2\varphi(t)P_{\sigma}\right)+\frac{I}{\xi}+\sum_{j=1}^{N}\varepsilon\max_{1\le i\le N}(c_{ji}(t))\right]\right\}, \tag{8}$$

$$\theta=\max_{1\le\sigma\le m}\left\{\lambda_{\max}\left[P_{\sigma-\mathrm{mode}(\tau)}^{-1}(\xi B_{\sigma}^{T}P_{\sigma}^{2}B_{\sigma})\right]\right\}, \tag{9}$$

$$\hat{\gamma}(t)=\gamma(t)+\theta(\mu\rho\eta_k)^{-1}, \tag{10}$$

where $\varepsilon>0$, $\xi>0$ and $\mu>0$ are constants.

Theorem 1. Suppose that Assumptions 1-3 are satisfied.

i) If there exist two constants α, β satisfying $\beta\ge\alpha\ge0$ and $\hat{\gamma}(t)\le-\beta<0$ such that

$$\ln\rho\eta_k-\alpha(t_k-t_{k-1})\le0, k=1,2,\cdots, \tag{11}$$

then the trivial solution of the error system (5) is globally exponentially stable, which implies network synchronization is reached exponentially by every node in the network (1) under control (4).

ii) If $\hat{\gamma}(t)\ge0$ and there exists a constant $\alpha\ge1$ such that

$$\ln\alpha\rho\eta_k+\int_{t_k}^{t_{k+1}}\hat{\gamma}(s)ds\le0, k=1,2,\cdots, \tag{12}$$

then $\alpha=1$ implies that the trivial solution of the error system (5) is stable, and $\alpha>1$ implies that the trivial solution of the error system (5) is asymptotically stable which implies that network synchronization is reached asymptotically by every node in the network (1) under control (4).

Proof. Construct the Lyapunov function as

$$V(t):V(e(t))=\sum_{i=1}^{N}e_i^{T}(t)P_{\sigma}e_i(t). \tag{13}$$

For any $t\in(t_{k-1},t_k]$, the total derivative of $V(t)$ with respect to (5) is

$$\dot{V}(t)=\sum_{i=1}^{N}\left\{\dot{e}_i^{T}(t)P_{\sigma}e_i(t)+e_i^{T}(t)P_{\sigma}\dot{e}_i(t)\right\}$$
$$=\sum_{i=1}^{N}\left\{e_i^{T}(t)\left(A^{T}P_{\sigma}+P_{\sigma}A\right)e_i(t)\right.$$
$$+2\tilde{g}^{T}(t,x_i,s)P_{\sigma}e_i(t)+2e_i^{T}(t)P_{\sigma}B_{\sigma}e_i(t-\tau)$$
$$\left.+\sum_{j=1}^{N}c_{ij}(t)\left(e_j^{T}(t)\Gamma^{T}(t)P_{\sigma}e_i(t)+e_i^{T}(t)P_{\sigma}\Gamma(t)e_j(t)\right)\right\}.$$

According to Lemma 1 and Assumption 1, there exist a constant $\varepsilon>0$ and continuous function $\varphi(t)\ge0$, such that

$$\dot{V}(t)\le\sum_{i=1}^{N}\left\{e_i^{T}(t)\left(A^{T}P_{\sigma}+P_{\sigma}A+2\varphi(t)P_{\sigma}\right)e_i(t)\right.$$
$$+2e_i^{T}(t)P_{\sigma}B_{\sigma}e_i(t-\tau)$$
$$\left.+\sum_{j=1}^{N}c_{ij}(t)\left(\varepsilon e_j^{T}(t)e_j(t)+\varepsilon^{-1}e_i^{T}(t)P_{\sigma}\Gamma(t)\Gamma^{T}(t)P_{\sigma}e_i(t)\right)\right\}.$$

From Lemma 2, there exist a constant $\xi>0$, such that

$$2e_i^{T}(t)P_{\sigma}B_{\sigma}e_i(t-\tau)$$
$$\le e_i^{T}(t)\frac{I}{\xi}e_i(t)+e_i^{T}(t-\tau)\xi B_{\sigma}^{T}P_{\sigma}^{2}B_{\sigma}e_i(t-\tau),$$

from $\sum_{j=1}^{N}c_{ij}(t)=0$,

$$\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}(t)\varepsilon e_j^{T}(t)e_j(t)=\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ji}(t)\varepsilon e_i^{T}(t)e_i(t).$$

Then, we obtain

$$\dot{V}(t)\le\sum_{i=1}^{N}\left\{\lambda_{\max}P_{\sigma}^{-1}\left[\left(A^{T}P_{\sigma}+P_{\sigma}A+2\varphi(t)P_{\sigma}\right)\right.\right.$$
$$\left.+\sum_{j=1}^{N}\left(c_{ji}(t)\varepsilon\right)+\frac{I}{\xi}\right]e_i^{T}(t)P_{\sigma}e_i(t)$$
$$\left.+\lambda_{\max}\left[P_{\sigma}^{-1}(\xi B_{\sigma}^{T}P_{\sigma}^{2}B_{\sigma})\right]e_i^{T}(t-\tau)P_{\sigma}e_i(t-\tau)\right\}$$
$$\le\max_{1\le\sigma\le m}\left\{\lambda_{\max}P_{\sigma}^{-1}\left[\left(A^{T}P_{\sigma}+P_{\sigma}A+2\varphi(t)P_{\sigma}\right)+\frac{I}{\xi}+\sum_{j=1}^{N}\varepsilon\max_{1\le i\le N}(c_{ji}(t))\right]\right\}\sum_{i=1}^{N}e_i^{T}(t)P_{\sigma}e_i(t)$$
$$+\max_{1\le\sigma\le m}\left\{\lambda_{\max}\left[P_{\sigma-\mathrm{mode}(\tau)}^{-1}(\xi B_{\sigma}^{T}P_{\sigma}^{2}B_{\sigma})\right]\right\}$$
$$\times\sum_{i=1}^{N}e_i^{T}(t-\tau)P_{\sigma-\mathrm{mode}(\tau)}e_i(t-\tau)$$
$$=\gamma(t)V(t)+\theta V(t-\tau).$$

From assumption 2, we have

$$V(t)\ge\sum_{i=1}^{N}\lambda_{\min}(P_{\sigma})e_i^{T}(t)e_i(t)$$
$$\ge\sum_{i=1}^{N}\mu\lambda_{\max}(P_{\sigma-\mathrm{mode}(\tau)})\rho\eta_k e_i^{T}(t-\tau)e_i(t-\tau)$$

$$\geq \sum_{i=1}^{N} \mu\rho\eta_k e_i^T(t-\tau) P_{\sigma-\mathrm{mode}(\tau)} e_i(t-\tau)$$

$$= \mu\rho\eta_k V(t-\tau).$$

Consequently, $V(t-\tau) \leq (\mu\rho\eta_k)^{-1} V(t)$.

Thus, $\dot{V}(t) = (\gamma(t) + \theta(\mu\rho\eta_k)^{-1}) V(t) = \hat{\gamma}(t) V(t)$, which implies that

$$V(t) \leq V(t_{k-1}^+) \exp \int_{t_{k-1}}^{t} \hat{\gamma}(s) ds, t \in (t_{k-1}, t_k]. \quad (14)$$

From (13) and (14), for $t \in (t_{k-1}, t_k]$, we can obtain

$$\max_{1\leq\sigma\leq m} \lambda_{\max}(P_\sigma) \sum_{i=1}^{N} e_i^T(t_{k-1}^+) e_i(t_{k-1}^+) \exp \int_{t_{k-1}}^{t} \hat{\gamma}(s) ds$$

$$\geq \min_{1\leq i\leq N} \lambda_{\min}(P_\sigma) \sum_{i=1}^{N} e_i^T(t) e_i(t),$$

or

$$\varpi(t) \leq \rho \varpi(t_{k-1}^+) \exp \int_{t_{k-1}}^{t} \hat{\gamma}(s) ds, t \in (t_{k-1}, t_k], \quad (15)$$

where $\varpi(t) = \sum_{i=1}^{N} e_i^T(t) e_i(t)$.

On the other hand, when $t = t_k^+$,

$$\varpi(t_k^+) = \sum_{i=1}^{N} e_i^T(t_k^+) e_i(t_k^+)$$

$$= \sum_{i=1}^{N} [(1+E_k) e_i(t_k)]^T [(1+E_k) e_i(t_k)]$$

$$\leq \lambda_{\max}[(1+E_k)^T (1+E_k)] \sum_{j=1}^{N} e_i^T(t_k) e_i(t_k)$$

$$\leq \eta_k \varpi(t_k), k = 1, 2, \cdots \quad (16)$$

From (15) and (16), for any $t \in (t_0, t_1]$,

$$\varpi(t) \leq \rho \varpi(t_0^+) \exp \int_{t_0}^{t} \hat{\gamma}(s) ds,$$

which leads to

$$\varpi(t_1) \leq \rho \varpi(t_0^+) \exp \int_{t_0}^{t_1} \hat{\gamma}(s) ds$$

and

$$\varpi(t_1^+) \leq \eta_1 \varpi(t_1) \leq \rho \eta_1 \varpi(t_0^+) \exp \int_{t_0}^{t_1} \hat{\gamma}(s) ds.$$

Similarly, for $t \in (t_1, t_2]$,

$$\varpi(t) \leq \rho \varpi(t_1^+) \exp \int_{t_1}^{t} \hat{\gamma}(s) ds$$

$$\leq \rho \eta_1 \varpi(t_1) \exp \int_{t_1}^{t} \hat{\gamma}(s) ds$$

$$\leq \rho^2 \eta_1 \varpi(t_0^+) \exp[\int_{t_0}^{t_1} \hat{\gamma}(s) ds] \exp[\int_{t_1}^{t} \hat{\gamma}(s) ds]$$

$$= \rho^2 \eta_1 \varpi(t_0^+) \exp \int_{t_0}^{t} \hat{\gamma}(s) ds.$$

In general, for $t \in (t_k, t_{k+1}]$,

$$\varpi(t) \leq \rho^{k+1} \varpi(t_0^+) \eta_1 \cdots \eta_k \exp \int_{t_0}^{t} \hat{\gamma}(s) ds. \quad (17)$$

i) If there exist two constants α, β satisfying $\beta \geq \alpha \geq 0$, $\hat{\gamma}(t) \leq -\beta < 0$ and (11), it follows from (14) and (17) that

$$\varpi(t) \leq \varpi(t_0^+) \rho^{k+1} \eta_1 \eta_2 \cdots \eta_k \exp[-\beta(t-t_0)]$$

$$= \varpi(t_0^+) \rho^{k+1} \eta_1 \eta_2 \cdots \eta_k \exp[-\alpha(t-t_0)] \cdot \exp[-(\beta-\alpha)(t-t_0)]$$

$$\leq \varpi(t_0^+) \rho^{k+1} \eta_1 \eta_2 \cdots \eta_k \exp[-\alpha(t_k-t_0)] \cdot \exp[-(\beta-\alpha)(t-t_0)]$$

$$= \rho \varpi(t_0^+) \rho \eta_1 \exp[-\alpha(t_1-t_0)] \rho \eta_2 \exp[-\alpha(t_2-t_1)] \cdots \rho \eta_k \exp[-\alpha(t_k-t_{k-1})] \exp[-(\beta-\alpha)(t-t_0)]$$

$$\leq \varpi(t_0^+) \rho \exp[-(\beta-\alpha)(t-t_0)], t \in (t_k, t_{k+1}].$$

Thus, the trivial solution of the error system (5) is globally exponentially stable, which implies network synchronization is reached exponentially by every node in the network (1) under control (4).

ii) If $\hat{\gamma}(t) \geq 0$, and there exists a constant $\alpha \geq 1$ satisfying (12), from (17) we obtain

$$\varpi(t) \leq \varpi(t_0^+) \rho^{k+1} \eta_1 \eta_2 \cdots \eta_k \exp \int_{t_0}^{t} \hat{\gamma}(s) ds$$

$$\leq \varpi(t_0^+) \rho^{k+1} \eta_1 \eta_2 \cdots \eta_k \exp \int_{t_0}^{t_{k+1}} \hat{\gamma}(s) ds$$

$$= \varpi(t_0^+) \rho \alpha^{-k} \exp[\int_{t_0}^{t_1} \hat{\gamma}(s) ds] \cdot \rho \eta_1 \alpha \exp[\int_{t_1}^{t_2} \hat{\gamma}(s) ds] \cdots \rho \eta_k \alpha \exp[\int_{t_k}^{t_{k+1}} \hat{\gamma}(s) ds]$$

$$\leq \varpi(t_0^+) \rho \alpha^{-k} \exp[\int_{t_0}^{t_1} \hat{\gamma}(s) ds], t \in (t_k, t_{k+1}],$$

then $\alpha = 1$ implies that the trivial solution of the error system (5) is stable, $\alpha > 1$ implies that the trivial solution of the error system (5) is asymptotically stable, therefore, network synchronization is reached asymptotically by every node in the network (1) under control (4).

The proof is completed.

III. SYNCHRONIZATION ANALYSIS OF HYBRID CONTROL COMPLEX NETWORK WITH INVARIANT TOPOLOGY

In section 2, we discuss a class of directed time-varying network. But some of real networks are with time-invariant topology. Therefore, in this section, we discuss a class of undirected time-invariant network, which is described by

$$\dot{x}_i(t) = Ax_i(t) + g(t, x_i(t)) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j(t),$$
$$i = 1,2,3,\cdots,N, \qquad (18)$$

where $t \in R_+$, $x_i(t) = (x_{i1}(t),\cdots,x_{in}(t))^T \in R^n$ is the state variable of node i. $\Gamma \in R^{n\times n}$ is a $0-1$ coupling link matrix ,the constant $c > 0$ represents the time-invariant coupling strength. If there is a connection between node i and node j $(i \neq j)$, $a_{ij} = a_{ji} = 1$; else, $a_{ij} = a_{ji} = 0$ $(i \neq j)$, and the diagonal elements are defined by

$$a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij} = -\sum_{i=1, j\neq i}^{N} a_{ji}, i = 1,2,\cdots,N.$$

The network (18) is said to achieve synchronization when $x_i(t) = s(t)$, $(i = 1,\cdots,N)$ as $t \to \infty$. Similarly, $s(t)$ and the synchronization error $e_i(t)$ can be described as (2) and (3), respectively, and the controller $u = u_1 + u_2$ is the same as (4). Then from (18), (2), (3) and (4), the synchronization error system is described as:

$$\begin{cases} \dot{e}_i(t) = Ae_i(t) + B_\sigma e_i(t-\tau) + \tilde{g}(t, x_i, s) \\ \qquad + c\sum_{j=1}^{N} a_{ij}\Gamma e_j(t), t \in (t_{k-1}, t_k], \\ \Delta e_i(t) = e_i(t_k^+) - e(t_k) = E_k e_i(t), t = t_k, \\ e_i(t_0^+) = e_{i0}, k = 1,2,\cdots, i = 1,2,\cdots,N, \end{cases} \qquad (19)$$

where $\tilde{g}(t, x_i, s) = g(t, x_i(t)) - g(t, s(t))$.

For convenience, define

$$\gamma'(t) = \max_{1\le l\le m}\{\lambda_{\max}[P_l^{-1}(A^T P_l + P_l A + 2\varphi(t) P_l + \frac{I}{\xi'})]\},$$

$$\theta' = \max_{1\le l\le m}\left\{\lambda_{\max}\left[P_{l-\mathrm{mode}(\tau)}^{-1}(\xi B_l^T P_l^2 B_l)\right]\right\},$$

$$\hat{\gamma}(t) = \gamma(t) + \theta(\mu\rho\eta_k)^{-1},$$

where $\xi' > 0$ and $\mu' > 0$ are constants.

Theorem 2. Suppose Assumptions 1-2 are satisfied.

i) If there exist two constants β', α' satisfying $\beta' \ge \alpha' \ge 0$ and $\hat{\gamma}'(t) \le -\beta' < 0$ such that

$$\ln \rho\eta_k - \alpha'(t_k - t_{k-1}) \le 0, k = 1,2,\cdots,$$

then the trivial solution of the error system (19) is globally exponentially stable, which implies network synchronization is reached exponentially by every node in the network (18) under control (4).

ii) If $\hat{\gamma}'(t) \ge 0$ and there exists a constant $\alpha' \ge 1$ such that

$$\ln \alpha'\rho\eta_k + \int_{t_k}^{t_{k+1}} \hat{\gamma}'(s)ds \le 0, k = 1,2,\cdots,$$

then $\alpha = 1$ implies that the trivial solution of the error system (19) is stable, and $\alpha > 1$ implies that the trivial solution of the error system (19) is asymptotically stable which implies that network synchronization is reached asymptotically by every node in the network (18) under control (4).

The proof is similar with Theorem 1, the details are omitted.

Remark 2. Theorem 1 and Theorem 2 don't impose any bound on the delay constant τ. Thus, our synchronization results are independent of delay.

IV. AN ILLUSTRATIVE EXAMPLE

In this section, we give an example to illustrate the effectiveness of the proposed method.

Consider a nearest-neighbor coupled network with 5 coupled nodes, in which each node is a Chen chaotic system [15]. The network achieves synchronization when $x_5(t) = x_4(t) = x_3(t) = x_2(t) = x_1(t) = s(t)$ as $t \to \infty$, where $s(t)$ is an isolate node without coupling.

The network can be described as (1), where $x_i = (x_{i1}, x_{i2}, x_{i3})^T$, $g(t, x_i) = (0, -x_{i1}x_{i3}, x_{i1}x_{i2})^T$,

$$A = \begin{bmatrix} -35 & 35 & 0 \\ -7 & 28 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \quad \Gamma(t) = I, \; C(t) =$$

$$\begin{bmatrix} -0.5\sin t - 0.5 & 0.5\sin t & 0 & 0 & 0.5 \\ 0.5 & -1 & 0.5 & 0 & 0 \\ 0 & 0.5 & -1 & 0.5 & 0 \\ 0 & 0 & 0.5 & -0.5\sin t - 0.5 & 0.5\sin t \\ 0.5\sin t & 0 & 0 & 0.5\sin t & -\sin t \end{bmatrix}.$$

Let the impulsive interval $t_k - t_{k-1} = 0.002$, $E_k = diag\{-0.69, -0.69, -0.69\}$. Suppose $l = 1,2$ and

$$B_1 = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, B_2 = \begin{bmatrix} 2 & 0 & 1 \\ 2 & -2 & 0 \\ 1 & 0 & -1 \end{bmatrix}.$$

Switching signal $\sigma := \sigma(i,k)$ changes with time and node, such as, for $h = 0,1,\cdots$, when $t \in (0.002(2h), 0.002(2h+1)]$, $i = 1,3,5$, $B_\sigma = B_1$, $i = 2,4$, $B_\sigma = B_2$. When $t \in (0.002(2h+1), 0.002(2h+2)]$, $i = 1,3,5$, $B_\sigma = B_2$, $i = 2,4$, $B_\sigma = B_1$.

Let $P_\sigma = I$, $\sigma = 1,2$, we have $\tilde{g}(t, x_i, s)$ $= (0, x_{11}x_{13} - x_{i1}x_{i3}, x_{i1}x_{i2} - x_{11}x_{12})^T$ in error system and Ref. [9] shows that there exists a positive constant $\delta = 60$, satisfying $\|s\| \le \delta, \|x_i\| \le \delta$. Then we can obtain $\tilde{g}^T(t, x_i, s) P_\sigma e_i(t) \le 2\delta e_i(t) P_\sigma e_i(t)$, $\varphi(t) = 2\delta$. From (6) to (11), we obtain $\rho = 1$, $\gamma(t) = 389.4991$, $\beta = 11.2995$, $\hat{\gamma}(t) = 507.0795$, $\eta_k = 0.0961 > 0$, $\rho_k = 10.4058$,with $\varepsilon = 1, \xi = 1, \mu = 1$. Assumptions 1-2 are satisfied.

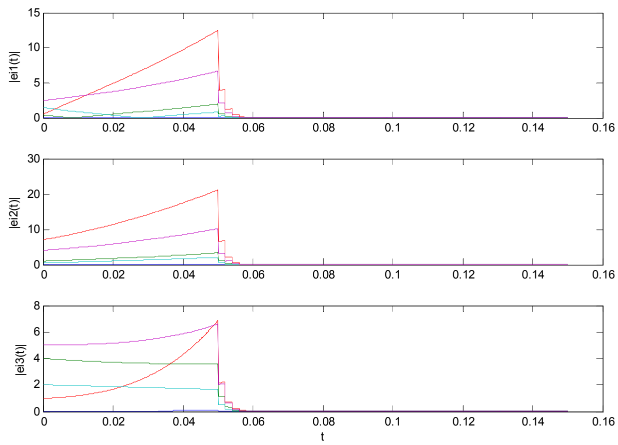

Fig.1.States error of the directed network under control (4) with $\tau = 0.05$.

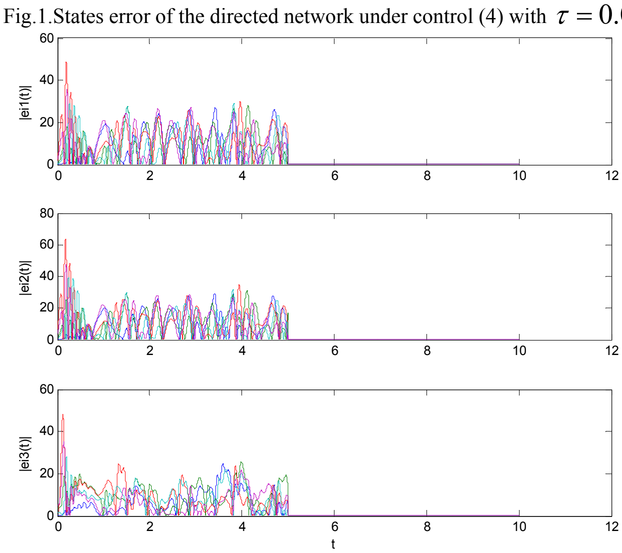

Fig.2. States error of the directed network under control (4) with $\tau = 5$.

Let $\alpha = 1.02$, then $\ln \alpha\rho\eta_k + \int_{t_k}^{t_{k+1}} \hat{\gamma}(s)ds$ $= -1.3084 < 0$. With this, from Theorem 1, the trivial solution of error system (5) is asymptotically stable. The complex network achieves asymptotical synchronization. Fig.1 and Fig.2 show the states error of each node in the nearest-neighbor coupled network with control delay $\tau = 0.05$ and $\tau = 5$, respectively. The illustrative figures show that our synchronization results are independent of delay τ.

V. Conclusion

In this paper, we studied synchronization of two classes of complex networks, which are a class of directed time-varying network and a class of undirected time-invariant network with constant edge weights, respectively. The control strategy introduced is a hybrid control method, which is a combination of an impulsive controller and a switching controller with constant time delay. Sufficient conditions for globally exponential and asymptotical synchronization of both of the two networks are established. An illustrative example has been given to show the effectiveness of the theoretical results.

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