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State-feedback Control Design for Continuous-time Piecewise Linear Systems: an LMI Approach

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***Abstract*— This paper studies the problem of state-feedback control design for a class of continuous-time piecewise linear systems. A new framework for the synthesis of piecewise linear control systems is given based on piecewise quadratic Lyapunov function and Reciprocal Projection Lemma. Under the new framework, Lyapunov variables and system dynamic matrices are separated by adding extra variables. And state-feedback control design for piecewise linear systems can be expressed in terms of linear matrix inequalities (LMIs). An example is given to illustrate the effectiveness of the proposed method.**

Key words: Piecewise linear systems; piecewise quadratic Lypaunov function; linear matrix inequalities; Reciprocal Projection Lemma.

I. Introduction

Piecewise linear systems arise often in practical control systems when piecewise linear components such as dead-zone, saturation, relays and hysteresis are encountered. In addition, many other classes of nonlinear systems can also be approximated by piecewise linear systems. In fact, piecewise linear systems are a broad modeling class in the sense that they have been shown to be equivalent to many other classes of systems, such as mixed logic dynamical systems [1] and extended linear complementary systems [2].

Since the author in [6] presented a pioneering work on the analysis of discrete-time piecewise linear systems in the early 1980s, numerous results [5]-[19] have been obtained on analysis and synthesis of piecewise linear systems. For example, the authors in [7], [8] presented results on stability and optimal performance analysis for piecewise linear systems based on a piecewise quadratic Lyapunov function. The author in [10] extended the stability analysis method [7] to discrete-time piecewise linear systems. Meanwhile, controller design for piecewise linear systems based on piecewise quadratic Lyapunov function arises, such as [11]-[16] for continuous-time systems and [17]-[19] for discrete-time systems.

By allowing the Lyapunov function to be piecewise quadratic and introducing S-procedure, piecewise quadratic Lyapunov function [7] is less conservative than quadratic Lyapunov function [4], [24] for stability analysis of piecewise linear systems. However, for state-feedback control design, it is hard to deduce an LMI condition to obtain the controller gain using piecewise quadratic Lyapunov function, due to parametrization of Lyapunov variables and existence of S-procedure. Usually, we can only obtain a nonconvex condition and achieve the controller gain by solving a set of bilinear matrix inequalities (BMIs) using kinds of methods [12], [14].

To solve this problem, a new framework for the synthesis of piecewise linear control systems is given based on piecewise quadratic Lyapunov function and Reciprocal Projection Lemma. Under the new framework, Lyapunov variables and system dynamic matrices are separated by adding extra variables. By a simple change of variables, state-feedback control design can be addressed in terms of LMIs, which is tractable with numerical software [24], [25]. Similar ideas have been presented for linear systems [22], [23].

The rest of the paper is organized as follows. Section 2 introduces the piecewise linear system model and piecewise quadratic stability, and presents the difficulty of state-feedback control design using piecewise quadratic Lyapunov function. Section 3 is the main result of this paper, which presents LMI conditions for state-feedback control design. Section 4 gives an example to illustrate the effectiveness of the proposed method. Finally, conclusions are given in Section 5.

Notations: We use standard notations throughout this note. M^T is the transpose of the matrix M. $M > 0 (M < 0)$ means that M is positive definite (negative definite). The symbol $*$ will be used in some matrix expressions to induce a symmetric structure. For example, if N is symmetric matrix, then

$$\begin{bmatrix} L & N \\ * & M \end{bmatrix} := \begin{bmatrix} L & N \\ N^T & M \end{bmatrix}$$

.

II. Preliminaries and Problem Statement

Consider piecewise linear systems

$$\dot{x}(t) = A_i x(t) + B_i u(t), \quad \text{for } x(t) \in X_i \tag{1}$$

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where $\{X_i\}_{i\in I} \subseteq R^n$ is a partition of the state space into a number of closed (possibly unbounded) polyhedral cells with pairwise disjoint interior. The index set of the cells is denoted $I = \{1, 2, \ldots, l\}$. Let $x(t) \in \cup_{i\in I} X_i$ be a continuous piecewise C^1 function on the time interval $[t_0, t_1]$ such that the derivative $\dot{x}(t)$ is defined, the equation $\dot{x}(t) = A_i x(t) + B_i u(t)$ holds for all i with $x(t) \in X_i$. $x(t) \in R^n$ is the system state variable, and $u(t) \in R^m$ the system input variable. Here, we assume there is no sliding mode, and all the local systems are controllable.

The state cells X_i are polyhedrons, so we can construct matrices $E_i \in R^{l\times n}, F_i \in R^{(l+n)\times n}$ such that

$$E_i x \geq 0 \qquad x \in X_i, \quad i \in I \tag{2}$$

$$F_i x = F_j x \quad x \in X_i \cap X_j, \quad i, j \in I \tag{3}$$

Here, (2),(3) describe state cells and cell boundaries respectively [8].

Quadratic stability (Proposition 3 in Appendix) of piecewise linear systems is attractive, since stability follows independently of cell partition and for a large class of switching schemes [7]. Meanwhile, state feedback control design based on quadratic Lyapunov is easy and can be addressed in terms of LMIs [11]. However, quadratic stability is very conservative for piecewise linear systems. To reduce the conservatism, the authors in [7] introduced a piecewise quadratic Lyapunov function and developed piecewise quadratic stability. The key idea is to make the piecewise Lyapunov function continuous across the cell boundaries and introduce the S-procedure to reduce the conservatism of the stability result.

The so-called piecewise quadratic Lyapunov function has the form

$$V(x) = x^T P_i x, \quad x \in X_i, \quad i \in I \tag{4}$$

with $P_i = F_i^T T F_i$, and F_i satisfies (3). Conditions for the existence of piecewise quadratic Lyapunov functions for the piecewise linear system (1) are formulated in the following proposition.

Proposition 1 [7]: (*Piecewise Quadratic Stability*) Consider symmetric matrices T, U_i and W_i, such that U_i, W_i have nonnegative entries, while $P_i = F_i^T T F_i$ satisfy

$$\begin{cases} A_i^T P_i + P_i A_i + E_i^T U_i E_i < 0 \\ P_i - E_i^T W_i E_i > 0 \end{cases} \tag{5}$$

Then $x(t)$ tends to zero exponentially for every continuous piecewise C^1 trajectory in $\cup_{i\in I} X_i$ satisfying (1) with $u \equiv 0$ for $t \geq 0$.

Note that condition (5) is LMIs with respect to T, which can be numerically feasible with commercially available software [24], [25]. Although less conservative, Proposition 1 can't easily be used to design state-feedback control for piecewise linear systems (1). In fact, it is a nonconvex problem due to the parametrization of P_i and the existence of S-procedure.

Consider the following piecewise linear state-feedback controller

$$u(t) = K_i x(t), \quad for\ x(t) \in X_i \tag{6}$$

Then the closed-loop piecewise linear system (1) becomes

$$\dot{x}(t) = (A_i + B_i K_i) x(t), \quad for\ x(t) \in X_i \tag{7}$$

Based on Proposition 1, if the following conditions are satisfied

$$(A_i + B_i K_i)^T P_i + P_i (A_i + B_i K_i) + E_i^T U_i E_i < 0 \tag{8}$$

$$P_i - E_i^T W_i E_i > 0 \tag{9}$$

the closed-loop system (7) is stable, in other words, the piecewise linear system (1) is stabilized by the controller (6).

Note that (8) and (9) are BMIs. In general, the conversion from BMIs into LMIs is performed by multiplying the inequalities on the left and right by P_i^{-1}. However, to make piecewise quadratic Lyapunov function continuous across the cell boundaries, $P_i = F_i^T T F_i$ with $F_i \in R^{(l+n)\times n}$ so that it is difficult to obtain P_i^{-1} parameterized in terms of F_i and T^{-1}. Meanwhile, due to existence of S-procedure, the usual conversion technique from BMIs into LMIs does not work in this case. This is the main problem when we design state feedback control for piecewise linear systems using piecewise quadratic Lyapunov function.

In this paper, to circumvent the nonlinearity problem, an alternative strategy is adopted to obtain an LMI condition for the state-feedback controller.

III. Main Results

In this section, we provide the main contribution of the paper. An LMI-based state-feedback controller will be designed based on piecewise quadratic Lyapunov function and Reciprocal Projection Lemma (see Lemma 2). The following two lemmas are essential for later developments.

Lemma 1[21]: For definite matrix P, the following inequality holds

$$G^T P^{-1} G \geq G^T + G - P \tag{10}$$

Lemma 2[23]: (*Reciprocal Projection Lemma*) Let Φ be any given positive definite matrix. The following statements are equivalent:

$$i): \Psi + S + S^T < 0 \tag{11}$$

$ii)$: the LMI problem

$$\begin{bmatrix} \Psi + \Phi - (W + W^T) & S^T + W^T \\ S + W & -\Phi \end{bmatrix} < 0 \tag{12}$$

is feasible with respect to W.

Now, we give the main result of the paper.

Theorem 1: For symmetric matrix T and U_i, W_i, such that U_i, W_i have nonnegative entries, and general matrix V_i, R_i, if the following LMIs are satisfied

$$\begin{bmatrix} -(V_i + V_i^T) & V_i^T A_i^T + R_i^T B_i^T + P_i & V_i^T \\ * & -P_i & 0 \\ * & * & E_i^T U_i E_i - P_i \end{bmatrix} < 0 \tag{13}$$

$$P_i - E_i^T W_i E_i > 0 \tag{14}$$

with $P_i = F_i^T T F_i, i \in I$, then the closed-loop piecewise linear system (7) is globally stable, and the controller gain for each local subsystem is given by

$$K_i = R_i V_i^{-1} \tag{15}$$

Proof: It follows from Proposition 1 that if the following conditions are satisfied

$$A_{cli}^T P_i + P_i A_{cli} + E_i^T U_i E_i < 0 \tag{16}$$

$$P_i - E_i^T W_i E_i > 0 \tag{17}$$

with $A_{cli} = A_i + B_i K_i$, then the closed-loop piecewise linear systems (7) is globally stable.

The inequality (16) is equivalent to

$$Q_i A_{cl_i}^T + A_{cl_i} Q_i + Q_i S_i Q_i < 0$$

with $Q_i := P_i^{-1}, S_i := E_i^T U_i E_i$.

The use of Lemma 2 with $\Psi = Q_i S_i Q_i$ and $S = Q_i A_{cl_i}^T$ yields

$$\begin{bmatrix} Q_i S_i Q_i + \Phi_i - (W_i + W_i^T) & A_{cl_i} Q_i + W_i^T \\ * & -\Phi_i \end{bmatrix} < 0 \tag{18}$$

By the congruence transformation

$$\begin{bmatrix} V_i & 0 \\ 0 & P_i \end{bmatrix}, \text{ with } V_i := W_i^{-1},\ P_i = Q_i^{-1}$$

the inequality (18) becomes

$$\begin{bmatrix} \Xi_i & V_i^T A_{cl_i} + P_i \\ * & -P_i \Phi_i P_i \end{bmatrix} < 0$$

where

$$\Xi_i = V_i^T (P_i^{-1} S_i P_i^{-1} + \Phi_i) V_i - (V_i^T + V_i)$$

By a Schur complement operation with the term $V_i^T (P_i^{-1} S_i P_i^{-1} + \Phi_i) V_i$, we obtains the following inequality

$$\begin{bmatrix} -(V_i^T + V_i) & V_i^T A_{cl_i} + P_i & V_i^T \\ * & -P_i \Phi_i P_i & 0 \\ * & * & -(P_i^{-1} S_i P_i^{-1} + \Phi_i)^{-1} \end{bmatrix} < 0 \tag{19}$$

The use of Lemma 1 with $\Phi_i = P_i^{-1}$ yields

$$(P_i^{-1} S_i P_i^{-1} + \Phi_i)^{-1} = P_i (S_i + P_i)^{-1} P_i \geq P_i - S_i$$

So, if the following inequality holds

$$\begin{bmatrix} -(V_i + V_i^T) & V_i^T A_{cl_i} + P_i & V_i^T \\ * & -P_i & 0 \\ * & * & E_i^T U_i E_i - P_i \end{bmatrix} < 0 \tag{20}$$

the inequality (16) holds.

The dual of (20) in the transformation $A_{cl_i} \to A_{cl_i}^T$ is

$$\begin{bmatrix} -(V_i + V_i^T) & V_i^T A_{cl_i}^T + P_i & V_i^T \\ * & -P_i & 0 \\ * & * & E_i^T U_i E_i - P_i \end{bmatrix} < 0 \tag{21}$$

Substituting $A_{cl_i} = A_i + B_i K_i$ and $R_i = K_i V_i$ into (21), we obtains (13).

So, if (13) holds, (16) holds. And (17) is the same as (14). Based on Proposition 1, the piecewise closed-loop linear system (7) is globally stable. In other words, the piecewise linear system (1) is stabilized by controller (6). And the control gain is given by $K_i = R_i V_i^{-1}$. Thus the proof is completed. ■

Remark 1: Conditions (13), (14) are LMIs with respect to T, U_i, W_i, V_i and R_i, which are well tractable with commercially available software.

Remark 2: Note that Proposition 1 doesn't apply to piecewise affine systems that have multiple equilibria and therefore, the method in Theorem 1 doesn't apply in this case. This is a limitation of the proposed method.

Based on Reciprocal Projection Lemma and piecewise quadratic stability without S-procedure (Proposition 2 in Appendix), we obtain the following corollary.

Corollary 1: For symmetric matrix T, and general matrix V_i and R_i, if the following LMI is satisfied, with $P_i = F_i^T T F_i$ for $i \in I$,

$$\begin{bmatrix} -(V_i + V_i^T) & V_i^T A_i^T + R_i^T B_i^T + P_i & V_i^T \\ * & -P_i & 0 \\ * & * & -P_i \end{bmatrix} < 0 \tag{22}$$

Then the state-feedback controller $u(t) = K_i x(t)$ stabilizes the piecewise linear system (7), and the controller gain for each local subsystem is given by

$$K_i = R_i V_i^{-1} \tag{23}$$

Proof: It follows from Proposition 2 that if the following two inequalities are satisfied

$$A_{cli}^T P_i + P_i A_{cli} < 0 \tag{24}$$

$$P_i = P_i^T > 0 \tag{25}$$

with $A_{cli} = A_i + B_i K_i$, then the closed-loop piecewise linear systems (7) is stable.

Based on Lemma 2, (24) is equivalent to

$$Q_i A_{cl_i}^T + A_{cl_i} Q_i < 0$$

with $Q_i := P_i^{-1}$.

The use of Lemma 2 with $\Psi = 0$ and $S = Q_i A_{cl_i}^T$ yields

$$\begin{bmatrix} \Phi_i - (W_i + W_i^T) & A_{cl_i} Q_i + W_i^T \\ * & -\Phi_i \end{bmatrix} < 0$$

By the congruence transformation

$$\begin{bmatrix} V_i & 0 \\ 0 & P_i \end{bmatrix}, \text{ with } V_i := W_i^{-1},\ P_i = Q_i^{-1}$$

the inequality above becomes

$$\left[\begin{array}{cc} V_i^T\Phi_i V_i-(V_i^T+V_i) & V_i^T A_{cl_i}+P_i \\ * & -P_i\Phi_i P_i \end{array}\right]<0$$

By a Schur complement operation with the term $V_i^T\Phi_i V_i$, we obtains the following inequality

$$\left[\begin{array}{ccc} -(V_i^T+V_i) & V_i^T A_{cl_i}+P_i & V_i^T \\ * & -P_i\Phi_i P_i & 0 \\ * & * & -\Phi_i^{-1} \end{array}\right]<0$$

Let $\Phi_i=P_i^{-1}$, then the inequality above becomes

$$\left[\begin{array}{ccc} -(V_i+V_i^T) & V_i^T A_{cl_i}+P_i & V_i^T \\ * & -P_i & 0 \\ * & * & -P_i \end{array}\right]<0 \quad (26)$$

The dual of (26) in the transformation $A_{cl_i}\to A_{cl_i}^T$ is

$$\left[\begin{array}{ccc} -(V_i+V_i^T) & V_i^T A_{cl_i}^T+P_i & V_i^T \\ * & -P_i & 0 \\ * & * & -P_i \end{array}\right]<0 \quad (27)$$

Substituting $A_{cl_i}=A_i+B_iK_i$ and $R_i=K_iV_i$ into (27) obtains (22).
Thus (22) is equivalent to (24) and (25), and the proof is completed. ■

Remark 3: On the one hand, Theorem 1 is less conservative than Corollary 1, since Proposition 1 is less conservative than Proposition 2. And it is easy to see that Theorem 1 reduces to Corollary 1 without S-procedure "$E_i^T U_i E_i$" and "$E_i^T W_i E_i$" in (13) and (14). On the other hand, both Theorem 1 and Corollary 1 are less conservative than quadratic stabilization [11].

IV. Numerical Examples

This example is given to illustrate that Theorem 1 is less conservative than quadratic stabilization [11] for piecewise linear systems.

Consider a continuous-time piecewise linear system

$$\dot{x}(t)=\begin{cases} A_1x(t)+B_1u(t), & x_1-x_2\geq 0,\ x_1+x_2\geq 0 \\ A_2x(t)+B_2u(t), & x_1-x_2<0,\ x_1+x_2>0 \\ A_3x(t)+B_3u(t), & x_1-x_2\leq 0,\ x_1+x_2\leq 0 \\ A_4x(t)+B_4u(t), & x_1-x_2>0,\ x_1+x_2<0 \end{cases} \quad (28)$$

The system matrices are given by:

$$A_1=\left[\begin{array}{cc} 0.8143 & -1.5415 \\ -0.0601 & 0.6594 \end{array}\right],$$
$$A_2=\left[\begin{array}{cc} -0.5385 & 0.2671 \\ -1.4398 & 1.2920 \end{array}\right],$$
$$A_3=\left[\begin{array}{cc} 0.6958 & 1.8465 \\ 1.9978 & -1.7646 \end{array}\right],$$
$$A_4=\left[\begin{array}{cc} -0.5588 & -0.9529 \\ 0.1941 & 0.3894 \end{array}\right],$$

$$B_1=\left[\begin{array}{c} -1.8029 \\ 0.2842 \end{array}\right], B_2=\left[\begin{array}{c} 0.8034 \\ 1.8492 \end{array}\right],$$
$$B_3=\left[\begin{array}{c} 1.0021 \\ 0.9600 \end{array}\right], B_4=\left[\begin{array}{c} -0.2725 \\ 0.5371 \end{array}\right].$$

The matrices describing the cells of the state are given as follows:

$$E_1=-E_3=\left[\begin{array}{cc} -1 & 1 \\ -1 & -1 \end{array}\right],$$
$$E_2=-E_4=\left[\begin{array}{cc} -1 & 1 \\ 1 & 1 \end{array}\right],$$
$$F_1=\left[\begin{array}{c} E_1 \\ I \end{array}\right], F_2=\left[\begin{array}{c} E_2 \\ I \end{array}\right],$$
$$F_3=\left[\begin{array}{c} E_3 \\ I \end{array}\right], F_4=\left[\begin{array}{c} E_4 \\ I \end{array}\right].$$

Simulation shows that the open-loop piecewise linear system is unstable. Here, we design a state-feedback controller $u(t)=K_ix(t), i=1,\dots 4$, to stabilize the unstable piecewise linear system. Quadratic stabilization [11] has no solution. However, using Theorem 1, the following solutions have been found:

$$T=\left[\begin{array}{cccc} 0.5507 & 0.1176 & 0.0300 & -0.1761 \\ 0.1176 & 0.5682 & 0.0679 & 0.2618 \\ 0.0300 & 0.0679 & 0.2084 & 0.0088 \\ -0.1761 & 0.2618 & 0.0088 & 0.3512 \end{array}\right],$$

$$P_1=\left[\begin{array}{cc} 1.3668 & -0.0973 \\ -0.0973 & 0.3592 \end{array}\right],$$
$$P_2=\left[\begin{array}{cc} 1.1679 & 0.5620 \\ 0.5620 & 1.8766 \end{array}\right],$$
$$P_3=\left[\begin{array}{cc} 1.7582 & 0.1498 \\ 0.1498 & 2.1107 \end{array}\right],$$
$$P_4=\left[\begin{array}{cc} 1.0163 & -0.5095 \\ -0.5095 & 1.5341 \end{array}\right],$$

$$K_1=\left[\begin{array}{cc} 1.3836 & -1.2081 \end{array}\right],$$
$$K_2=\left[\begin{array}{cc} 0.6109 & -1.9962 \end{array}\right],$$
$$K_3=\left[\begin{array}{cc} -2.8732 & -1.5435 \end{array}\right],$$
$$K_4=\left[\begin{array}{cc} 1.8866 & -4.2790 \end{array}\right].$$

Let the initial system state be $x_0=\left[\begin{array}{cc} 1 & 0.9 \end{array}\right]$. The response of the closed-loop control system is shown in Fig.1. From the simulation result, we know that the piecewise linear system has been stabilized. This example shows that Theorem 1 is less conservative than quadratic stabilization.

V. Conclusions

State-feedback control design for continuous-time piecewise linear systems using piecewise quadratic Lyapunov function is usually nonconvex. To solve this problem, a new framework for the synthesis of piecewise linear control systems has been given based on piecewise quadratic Lyapunov function and Reciprocal Projection Lemma. Under

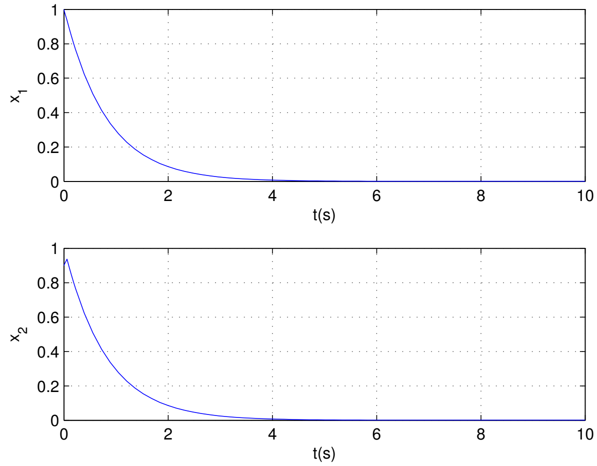

Fig. 1. Response of the closed-loop control system

the new framework, Lyapunov variables and system dynamic matrices are separated by adding extra variables. And state-feedback control design for continuous-time piecewise linear systems can be expressed in terms of LMIs. An example has been given to illustrate the effectiveness of the proposed method.

VI. Appendix

Proposition 2: (*Piecewise quadratic stability without S-procedure*) For symmetric matrix T, if $P_i = F_i^T T F_i$ satisfy

$$\begin{cases} A_i^T P_i + P_i A_i < 0 \\ P_i > 0 \end{cases} \tag{29}$$

Then $x(t)$ tends to zero exponentially for every continuous piecewise C^1 trajectory in $\cup_{i\in I} X_i$ satisfying (1) with $u \equiv 0$ for $t \geq 0$.

Proposition 3: (*Quadratic stability*) If there exists a matrix $P = P^T > 0$ such that $A_i^T P + P A_i < 0, i \in I$, then $x(t)$ tends to zero exponentially for every continuous piecewise C^1 trajectory in $\cup_{i\in I} X_i$ satisfying (1) with $u \equiv 0$ for $t \geq 0$.

References

[1] A. Bemporad and M. Morari, "Control of systems integrating logic, dynamics, and constraints". *Automatica*, vol. 35, 1999, pp 407-427.

[2] B. D. Schutter and B. D. Moor, "The extended linear complementarity problem and the modeling and analysis of hybrid systems". In P. Antsaklis, W. Kohn, M. Lemmon, A. Nerode and S.Satry(Eds) *Hybrid Systems Verification and Control: Lecture Notes in Computer Science*, vol.1567, pp 70-85. (Springer-Verlag).

[3] E. D. Sontag, "Nonlinear regulation: the piecewise linear approach", *IEEE Trans. Automat. Contr.*,Vol. 26, pp. 346-357, 1981.

[4] M. Corless, "Robust stability analysis and controller design with quadratic Lyapunov functions", in *Variable Structure and Lyapunov Control*, A. S. I. Zinober, Ed., 1994, ch. 9, pp. 181-203. Lecture notes in Control and Information Sciences.

[5] J. I. Imura and Van der Schaft, "Characterization of well-posedness of piecewise-linear systems", *IEEE Trans. Automat. Contr.*, Vol. 45, pp. 1600-1619. 2000.

[6] E. D. Sontag, "Nonlinear regulation: the piecewise linear approach", *IEEE Trans. Automat. Contr.*,Vol. 26, pp. 346-357, 1981.

[7] M. Johansson and A. Rantzer, "Computation of piecewise quadratic Lyapunov functions of hybrid systems", *IEEE Trans. Automat. Contr.*, Vol. 43, no.4, pp. 555 - 559 , 1998.

[8] A. Rantzer and M. Johansson, "Piecewise linear quadratic optimal control", *IEEE Trans. Automat. Contr.*, Vol. 45, no.4, pp. 629-637, 2000.

[9] M. Johansson, *Piecewise Linear Control Systems*, Springer-verlag, 2003.

[10] G. Feng, "Stability analysis of piecewise discret-time linear systems", *IEEE Trans. Automat. Contr.*, Vol. 47, no. 7, pp. 1108-1112, 2002.

[11] A. Hassibi, and S. Boyd, "Quadratic stabilization and control of piecewise-linear systems", *Proc. American Contol Confenrence*, Philadephia, Pennsylvania, pp. 3659-3664, 1998.

[12] L. Rodrigues and J. H. How, "Observer-based control of piecewise-affine systems", *Int. J. Control.*, vol. 76, pp. 459-477, 2003.

[13] L. Rodrigues and S. Boyd, "Piecewise-affine state feedback for piecewise-affine slab systems using convex optimization", *Systems and Control Letters*, vol. 54, pp. 835-853, 2005.

[14] G. Feng, "Controller design and analysis of uncertain piecewise-linear systems", *IEEE Trans. Circuits. Syst. I*, Vol. 49, no.2, pp. 224- 232 , 2002.

[15] Y. Zhu, D. Q. Li and G. Feng, "H_∞ Controller synthesis of uncertain piecewise continuous-time linear systems", *IEE Proc. Control Theory Appl.*, Vol. 152, no.5, pp. 513- 519 , 2005.

[16] M. Chen, C. R. Zhu and G. Feng, "Linear-matrix-inequality-based approach to H_∞ controller synthesis of uncertain continuous-time piecewise linear systems", *IEE Proc. Control Theory Appl.*, Vol. 151, No. 3, 2004.

[17] D. Mignone, G. Ferrari-Trecate, and M. Morari, "Stability and stabilization of piecewise affine and hybrid systems: an LMI approach", *IEEE Conference on Decision and Contol*, Sydney, Australia, 2000.

[18] G. Ferrari-Trecate, F. A. Cuzzola, D. Mignone, and M. Morari, "Analysis of discret-time piecewise affine and hybrid systems", *Automatica*, vol. 38, pp. 2139-2146, 2002.

[19] F. A. Cuzzola and M. Morari, "An LMI approach for H_∞ analysis and control of discrete-time piecewise affine systems", *Int. J. Control.*, vol. 75, pp. 2193-1301, 2002.

[20] J. Xu and L. Xie, "Null controllability of discrete-time planar bimodal piecewise linear systems", *Int. J. Control*, Vol 78, No. 18, pp. 1486-1496, 2005.

[21] M. C. de Oliveira, J. Bernussou and J. C. Geromel, "A new discrete-time robust stability condition", *Systems and Control Letters*, vol. 37, pp. 261-265, 1999.

[22] M. C. de Oliveira, J. C. Geromel and J. Bernussou, "Extended H_2 and H_∞ norm characterizations and controller parametrizations for discrete-time systems", *Int. J. Control*, vol. 75, pp. 666-679, 2002.

[23] P. Apkarian, Hoang Duong Tuan, and J. Bernussou , "Continuous-time analysis, eigenstructure assignment, and H_2 synthesis with enhanced linear matrix inequalities (LMI) characterizations", *IEEE Trans. Automat. Contr.*, Vol. 46, no.12, pp. 1941 - 1946 , 2001.

[24] S. Boyd, L. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in Systems and Control Theory*. Philadelphia, PA: SIAM, 1994.

[25] P. Gahinet, A. Nemirovski, A. Laub, and M. Chilali, *The LMI Control Toolbox*. Natick, MA: Mathworks, 1995.