2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

A New Algorithm for Simultaneous Input and State Estimation

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Abstract— **Input and state estimation of a dynamic system can find many applications in fault detection and diagnosis, target tracking, and so on. This paper presents a new simultaneous input and state estimation scheme. A recursive algorithm is developed based on the idea of achieving minimum mean square error and minimum error variance. The convergence property of the proposed algorithm is also analyzed. Numerical examples are provided to illustrate the effectiveness of the algorithm.**

I. INTRODUCTION

The problem of simultaneous input and state estimation for dynamic systems has wide applications in fault detection and diagnosis [1], maneuvering target tracking [2], geophysics and environmentology [3], in which cases inputs are often unmeasurable or inaccessible. It is also potentially useful in networked control systems with unknown input package delays and even losses [4]. Due to its practical applications, simultaneous input and state estimation has received much attention during the past several decades.

For different applications, related research in the existing literature can be mainly classified into three types:

- **State estimation subject to unknown inputs:** Kitanidis develops an unbiased minimum-variance linear state filter that has the state estimation independently with the unknown inputs [5]. Darouacha *et. al.* extend Kitanidis's design by giving a more general filter structure and the convergence conditions for the time-invariant case [6]. Further, in [7], Darouacha *et. al.* consider the same problem for a system with direct feedthrough, and present an optimal filter design as well as stability conditions. Without much optimization involved, matrix calculations are also popular in state observer design with unknown inputs, see [8], [9], [10]. Sliding mode observer is another promising way to estimate states of a system subject to unknown inputs. In [11], it is proven that the proposed sliding mode observer can converge asymptotically or in finite time.
- **Unknown input estimation:** In many cases, it is practically demanding to determine the unknown inputs of a dynamic system, e.g., in fault detection and diagnosis. In literature there exist numerous works on this topic, see [12]–[15] and the references therein.
- **Simultaneous input and state estimation:** It is worth noting that state and input estimations are inherently interconnected and coupled. In [16], a two-stage Kalman filter and an input filtering technique are combined to achieve joint estimation. Gillijns and Moor propose a set of multi-step recursive filters to jointly estimate inputs and states by minimizing the error variance for discrete-time linear systems without and with the direct feedthrough [17] [18], respectively. However, convergence analysis of the proposed algorithms is not discussed in [17] [18].

The goal of this paper is to present an estimator design to simultaneously predict the input and state variables for an LTI system. An algorithm for input and state co-estimation for linear discrete-time systems is developed. The estimation algorithm is unbiased and minimizes both mean square errors and error variances. Comparing the approach given in [18] for the similar problem, the approach in this paper is completely different and gives a more straightforward solution.

The rest of the paper is organized as follows. In Section II, we briefly formulate the problem of interest. Section III presents the algorithm, providing the proof of optimality and some insights into convergence properties. Section IV gives illustrative examples to demonstrate the effectiveness of the proposed algorithm. Finally, some concluding remarks are offered in Section V.

II. PROBLEM FORMULATION

The problem setting is shown in Fig. 1. Consider the linear time-invariant dynamic system

$$\left\{ \begin{array}{rcl} x_{k+1} & = & Ax_k + Bu_k + w_k, \\ y_k & = & Cx_k + Du_k + v_k, \end{array} \right. \tag{1}$$

where $x_k \in \mathbb{R}^n$ denotes the system state at time instant k, $u_k \in \mathbb{R}^m$ is the unknown input and $y_k \in \mathbb{R}^p$ is the measurement. The transition matrices, A, B, C and D, are assumed observable and have compatible dimensions. The process noise w_k and measurement noise v_k are mutually uncorrelated zero-mean white noises with known covariances, i.e.,

$$E\{w_k w_l^{\mathrm{T}}\} = R_w \delta_{k-l},\ E\{v_k v_l^{\mathrm{T}}\} = R_v \delta_{k-l},\ E\{w_k v_l^{\mathrm{T}}\} = 0,$$

where δ_k is the Kronecker delta function.

This paper is to build recursive input and state estimators for the system in (1). The estimators are expected to be convergent and as optimal as possible. Here the optimality is defined in the sense of both MMSE and MV. According to the theory of input and state observer design for deterministic linear systems [23], we assume that the input and state

This research was supported by the Natural Sciences and Engineering Research Council of Canada and the Canadian Foundation of Innovation.

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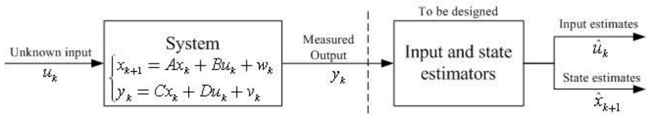


Fig. 1. Blockdiagram for simultaneous input and state estimation.

estimators have the following form

$$\hat{u}_k = H_k\left(y_k - C\hat{x}_k\right), \tag{2}$$

$$\hat{x}_{k+1} = A\hat{x}_k + B\hat{u}_k + L_k\left(y_k - C\hat{x}_k - D\hat{u}_k\right), \tag{3}$$

where $\hat{x}_k$ represents the state estimate and $\hat{u}_k$ the input estimate. H_k and L_k are estimators' gain matrices that are needed to be defined later. The mean square errors of input and state estimation are defined, respectively, as

$$J_k^u = E\left\{\tilde{u}_k^{\mathrm{T}}\tilde{u}_k\right\}, \tag{4}$$

$$J_{k+1}^x = E\left\{\tilde{x}_{k+1}^{\mathrm{T}}\tilde{x}_{k+1}\right\}, \tag{5}$$

where $\tilde{u}_k$ and $\tilde{x}_{k+1}$ are estimation errors:

$$\tilde{u}_k = u_k - \hat{u}_k,\ \tilde{x}_{k+1} = x_{k+1} - \hat{x}_{k+1}.$$

We also define some estimation covariance matrices

$$P_k^u = E\{\tilde{u}_k\tilde{u}_k^{\mathrm{T}}\}, \tag{6}$$

$$P_k^{ux} = E\{\tilde{u}_k\tilde{x}_k^{\mathrm{T}}\}, \tag{7}$$

$$P_{k+1}^x = E\{\tilde{x}_{k+1}\tilde{x}_{k+1}^{\mathrm{T}}\}, \tag{8}$$

where P_k^u and P_{k+1}^x are symmetric and positive definite.

Centered around developing the algorithm for simultaneous input and state estimation with MMSE and MV, this paper focuses on three tasks:

1. Design the optimal input estimator given in (3) by determining H_k that simultaneously minimizes J_{k+1}^x and P_{k+1}^x.
2. Design the optimal state estimator given in (2) by determining L_k that simultaneously minimizes state estimation error and variance, i.e., J_k^u and P_k^u;
3. Analyze the convergence properties of the proposed algorithm.

III. Main Results

This section begins with some preliminary lemmas, followed immediately by development of the state and input estimators. An algorithm will then be presented and its convergence will be analyzed.

A. Preliminaries

Some facts about matrix traces will be used in this section and stated as follows.

Lemma 1: [22] For matrices $X \in \mathbb{R}^{q\times l}$ and $Y \in \mathbb{R}^{l\times q}$, we have

$$\mathrm{tr}(XY) = \mathrm{tr}(YX), \quad \frac{\partial \mathrm{tr}(XY)}{\partial X} = Y^{\mathrm{T}},$$

$$\frac{\partial \mathrm{tr}(YX^T)}{\partial X} = Y, \quad \frac{\partial \mathrm{tr}(XYX^T)}{\partial X} = X(Y^{\mathrm{T}} + Y),$$

where tr represents the trace of a matrix.

The following property of nonnegative definite matrices will also be used.

Lemma 2: If matrix $X \in \mathbb{R}^{q\times q}$ is nonnegative definite, for any matrix $Y \in \mathbb{R}^{l\times q}$, YXY^{T} is also nonnegative definite, namely, $YXY^{\mathrm{T}} \geq 0$.

We are looking for an input and state estimation scheme that minimizes both mean square error and error variance. A necessary condition is that the estimates are unbiased.

Lemma 3: For the considered system (1), in order for the state and input estimators in (2) and (3) to be unbiased, D must be of full column rank, and the following initial condition must be satisfied:

$$E(\hat{x}_0) = E(x_0). \tag{9}$$

Proof: Substituting equations in (1) to (2) and (3), we obtain

$$\tilde{u}_k = -H_k\left(C\tilde{x}_k + v_k\right) + (I - H_kD)u_k, \tag{10}$$

$$\tilde{x}_{k+1} = \left(A - L_kC\right)\tilde{x}_k + \left(B - L_kD\right)\tilde{u}_k - L_kv_k + w_k, \tag{11}$$

where I is the identity matrix. Recursively using the above dynamics until $k = 0$, it is then straightforward to obtain that the estimates are unbiased, that is, $E(\tilde{u}_k) = 0$ and $E(\tilde{x}_k) = 0$, when both (9) and the following input unbiasedness constraint is satisfied

$$H_kD = I, \tag{12}$$

where $H_k \in \mathbb{R}^{m\times p}$ and $I \in \mathbb{R}^{m\times m}$. Only when D has a full column rank, there exists an H_k such that (12) holds. Proof of Lemma 3 is complete. ■

With (12) satisfied, (10) becomes

$$\tilde{u}_k = -H_k\left(C\tilde{x}_k + v_k\right). \tag{13}$$

B. Input Estimation

The optimal H_k – H_k^* – can be found by solving the following constrained *simultaneous* optimization problem on J_k^u and P_k^u:

$$\min_{H_k}\left\{J_k^u, P_k^u\right\},$$
$$\text{s.t. } H_kD = I.$$

Theorem 1 in the following presents a solution that minimizes J_k^u subject to (12). Theorem 2 further shows that, with the proposed solution H_k^*, P_k^u will be minimized as well.

Theorem 1: Assume that input estimation is unbiased. Then if H_k is designed as

$$H_k^* = \left(D^{\mathrm{T}}Q_k^{-1}D\right)^{-1}D^{\mathrm{T}}Q_k^{-1}, \tag{14}$$

where $Q_k = CP_k^xC^{\mathrm{T}} + R_v$, the mean square error J_k^u is minimized.

Proof: Using (13), J_k^u can be expanded as

$$\begin{aligned} J_k^u &= E\left\{\left(C\tilde{x}_k + v_k\right)^{\mathrm{T}}H_k^{\mathrm{T}}H_k\left(C\tilde{x}_k + v_k\right)\right\} \\ &= E\left\{\tilde{x}^{\mathrm{T}}{}_kC^{\mathrm{T}}H_k^{\mathrm{T}}H_kC\tilde{x}_k\right\} + E\left\{v^{\mathrm{T}}{}_kH^{\mathrm{T}}{}_kH_kv_k\right\} \\ &= \mathrm{tr}\left\{H_kCP_k^xC^{\mathrm{T}}H_k^{\mathrm{T}}\right\} + \mathrm{tr}\left\{H_kR_vH^{\mathrm{T}}{}_k\right\} \\ &= \mathrm{tr}\left\{H_kQ_kH_k^{\mathrm{T}}\right\}. \end{aligned}$$

The Lagrange multipliers with an equality constraint can be applied here. Let λ be a matrix of appropriate dimensions, and rewrite the expression of J_k^u equivalently as

$$J_k^u = \mathrm{tr}\left\{H_k Q_k H^{\mathrm{T}}{}_k + \lambda (I - H_k D)\right\}. \tag{15}$$

Since J_k^u is wanted to be minimal, we equate its partial derivative with respect to (w.r.t.) H_k to 0. Then from Lemma 1 it follows that

$$\frac{\partial J_k^u}{\partial H_k} = 2H_k Q_k - \lambda^{\mathrm{T}} D^{\mathrm{T}} = 0. \tag{16}$$

Combining (16) with (12), we can easily get the final solution

$$H_k = \left(D^{\mathrm{T}} Q_k^{-1} D\right)^{-1} D^{\mathrm{T}} Q_k^{-1}. \tag{17}$$

This proves Theorem 1. ■

Let us consider the input estimation covariance, P_k^u. From the definition of P_k^u and (13), it follows that

$$P_k^u = H_k Q_k H_k^{\mathrm{T}}. \tag{18}$$

The next theorem indicates that P_k^u has a lower bound, which can be achieved with $H_k = H_k^*$.

Theorem 2: For any H_k satisfying the unbiasedness constraint, the following relation holds true:

$$P_k^u \geq \left(D^{\mathrm{T}} Q_k^{-1} D\right)^{-1}, \tag{19}$$

where the equality is held if and only if $H_k = H_k^*$.

Proof: Using (12), (14), (18) and Lemma 2, we obtain

$$\begin{aligned} [H_k - H_k^*]\, Q_k\, [H_k - H_k^*]^{\mathrm{T}} &= H_k Q_k H_k^{\mathrm{T}} - \left(D^{\mathrm{T}} Q_k^{-1} D\right)^{-1} \\ &= P_k^u - \left(D^{\mathrm{T}} Q_k^{-1} D\right)^{-1} \geq 0. \end{aligned}$$

Hence (19) is proven. The two sides obviously are equal when $H_k = H_k^*$. The uniqueness of H_k^* comes directly from the fact that Q_k is positive definite. ■

C. State Estimation

Now consider the state estimation problem. Likewise we define L_k^* by

$$L_k^* = \arg\min_{L_k} \left\{J_{k+1}^x, P_{k+1}^x\right\}.$$

The equation above indicates that L_k^* is the optimal L_k for minimization of J_{k+1}^x and P_{k+1}^x produced by the state estimator in (3).

Denote $M = [A\ B]$ and $N = [C\ D]$ and define the following matrices

$$\begin{aligned} S_k &= M O_k M^{\mathrm{T}}, \\ T_k &= M O_k N^{\mathrm{T}} - B H_k^* R_v, \\ U_k &= N O_k N^{\mathrm{T}} + R_v - D H_k^* R_v - R_v H_k^{*\mathrm{T}} D^{\mathrm{T}}, \end{aligned}$$

where

$$O_k = \begin{bmatrix} P_k^x & (P_k^{ux})^{\mathrm{T}} \\ P_k^{ux} & P_k^u \end{bmatrix}.$$

We are now ready to show the optimal state estimation design in the following two theorems in order.

Theorem 3: If the state estimator gain L_k is designed as

$$L_k^* = T_k U_k^{-1}. \tag{20}$$

then the mean square error of state estimation, J_{k+1}^x, achieves its minimum value.

Proof: The proof is analogous to that of Theorem 1. From (5) and (11) it follows that

$$\begin{aligned} J_{k+1}^x &= E\left\{\tilde{x}_k^{\mathrm{T}}[A - L_k C]^{\mathrm{T}}[A - L_k C]\tilde{x}_k\right\} \\ &\quad + E\left\{\tilde{u}_k^{\mathrm{T}}[B - L_k D]^{\mathrm{T}}[B - L_k D]\tilde{u}_k\right\} \\ &\quad + 2E\left\{\tilde{x}_k^{\mathrm{T}}[A - L_k C]^{\mathrm{T}}[B - L_k D]\tilde{u}_k\right\} \\ &\quad - 2E\left\{\tilde{u}_k^{\mathrm{T}}[B - L_k D]^{\mathrm{T}} L_k v_k\right\} \\ &\quad + E\left\{v_k^{\mathrm{T}} L_k^{\mathrm{T}} L_k v_k\right\} + E\left\{w_k^{\mathrm{T}} w_k\right\} \\ &= \mathrm{tr}\left\{[A - L_k C] P_k^x [A - L_k C]^{\mathrm{T}}\right\} \\ &\quad + \mathrm{tr}\left\{[B - L_k D] P_k^u [B - L_k D]^{\mathrm{T}}\right\} \\ &\quad + 2\mathrm{tr}\left\{[B - L_k D] P_k^{ux} [A - L_k C]^{\mathrm{T}}\right\} \\ &\quad - 2\mathrm{tr}\left\{L_k v_k \tilde{u}_k^{\mathrm{T}} [B - L_k D]^{\mathrm{T}}\right\} \\ &\quad + \mathrm{tr}\left\{L_k R_v L_k^{\mathrm{T}}\right\} + \mathrm{tr}(R_w) \\ &= \mathrm{tr}\left\{S_k - T_k L_k^{\mathrm{T}} - L_k T_k^{\mathrm{T}} + L_k U_k L_k^{\mathrm{T}} + R_w\right\}. \end{aligned}$$

We note that the partial derivative of J_{k+1}^x w.r.t L_k is

$$\frac{\partial J_{k+1}^x}{\partial L_k} = -2T_k + 2L_k U_k.$$

Replacing L_k with L_k^*, the partial derivative above will be equal to 0. This shows that L_k^* minimizes J_{k+1}^x and concludes the proof. ■

Theorem 4: Assume that $L_k = L_k^*$ holds. Then the variance of state estimation, P_{k+1}^x, is minimized.

Proof: Using (8) and (11), we expand P_{k+1}^x as follows:

$$P_{k+1}^x = S_k - L_k T_k^{\mathrm{T}} - T_k L_k^{\mathrm{T}} + L_k U_k L_k^{\mathrm{T}} + R_w,$$

which can be written equivalently as

$$P_{k+1}^x = S_k - T_k U_k^{-1} T_k^{\mathrm{T}} + \left(L_k - T_k U_k^{-1}\right) U_k \left(L_k - T_k U_k^{-1}\right)^{\mathrm{T}} + R_w.$$

Because U_k is positive definite, if setting $L_k = L_k^* = T_k U_k^{-1}$, P_{k+1}^x will achieve minimum

$$P_{k+1}^x = S_k - T_k U_k^{-1} T_k^{\mathrm{T}} + R_w = S_k - L_k^* T_k^{\mathrm{T}} + R_w. \tag{21}$$

This completes the proof. ■

It is noteworthy that the calculation of O_k involves updating P_k^{ux}. From its definition, we have

$$P_k^{ux} = E\left\{-H_k\,[C\tilde{x}_k + v_k]\,\tilde{x}^{\mathrm{T}}{}_k\right\} = -H_k C P_k^x. \tag{22}$$

Theorems 3 and 4 establish an optimal design procedure for the state estimator. However, the proposed state estimator has two potential problems: 1) its convergence is hard to analyze with a complex structure; and 2) U_k may be singular in numerical simulation. Thus we would introduce slight modifications to T_k and U_k:

$$\begin{aligned} T_k &= M O_k N^{\mathrm{T}}, \\ U_k &= N O_k N^{\mathrm{T}} + R_v. \end{aligned}$$

With new T_k and U_k, the proofs of Theorems 3 and 4 still proceed identically if the loose correlation between $\tilde{u}_k$ and v_k is ignored.

D. Algorithm summary

The input and state estimation scheme is summarized in Algorithm 1.

Algorithm 1: The simultaneous input and state estimation algorithm

Initialization: $E(\hat{x}_0) = E(x_0)$, $P_0^x = p_0 I$, where p_0 is a large positive value

for $k = 1$ *to* N **do**

$Q_k = C P_k^x C^{\mathrm{T}} + R_v$

$H_k^* = \left[D^{\mathrm{T}} Q_k D\right]^{-1} D^{\mathrm{T}} Q_k^{-1}$

$\hat{u}_k = H_k^* [y_k - C\hat{x}_k]$

$P_k^u = H_k^* Q_k H_k^{*\mathrm{T}}$,

if $k < N$ **then**

$P_k^{ux} = -H_k^* C P_k^x$

$O_k = \begin{bmatrix} P_k^x & (P_k^{ux})^{\mathrm{T}} \\ P_k^{ux} & P_k^u \end{bmatrix}$

$S_k = [\ A \quad B\ ] O_k [\ A \quad B\ ]^{\mathrm{T}}$

$T_k = [\ A \quad B\ ] O_k [\ C \quad D\ ]^{\mathrm{T}}$

$U_k = [\ C \quad D\ ] O_k [\ C \quad D\ ]^{\mathrm{T}} + R_v$

$L_k^* = T_k U_k^{-1}$

$\hat{x}_{k+1} = A\hat{x}_k + B\hat{u}_k + L_k^* [y_k - C\hat{x}_k - D\hat{u}_k]$

$P_{k+1}^x = S_k - L_k^* T_k^{\mathrm{T}} + R_w$

end

end

Remark 1: We can extend conveniently Algorithm 1 to linear time-varying discrete-time systems just by replacing A, B, C and D with their time-varying counterparts A_k, B_k, C_k and D_k. The proof can be done in analogy to the above. Note that the new system matrices are required to be observable for any k.

E. Convergence Analysis

The convergence properties of the proposed Algorithm 1 can be done by formulating a Riccati-like matrix equation and by analyzing its solutions. It is found that the convergence of both the state and input estimation depends on P_{k+1}^x. Therefore, convergence analysis of Algorithm 1 is reduced to that of P_{k+1}^x.

To analyze the convergence property of P_{k+1}^x, we first rewrite (21) as

$$P_{k+1}^x = M O_k M^{\mathrm{T}} - M O_k N^{\mathrm{T}} \left(N O_k N^{\mathrm{T}} + R_v\right)^{-1} N O_k M^{\mathrm{T}} + R_w, \tag{23}$$

where O_k can be considered a generalized function of P_k^x. From (23) we can define a generalized algebraic Riccati equation (GARE) as follows:

$$g(X) = M O(X) M^{\mathrm{T}} - M O(X) N^{\mathrm{T}} \left(N O(X) N' + R_v\right)^{-1} N O(X) M^{\mathrm{T}} + R_w. \tag{24}$$

Here $O(X)$ has exactly the same structure as O_k, with P_k^x replaced by X. As P_k^x is positive definite, we assume that X is also positive definite. From (24) and (23) it follows that

$$P_{k+1}^x = g(P_k^x).$$

Theorem 5: Consider the Riccati operator $\phi(K, X) = F O(X) F' + V$, where $F = M + KN$, $V = K R_v K' + R_w$. Assume that there exits a $\widetilde{K}$ and a $\widetilde{P} > 0$ such that

$$\widetilde{P} > \phi(\widetilde{K}, \widetilde{P}).$$

Then, for any $P_0^x > 0$, the sequence from $P_{k+1}^x = g(P_k^x)$ converges:

$$\lim_{k\to\infty} P_k = \overline{P},$$

where $\overline{P}$ satisfies

$$\overline{P} = g(\overline{P}).$$

Proof: The proof is omitted because of limited space, and will be included in an extended version. ∎

IV. Numerical Examples

In this section, we illustrate Algorithm 1 through two numerical examples.

Example 1: Consider an LTI system described by

$$A = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.8 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix};$$
$$C = \begin{bmatrix} 0.3 & 1 \\ 0 & 0.5 \end{bmatrix}, \quad D = \begin{bmatrix} 0.4 \\ 0.3 \end{bmatrix},$$
$$R_w = \begin{bmatrix} 0.08^2 & 0 \\ 0 & 0.08^2 \end{bmatrix}, \quad R_v = \begin{bmatrix} 0.07^2 & 0 \\ 0 & 0.07^2 \end{bmatrix}.$$

In this example, the input $\{u_k\}$ is taken as a uniformly-distributed sequence that satisfies:

$$E(u_k) = 0, \quad E(u_k^2) = 10, \quad E(u_k u_l) = 0 \text{ for } k \neq l.$$

No information about $\{u_k\}$ is available. Present to us is only $\{y_k\}$, from which Algorithm 1 is applied to estimate simultaneously the system inputs and states. The input estimation results are shown in Fig 2. It is seen that the input estimates are close to the original inputs. We also make a comparison between the state estimates and their true values in Figs. 3(a) and 3(b), and observe that only trivial differences exist.

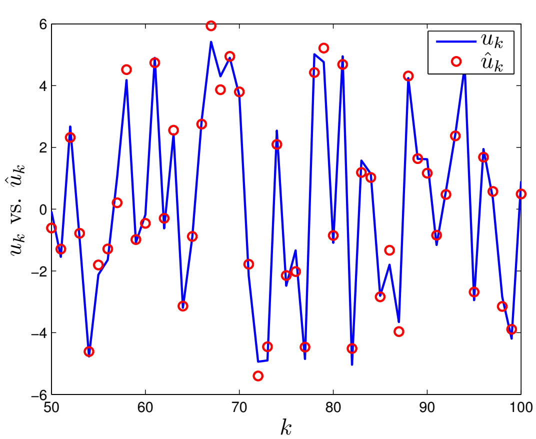
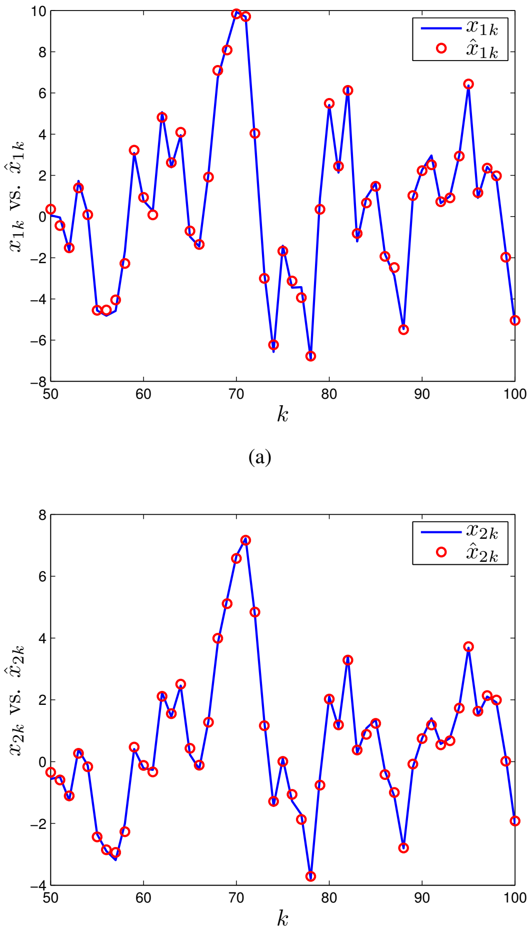


Fig. 2. Example 1: Comparison between the original input u_k and input estimates $\hat{u}_k$.

On certain occasions such as in maneuvering target tracking, some information can be determined *a priori* or assumed



(a)

(b)

Fig. 3. Example 1: Comparisons between the true state values and their estimates. (a) x_{1k} and state estimate $\hat{x}_{1k}$. (b) x_{2k} and state estimates $\hat{x}_{2k}$.

about the input, then it is likely to help improve estimation performance.

Example 2: We use the same system as in Example 1. Instead of assuming the random binary-value signal as the input, we assume that the two values, -10 and 10, are known. It is shown in Fig 4 that the inputs and their estimates are accurately superimposed. The state estimation also becomes more accurate in accordance, as illustrated in Figs. 5(a) and 5(b).

V. Conclusion

Simultaneous estimation of system inputs and states is a challenge. This paper considers the problem in view of both MMSE and MV and develops optimal estimator design procedures. Theoretical analysis of the optimality is carried out. Further, we propose an input and state co-estimation algorithm and analyze its convergence. Simulation results demonstrate the effectiveness of the proposed algorithm..

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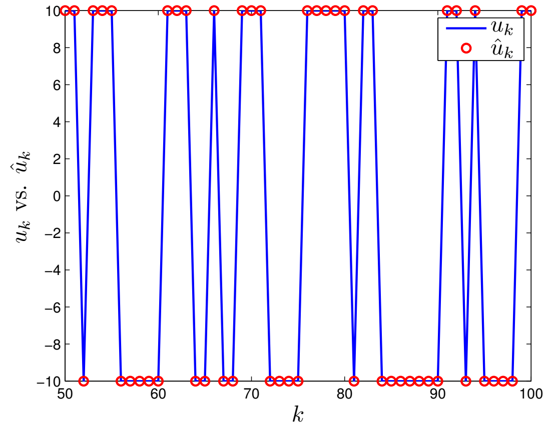
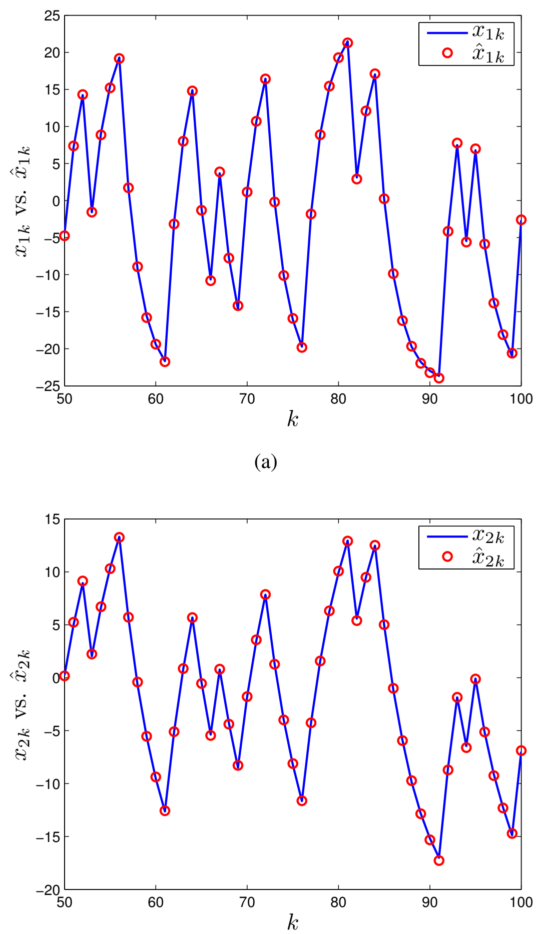


Fig. 4. Example 2: Comparison between the original input u_k and input estimates $\hat{u}_k$.



(a)

(b)

Fig. 5. Example 2: Comparisons between the true state values and their estimates. (a) x_{1k} and state estimate $\hat{x}_{1k}$. (b) x_{2k} and state estimates $\hat{x}_{2k}$.

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