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H_2 and H_∞ Dynamic Output Feedback Control of a Magnetic Bearing System via LMIs

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Abstract— **This paper presents a new design procedure for a robust stability, robust H_∞ control and robust H_2 control via dynamic output feedback for a class of uncertain linear systems. The uncertainties are norm bounded type. The state space matrices of the controllers are the solutions of some linear matrix inequalities problems. Finally, these procedures are applied to an active radial magnetic bearing system to support a high-speed energy storage flywheel.**

I. INTRODUCTION

An active magnetic bearing (AMB) is a collection of electromagnets used to suspend an object via feedback control. The principal benefits of AMBs, compared to mechanical and hydrostatic bearings, are a dramatic reduction in friction, which, in turn, allows efficient operation at extremely high speeds, the elimination of lubricants and their associated supply systems, the ability to operate in a vacuum and at high temperature, and the capability for actively controlling the stiffness of the bearing. Due to these advantages, AMBs have found usage in many industrial applications, such as electric auxiliaries for aircraft, energy storage flywheels, as well as high-speed turbines and compressors, etc. [1].

This paper considers a basic AMB system comprising an electromagnet on each side of a rigid rotor, as shown in Fig. 1. The model on which the controller design is based is described by a second-order linear interval system with unknown disturbances. The parameter uncertainty in the system is well described by the given parameter intervals, while the unmodeled dynamics may be included in the disturbance [2]. To eliminate the need for velocity feedback, an output feedback controller is proposed which uses only the rotor position signal.

Linear matrix inequalities (LMIs) have emerged as a powerful formulation and design technique for a variety of linear control problems. Since solving LMIs is a convex optimization problem, such formulations offer a numerically tractable means of attacking problems that lack an analytical solution. Consequently, reducing a control design problem to an LMI can be considered as a practical solution to this problem.

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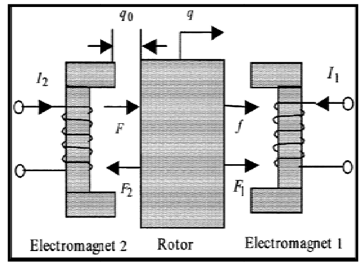

Fig. 1: Basic magnetic bearing.

Many researchers have considered the problem of designing robust stability, H_∞ performance and H_2 performance for nom-bounded uncertain systems in recent years. In this paper, we address robust stability, H_∞ and H_2 performance via output feedback control for the class of uncertain linear systems. We present the conditions of problem in terms of a number of linear matrix inequalities . Finally, these problems are applied to an active radial magnetic bearing system.

This paper is organized as follows: Section II includes description of system and required lemmas. In Section III, theorems for robust stability, H_2 and H_∞ performance are presented. The application of design methods in magnetic bearing system and simulation results are presented in Section IV. Finally, Concluding remarks are given in Section V.

II. PRELIMINARIES

This paper is concerned with the class of uncertain linear systems that can be described by state-space equations of the form:

$$\begin{cases} \dot{x} = \big(A_i + \Delta A(t)\big)x(t) + (B_1)\omega(t) + \big(B_2 + \Delta B_2(t)\big)u \\ Z(t) = C_1 x(t) + D_{11}\omega(t) + D_{12}u(t) \\ y(t) = C_2 x(t) + D_{21}\omega(t) \end{cases} \tag{1}$$

x is the state vector, ω is the disturbance input vector, u is the control input vector, Z is the controlled output vector, y is the measurement vector. $A, B_1, B_2, C_1, C_2, D_{11}, D_{12}, D_{21}$ are constant matrices with appropriate dimensions, and $\Delta A(t)$, $\Delta B_2(t)$ represent norm bounded parameter uncertainties which are in the following form:

$$\Delta A = H_1 F_1 E_1, \quad \Delta B_2 = H_2 F_2 E_2$$

where H_1, E_1, H_2, E_2 are known real constant matrices, and F_1, F_2 is an unknown matrix that belongs to the following set:

$$\Omega := \left\{ \begin{matrix} F(t)|F(t)^T F(t \leq I, \\ F(t) is\ lebesgue\ measurable \end{matrix} \right\}$$

By applying output feedback controller in the following form:

$$\begin{cases} \dot{x}_c(t) = A_c x_c(t) + B_c y(t) \\ u(t) = C_c x_c(t) + D_c y(t) \end{cases} \tag{2}$$

to (1), the closed loop system will be:

$$\begin{cases} \dot{X}_{cl} = \big(A_{cl} + \Delta A_{cl}(t)\big) X_{cl}(t) + \big(B_{cl} + \Delta B_{cl}(t)\big)\omega(t) \\ Z(t) = C_{cl} X_{cl}(t) + D_{cl}\omega(t) \end{cases} \tag{3}$$

where

$$A_{cl} = \begin{bmatrix} A + B_2 D_C C_2 & B_2 C_C \\ B_C C_2 & A_C \end{bmatrix},\quad B_{cl} = \begin{bmatrix} B_1 + B_2 D_C D_{21} \\ B_C D_{21} \end{bmatrix}$$
$$C_{cl} = [C_1 + D_{12} D_C C_2 \quad D_{12} C_C],\quad D_{cl} = D_{11} + D_{12} D_C D_{21}$$

and the closed loop uncertainties ΔA_{cl} and ΔB_{cl} are:

$$\Delta A_{cl} = H_{cl} F_{cl} E_{1cl}\,, \qquad \Delta B_{cl} = H_{cl} F_{cl} E_{2cl}$$

where

$$H_{cl} = \begin{bmatrix} H_1 & H_2 \\ 0 & 0 \end{bmatrix}, \qquad F_{cl} = \begin{bmatrix} F_1 & 0 \\ 0 & F_2 \end{bmatrix}$$
$$E_{1cl} = \begin{bmatrix} E_1 & 0 \\ E_2 D_C C_2 & E_2 C_C \end{bmatrix},\quad E_{2cl} = \begin{bmatrix} 0 \\ E_2 D_C D_{21} \end{bmatrix}$$

Lemma 1: Suppose that system (4) is asymptotically stable.

$$\begin{cases} \dot{x}(t) = Ax(t) + Bw(t) \\ z(t) = Cx(t) + Dw(t) \end{cases} \tag{4}$$

let $T_s = C(IS - A)^{-1}B + D$ denote its transfer function. if $D = 0$ then the following statements are equivalent:

(a). $\|T\|_2 < \gamma$

(b).There exists $X = X^T > 0$ and Z such that:

$$\begin{bmatrix} A^T X + XA & XB \\ B^T X & -\gamma I \end{bmatrix} < 0, \begin{bmatrix} X & C^T \\ C & Z \end{bmatrix} > 0,$$

$$trace(Z) < \gamma \tag{5}$$

Lemma 2 (Bounded Real Lemma): For system (4), H_∞ performance, with $\gamma > 0$ is equivalent to the existence of $X > 0$ satisfying:

$$\begin{bmatrix} AX + XA^T & B & XC^T \\ B^T & -\gamma I & D^T \\ CX & D & -\gamma I \end{bmatrix} < 0 \tag{6}$$

Lemma 3 (Schur Complement): The Linear inequality:

$$\begin{bmatrix} Q(X) & S(X) \\ S^T(X) & R(X) \end{bmatrix} > 0$$

with $Q = Q^T$, $R = R^T > 0$ and S is an affine function of X, is equivalent to:

$$\begin{cases} Q(X) - S(X)R^{-1}(X)S^T(X) > 0 \\ R(\mathrm{X}) > 0 \end{cases}$$

Lemma 4: Let Σ, Ω, Γ be matrices with appropriate dimensions which Ω is a symmetric matrix Then for every matrix F with $FF^T \leq I$, $\Omega + \Gamma F \Sigma + (\Gamma F \Sigma)^T \leq 0$ is equivalent to the $\Omega + \varepsilon \Gamma \Gamma^{\mathrm{T}} + \varepsilon^{-1} \Sigma^T \Sigma \leq 0$, if and only if there exist a constant $\varepsilon > 0$ [6].

III. Main Results

A. Robust Stability via Output Feedback

The following theorem proposes an LMI for designing output feedback controller satisfying robust stability.

Theorem 1: consider the change of controller variables as follows [3]:

$$\begin{aligned} \hat{A}_C &= SAR + NB_C C_2 R + SB_2 C_C M^T + NA_C M^T \\ &\quad + SB_2 D_C C_2 R \\ \hat{B}_C &= NB_C + SB_2 D_C \\ \hat{C}_C &= C_C M^T + D_C C_2 R \\ \widehat{D}_C &= D_C \end{aligned} \tag{7}$$

where M and N are invertible and should be chosen such that:

$$MN^T = I - RS \tag{8}$$

For system (1), there exists an output feedback controller in the form of system (2) such that closed loop system (3) for every admissible uncertainties, satisfies robust stability, if the following system of LMI's is feasible.

Find $R = R^T \in \boldsymbol{R}^{n\times n}$, $S = S^T \in \boldsymbol{R}^{n\times n}$, $\hat{A}_c, \hat{B}_c, \hat{C}_c, \widehat{D}_C$ and scalar $\varepsilon > 0$ such that:

$$\begin{bmatrix} R & I \\ I & S \end{bmatrix} > 0 \tag{9}$$

and

$$\begin{bmatrix} \psi_A + {\psi_A}^T & {\psi_E}^T & \varepsilon\psi_H \\ \blacksquare & -\varepsilon I & 0 \\ \blacksquare & \blacksquare & -\varepsilon I \end{bmatrix} < 0 \tag{10}$$

where:

$$\psi_A = \begin{bmatrix} AR + B_2 \hat{C}_C & A + B_2 \widehat{D}_C C_2 \\ \hat{A}_C & SA + \hat{B}_C C_2 \end{bmatrix},$$
$$\psi_E = \begin{bmatrix} E_1 R & E_1 \\ E_2 \hat{C}_C & E_2 \widehat{D}_C C_2 \end{bmatrix}, \qquad \psi_H = \begin{bmatrix} H_1 & H_2 \\ SH_1 & SH_2 \end{bmatrix}$$

The state matrices of the controller ($A_C, B_C, B_C\ and\ D_C$) can be recoverd from (7). Note that ■ is used to show symmetric terms.

Proof: The system (3) is said to be stable for perturbations Δ if there exists a matrix $X = X^T > 0$ such that:

$$(A_{cl} + \Delta A_{cl}(t))^T X + X\big(A_{cl} + \Delta A_{cl}(t)\big) < 0 \tag{11}$$

By separating uncertain part of inequality (11) and using lemma 4, the following inequality is obtained:

$$A_{cl}X + XA_{cl}^T + \varepsilon H_{cl}H_{cl}^T + \varepsilon^{-1}XE_{1cl}^T E_{1cl}X < 0 \tag{12}$$

Using Schur complement for inequality (12), yields:

$$\begin{bmatrix} A_{cl}X + XA_{cl}^T & XE_{1cl}^T & \varepsilon H_{cl} \\ \blacksquare & -\varepsilon I & 0 \\ \blacksquare & \blacksquare & -\varepsilon I \end{bmatrix} < 0 \tag{13}$$

Since matrices A_{cl} and X are multiplied in inequality (13), it is non convex. Therefore X and $X^{-1}$are partitiond as follow:

$$X = \begin{bmatrix} R & M \\ M^T & U \end{bmatrix},\ X^{-1} = \begin{bmatrix} S & N \\ N^T & V \end{bmatrix} \tag{14}$$

It is readily verified that X satisfies the identity

$$\Pi_1 = X\Pi_2\ ,\ \Pi_1 = \begin{bmatrix} R & I \\ M^T & 0 \end{bmatrix},\ \Pi_2 = \begin{bmatrix} I & S \\ 0 & N^T \end{bmatrix} \tag{15}$$

with pre-and post multiplying inequality $X > 0$ by ${\Pi_2}^T$ and Π_2 respectively, inequality (9) is obtained. Similarly, the last LMI condition (10) is derived from (13) by pre- and post multiplication by $\mathrm{diag}({\Pi_2}^T, I, I)$ and $\mathrm{diag}(\Pi_2, I, I)$ respectively.

B. Robust H_∞ Control via Output Feedback

The following theorem proposes an LMI for designing output feedback controller satisfying H_∞ performance.

Theorem 2: For system (1), there exists an output feedback controller in the form of system (2) such that closed loop system (3) for every admissible uncertainties, satisfies H_∞ performance with $\gamma > 0$, if the following system of LMI's is feasible.

Find $R = R^T \in \boldsymbol{R}^{n\times n}$, $S = S^T \in \boldsymbol{R}^{n\times n}$, $\hat{A}_c, \hat{B}_c, \hat{C}_c, \widehat{D}_C$ and scalar $\varepsilon > 0$ such that:

$$\begin{bmatrix} R & I \\ I & S \end{bmatrix} > 0 \tag{16}$$

and

$$\begin{bmatrix} \psi_A + {\psi_A}^T & \psi_B & {\psi_C}^T & {\psi_E}^T & \varepsilon\psi_H \\ \blacksquare & -\gamma I & D_{cl}^T & E_{2cl}^T & 0 \\ \blacksquare & \blacksquare & -\gamma I & 0 & 0 \\ \blacksquare & \blacksquare & \blacksquare & -\varepsilon I & 0 \\ \blacksquare & \blacksquare & \blacksquare & \blacksquare & -\varepsilon I \end{bmatrix} < 0 \tag{17}$$

where

$$\psi_A = \begin{bmatrix} AR + B_2\hat{C}_C & A + B_2\widehat{D}_C C_2 \\ \hat{A}_C & SA + \hat{B}_C C_2 \end{bmatrix},\ \psi_B = \begin{bmatrix} B_1 + B_2\widehat{D}_C D_{21} \\ SB_1 + \hat{B}_C D_{21} \end{bmatrix}$$
$$\psi_C = \begin{bmatrix} C_1R + D_{12}\hat{C}_C & C_1 + D_{12}\widehat{D}_C C_2 \end{bmatrix} \tag{18}$$
$$\psi_E = \begin{bmatrix} E_1R & E_1 \\ E_2\hat{C}_C & E_2\widehat{D}_C C_2 \end{bmatrix},\quad \psi_H = \begin{bmatrix} H_1 & H_2 \\ SH_1 & SH_2 \end{bmatrix}$$

where $\hat{A}_c, \hat{B}_c, \hat{C}_c, \widehat{D}_C$ are defined in (7).

Proof: By considering (6) for closed loop system (3), following inequality is obtained:

$$\begin{bmatrix} Herm(A_{cl} + \Delta A_{cl})X & (B + \Delta B_{cl}) & X{C_{cl}}^T \\ \blacksquare & -\gamma I & {D_{cl}}^T \\ \blacksquare & \blacksquare & -\gamma I \end{bmatrix} < 0 \tag{19}$$

where $Herm$ denotes the Hermitian transpose.

By separating uncertain part of inequality (19) and using lemma 4, the following inequality is obtained:

$$\begin{bmatrix} A_{cl}X + XA_{cl}^T + \varepsilon H_{cl}H_{cl}^T + \varepsilon^{-1}XE_{1cl}^T E_{1cl}X & \blacksquare & \blacksquare \\ {B_{cl}}^T + \varepsilon^{-1}E_{2cl}^T E_{1cl}X & -\gamma I + \varepsilon^{-1}E_{2cl}^T E_{2cl} & \blacksquare \\ C_{cl}X & D_{cl} & -\gamma I \end{bmatrix} < 0 \tag{20}$$

Using Schur complement for inequality (20), yields:

$$\begin{bmatrix} A_{cl}X + XA_{cl}^T & B_{cl} & XC_{cl}^T & XE_{1cl}^T & \varepsilon H_{cl} \\ \blacksquare & -\gamma I & D_{cl}^T & E_{2cl}^T & 0 \\ \blacksquare & \blacksquare & -\gamma I & 0 & 0 \\ \blacksquare & \blacksquare & \blacksquare & -\varepsilon I & 0 \\ \blacksquare & \blacksquare & \blacksquare & \blacksquare & -\varepsilon I \end{bmatrix} < 0 \tag{21}$$

Since matrices A_{cl} and X are multiplied in inequality (21), it is non convex. Therefore X and $X^{-1}$are partitioned the same as (14) and Π_1 and Π_2 are defined similar to (15). With pre-and post multiplying inequality $X > 0$ by ${\Pi_2}^T$ and Π_2 respectively, inequality (16) is obtained. Similarly, the last LMI condition (17) is derived from (21) by pre- and post multiplication by $\mathrm{diag}({\Pi_2}^T, I, I, I, I)$ and $\mathrm{diag}(\Pi_2, I, I, I, I)$ respectively.

Remark: Since ε and ψ_H are multiplied in (17), this inequality is non convex, but according to that ε is a scalar, this inequality can be easily solved by line search on ε. Suppose an arbitrary positive scalar, then if the problem was infeasible, change it with respect to this fact that the problem is feasible.

C. Robust H_2 Control via Output Feedback

The following theorem proposes an LMI for designing output feedback controller satisfying H_2 performance.

Theorem 3: For system (1), there exists an output feedback controller in the form of system (2) such that closed loop system (3) for every admissible uncertainties, satisfies H_2 performance, if the following system of LMI's is feasible.

Find $R = R^T \in \boldsymbol{R}^{n\times n}$, $S = S^T \in \boldsymbol{R}^{n\times n}$, matrices $\hat{A}_c, \hat{B}_c, \hat{C}_c$, scalar $\varepsilon > 0$ and Z such that:

$$\left[\begin{array}{cc|c} R & I & \\ I & S & \psi_C^T \\ \hline \multicolumn{2}{c|}{\psi_C} & Z \end{array}\right] > 0 \tag{22}$$

$$\begin{bmatrix} \psi_A + \psi_A^T & \psi_B & \psi_E^T & \varepsilon\psi_H \\ \blacksquare & -I & E_{2cl}^T & 0 \\ \blacksquare & \blacksquare & -\varepsilon I & 0 \\ \blacksquare & \blacksquare & \blacksquare & -\varepsilon I \end{bmatrix} < 0 \tag{23}$$

$$trace(Z) < \gamma, \quad D_{cl} = 0$$

where $\psi_A, \psi_B, \psi_C, \psi_E, \psi_H$ are defined in (18) and $\hat{A}_c, \hat{B}_c, \hat{C}_c, \hat{D}_C$ are defined in (7).

Proof: By considering (5) for closed loop system (3), following inequalities is obtained:

$$\begin{bmatrix} HermX(A_{cl} + \Delta A_{cl}) & X(B + \Delta B_{cl}) \\ \blacksquare & -\gamma I \end{bmatrix} < 0,$$
$$\begin{bmatrix} X & C_{cl}^T \\ C_{cl} & Z \end{bmatrix} > 0$$
$$trace(Z) < \gamma \tag{24}$$

By separating uncertain part of inequality (24) and using lemma 4, the following inequality is obtained:

$$\begin{bmatrix} A_{cl}^T X + XA_{cl} + \varepsilon X H_{cl} H_{cl}^T X + \varepsilon^{-1} E_{1cl}^T E_{1cl} & \blacksquare \\ B_{cl}^T X + \varepsilon^{-1} E_{2cl}^T E_{1cl} & \varepsilon^{-1} E_{2cl}^T E_{2cl} - I \end{bmatrix} < 0$$
$$\begin{bmatrix} X & C_{cl}^T \\ C_{cl} & Z \end{bmatrix} > 0,$$
$$trace(Z) < \gamma \tag{25}$$

Using Schur complement for inequality (25), yields:

$$\begin{bmatrix} A_{cl}^T X + XA_{cl} & XB_{cl} & E_{1cl}^T & \varepsilon X H_{cl} \\ \blacksquare & -I & E_{2cl}^T & 0 \\ \blacksquare & \blacksquare & -\varepsilon I & 0 \\ \blacksquare & \blacksquare & \blacksquare & -\varepsilon I \end{bmatrix} < 0 \tag{26}$$

$$\begin{bmatrix} X & C_{cl}^T \\ C_{cl} & Z \end{bmatrix} > 0, \quad trace(Z) < \gamma$$

Since matrices A_{cl} and X are multiplied in inequality (26), it is non convex. Therefore X and $X^{-1}$are partitiond the same as (14) and Π_1 and Π_2 are defined similar to (15). With pre-and post multiplying inequality $\begin{bmatrix} X & C_{cl}^T \\ C_{cl} & Z \end{bmatrix} > 0$ by $(\Pi_1, I)^T$ and (Π_1, I) respectively, inequality (22) is obtained. Similarly, LMI condition (23) is derived from (26) by pre- and post multiplication by diag$(\Pi_2{}^T, I, I, I)$ and diag(Π_2, I, I, I) respectively.

IV. Application to Magnetic Bearing

A. Problem Formulation

A dynamical mathematical model for the AMB shown in Fig. 1, can be established as follows:

$$m\ddot{q} = -\frac{\mu_0 A N^2}{4}\left[\left(\frac{I_1}{q_0 - q}\right)^2 - \left(\frac{I_2}{q_0 + q}\right)^2\right] + f + F \tag{27}$$

where

m Mass of the rotor (kg);
q Position displacement of the rotor (m);
q_0 Nominal air gap (m);
μ_0 Permeability of free space H/m;
A Total pole-face area of each electromagnet (m);
N Number of turns on each electromagnet coil;
I_1, I_2 Electromagnet coil currents (A);
f An unknown disturbance (N);
F Some known force acting on the rotor (N).

When (1) is linearized at the equilibrium point,

$$I_1 = I_2 = I_0 \quad , \quad q = 0$$

and augmented with the control structure shown in Fig. 2, the linearized model is obtained as the following second-order system:

$$\ddot{q} - \omega^2 q = \sigma u + \frac{1}{m}(f + F) \tag{28}$$

where

$$\omega = \frac{\mu_0 A N^2 I_0^2}{m q_0^3} \qquad \sigma = -\frac{\mu_0 A N^2 I_0}{m q_0^2} \tag{29}$$

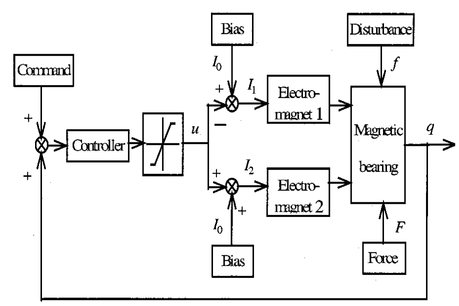

Fig. 2. Diagram of the control system.

Due to inaccuracies in the measurement of some of the physical parameters and changing environmental conditions, the system parameters ω and σ are generally uncertain. However, without loss of generality, it can be assumed that their values lie within some known intervals:

$$\omega \in [\omega_1 \quad \omega_2] \qquad \sigma \in [\sigma_1 \quad \sigma_2]$$

where, $\omega_1, \omega_2, \sigma_1$and σ_2 are known scalars satisfying:

$$\omega_2 \geq \omega_1 > 0 \qquad \sigma_1 \leq \sigma_2 < 0$$

In order to avoid the need for velocity feedback, while at the same time achieving satisfactory stability, this paper addresses the control of the system (28) using a output feedback controller. Assuming that only the rotor displacement position is measured, and denoting:

$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \qquad y = q$$

the system (28) can be converted into the following equivalent state-space form:[2]

$$\begin{cases} \dot{x} = (A + \Delta A)x + B_1(f + F) + (B_2 + \Delta B_2)u \\ y = cx \end{cases}$$

$$(A + \Delta A) = \begin{bmatrix} 0 & 1 \\ \varpi & 0 \end{bmatrix}, \; B_1 = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}, \; (B_2 + \Delta B_2) = \begin{bmatrix} 0 \\ \sigma \end{bmatrix}$$

$$c = [1 \quad 0] \tag{30}$$

where, $\varpi = \omega^2$, and the parameters ω and σ satisfy $\varpi_2 \geq \varpi \geq \varpi_1 > 0$ and $\sigma_1 \leq \sigma \leq \sigma_2 < 0$. By defining nominal values $\sigma_0 := \frac{\sigma_1+\sigma_2}{2}$, $\varpi_0 := \frac{\varpi_1+\varpi_2}{2}$ and scaled errors as $\Delta_1 = \frac{\sigma-\sigma_0}{\sigma_2-\sigma_0}$, $\Delta_2 = \frac{\varpi-\varpi_0}{\varpi_2-\varpi_0}$, then they implies: $\sigma = \sigma_0 + W_1\Delta_1$ with $W_1 = \sigma_2 - \sigma_0$ and $\varpi = \varpi_0 + W_2\Delta_2$ with $W_2 = \varpi_2 - \varpi_0$ by the class of uncertainties:

$$\Delta_i := \{\Delta_i \in R \mid -1 < \Delta_i < 1\}$$

Note that the original parameters $\omega \in [\omega_1 \quad \omega_2]$, $\sigma \in [\sigma_1 \quad \sigma_2]$ has been transformed into the new parameters $\Delta_1, \Delta_2 \in (-1, 1)$ by using a nominal values σ_0, ϖ_0 and W_1, W_2 as weights. Therefore:

$$A = \begin{bmatrix} 0 & 1 \\ \varpi_0 & 0 \end{bmatrix}, \Delta A = \begin{bmatrix} 0 & 0 \\ W_1\Delta_1 & 0 \end{bmatrix}, \; B_1 = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}, \; B_2 = \begin{bmatrix} 0 \\ \sigma_0 \end{bmatrix}$$

$$\Delta B_2 = \begin{bmatrix} 0 \\ W_2\Delta_2 \end{bmatrix}, c_1 = c_2 = [1 \quad 0], \; D_{11} = D_{12} = D_{21} = 0$$

and uncertain matrices are:

$$H_1 = \begin{bmatrix} 0 & 0 \\ W_1 & 0 \end{bmatrix}, H_2 = \begin{bmatrix} 0 & 0 \\ W_2 & 0 \end{bmatrix}, \; E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; E_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

A general output feedback controller for the system (30) can be written in the following form:

$$\begin{cases} \dot{x}_c(t) = A_c x_c(t) + B_c y(t) \\ u(t) = C_c x_c(t) + D_c y(t) + K_f F \end{cases} \tag{31}$$

where A_c, B_c, C_c and D_c are four scalar controller coefficients, to be designed, and the term K_f is introduced to compensate for the effect of the force F, the coefficient being given by [2]:

$$K_f = -\frac{1}{m\sigma} = \frac{q_0^2}{\mu_0 A N^2 I_0}$$

B. Simulation Results

The values of ω and, and those of the interval boundaries, ω_i and σ_i, $i = 1,2$, are given in Table 1.

TABLE 1
Parameters ω and σ and lower and their upper bounds

Parameter	Value
ω	359.6
σ	-10.35
ω_1	240
ω_2	390
σ_1	-22
σ_2	-4.5

TABLE 2
Magnetic Parameters

Parameter	value
m	6kg
A	$4\times2.610^{-4}m^2$
N	40
I_0	5 A
q_0	.4mm

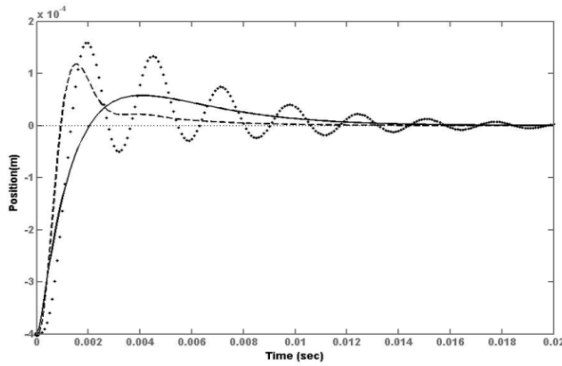

Fig. 3. Closed-loop system response using robust stabilizer controller: $\boldsymbol{W_1}$ =1.8, $\boldsymbol{W_2}$=6.5 (solid), $\boldsymbol{W_1}$ =0.1, $\boldsymbol{W_2}$=1 (dotted), $\boldsymbol{W_1}$ =0.4, $\boldsymbol{W_2}$=3 (dash).

The nominal parameters for the bearings are given in Table 2. The close loop system response with respect to robust stabilizer controller is shown in Fig. 3. The close loop system response with respect to robust H_∞ controller is shown in Fig. 4 and its bode-magnitude diagram is shown in Fig. 5, Fig. 6 shows the ratio of regulated output energy to The disturbance energy of this system. The close loop system response with respect to robust H_2 controller is shown in Fig. 7 and its bode-magnitude diagram is shown in Fig. 8. The obtained value of γ from H_∞ controller without uncertainty is 0.016, and the obtained value of γ from H_2 controller without uncertainty is 2.1538e-006.Comparing Fig. 3, Fig. 4 and Fig. 7 shows that H_∞ controller has a better robustness than the other controllers, but H_2 controller has a better transient response.

V. Conclusion

In this paper, we addressed robust stability, H_∞ and H_2 performance via output feedback control for the class of uncertain linear systems. We presented the conditions of problem in terms of a number of linear matrix inequalities. Then these problems were applied to an active radial magnetic bearing system to support a high-speed energy storage flywheel. The effectiveness of the design was shown in simulation results.

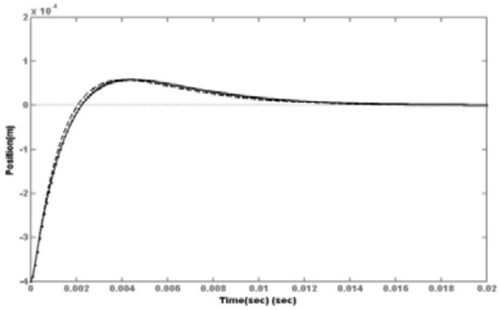

Fig 4. Closed-loop system response using robust H_∞ controller**:** W_1 =1.8, W_2=6.5 (solid), W_1 =0.1**,** W_2=1 (dotted), W_1 =0.4, W_2=3(dash).

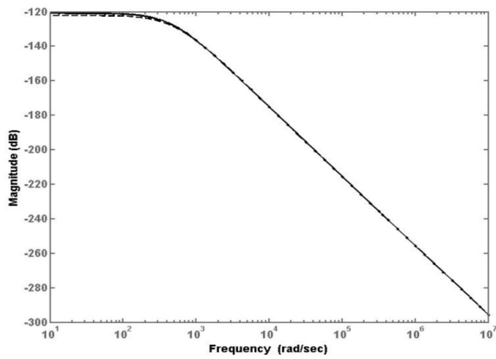

Fig 5. Bode-magnitude diagram using robust $\boldsymbol{H_\infty}$ controller: $\boldsymbol{W_1}$ =1.8, $\boldsymbol{W_2}$=6.5 (solid), $\boldsymbol{W_1}$ =0.1, $\boldsymbol{W_2}$=1 (dotted), $\boldsymbol{W_1}$ =0.4, $\boldsymbol{W_2}$=3(dash).

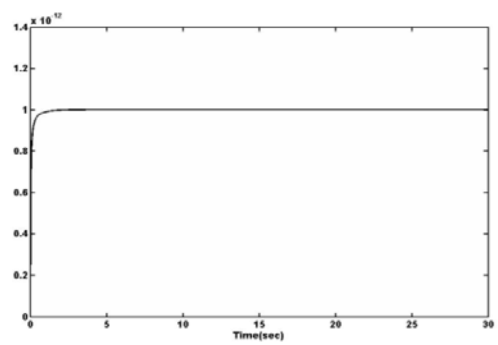

Fig 6. The ratio of regulated output energy to the disturbance energy of the system.

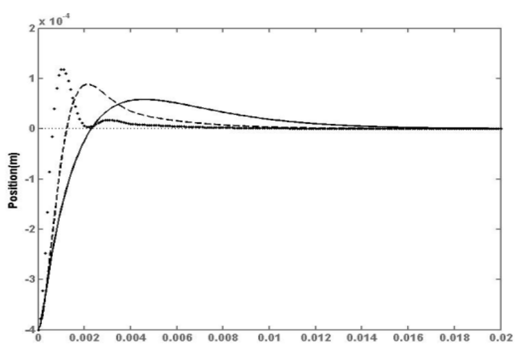

Fig 7. Closed-loop system response using robust $\boldsymbol{H_2}$ controller: $\boldsymbol{W_1}$ =1.8, $\boldsymbol{W_2}$=6.5 (solid), $\boldsymbol{W_1}$ =0.1, $\boldsymbol{W_2}$=1 (dotted), $\boldsymbol{W_1}$ =0.4, $\boldsymbol{W_2}$=3(dash).

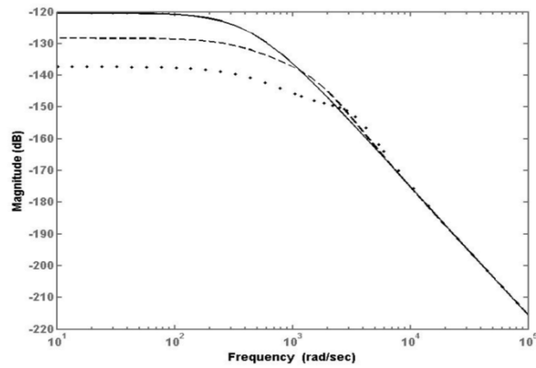

Fig 8. Bode-magnitude diagram using robust $\boldsymbol{H_2}$ controller: $\mathbf{W_1}$ =1.8, $\boldsymbol{W_2}$=6.5 (solid), $\boldsymbol{W_1}$ =0.1, $\boldsymbol{W_2}$=1 (dotted), $\boldsymbol{W_1}$ =0.4, $\boldsymbol{W_2}$=3(dash).

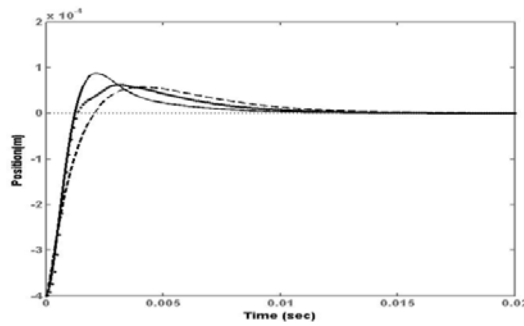

Fig 9. Closed-loop system response: Robust stabilizer (dotted), $\boldsymbol{H_\infty}$ controller (dash), $\boldsymbol{H_2}$ controller (solid).

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