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Adaptive Neural Network Satellite Attitude Control in the Presence of Inertia and CMG Actuator Uncertainties

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Abstract— **A neural network-based adaptive attitude tracking controller is developed in this paper, which achieves attitude tracking in the presence of parametric uncertainty, nonlinear actuator disturbances, and unmodeled external disturbance torques, which do not satisfy the linear-in-the-parameters assumption (i.e., non-LP). The satellite control torques are produced by means of a cluster of control moment gyroscopes (CMGs), which have uncertain dynamic and static friction in the gimbals in addition to unknown electromechanical disturbances. Some challenges encountered in the control design are that the control input is premultiplied by a non-square, time-varying, nonlinear, uncertain matrix and is embedded in a discontinuous nonlinearity. Controller performance is proven via Lyapunov stability analysis.**

I. Introduction[1]

Due to the exorbitant cost of launching large, heavy payloads into space, the aerospace industry is moving toward smaller satellites (small-sats) and the technology to support them [1], [2]. Certain challenges arise, however, in designing precision attitude control systems (ACS) for small-sats. Specifically, small-sats are more susceptible to external disturbances than their larger counterparts, and the actuator dynamics have more of an influence on the satellite dynamics than in the larger counterparts. These conflicting requirements necessitate novel solutions to ACS for small-sats. A neural network (NN) attitude control system is proposed in this paper for control moment gyroscope (CMG)-actuated small-sats, which are subject to parameteric uncertainty, uncertain actuator dynamics, and nonlinear disturbance torques.

The design of ACS for satellites is complicated due to parametric uncertainties, disturbances, and nonlinearities, which usually exist in the corresponding plant dynamics. To cope with these challenges, attitude controllers based on NNs are often utilized [3]–[5]. In [4], an attitude control approach based on the radial basis function neural network (RBFNN) is developed. The satellite dynamic model utilized in [4] includes no friction effects or disturbances in the reaction wheel actuators. Another NN attitude controller is presented in [5], which utilizes NNs to approximate the parametric uncertainties and nonlinearities present in the system dynamics. An online NN is used in [5] to re-optimize a Single Network Adaptive Critic, or SNAC-based optimal controller, which has been designed a priori for the nominal system. In [3], a NN attitude controller is developed based on a simplified nonlinear model of the Space Station Freedom. The dynamic model for the space station considered in [3] is simplified by assuming small roll/yaw attitude errors and small products of inertia. The attitude controller in [3] demonstrates the capability of the NN to adaptively compensate for varying inertia characteristics. The NN controllers presented in [3] and [5] are tested in attitude control problems under the assumption that a control torque can be directly applied about the spacecraft body-fixed axes.

For applications involving small satellites, the assumption that a torque can be directly applied about the satellite body-fixed axes may not be valid because the control torques are generated by actuators which have additional dynamics. For small-sats, control torques are usually generated by CMGs due to their low mass and power consumption properties. Unfortunately, the torque-producing capacity of CMGs can deteriorate over time due to bearing degradation and increased friction in the gimbals. In addition, electromechanical disturbances (e.g., tachometer ripple, motor cogging, motor back electromotive forces (BEMF), commutation or switching errors, and other electrical errors) in the CMG torquer control loop can hinder CMG performance [6]. The presence of these mechanical and electromechanical factors in the CMG actuators causes significant challenges in designing ACS for small-sats using CMG actuators.

An adaptive NN attitude tracking controller is developed in this paper for CMG-actuated small-sats, which compensates for uncertain satellite inertia, uncertain CMG gimbal friction, CMG actuator disturbances, and nonlinear disturbance torques, which do not satisfy the linear-in-the-parameters assumption (i.e., non-LP). The NN weights and thresholds are adjusted on-line, with no off-line learning phase required. In addition to the unknown CMG gimbal friction assumed present in the CMG torque model (e.g., see [2]), unknown, nonlinear electromechanical disturbances are assumed to be present in the CMG actuators. Some of the challenges encountered in the control design are that the control input (i.e., CMG gimbal angular rate) is premultiplied by a non-square, time-varying, nonlinear uncertain matrix due to dynamic gimbal friction and electromechanical disturbances and is embedded in a discontinuous nonlinearity due to static gimbal friction. Furthermore, due to the small size of the

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satellite considered in this development, the motion of the CMGs causes significant time-variation in the satellite inertia characteristics. The time-variation of the satellite inertia manifests itself as a nonlinear disturbance torque in the satellite dynamic model, which is handled via a Lyapunov-based adaptive law.

II. DYNAMIC MODEL AND PROPERTIES

The dynamic model for a rigid body CMG-actuated satellite can be expressed as [7], [8]

$$J\dot{\omega} = -\omega^{\times} J\omega + \tau_{cmg} - \dot{J}\omega + \tau_{d}. \tag{1}$$

In (1), $J(\delta) \in \mathbb{R}^{3\times3}$ represents the positive definite, symmetric satellite inertia matrix that is a function of the CMG gimbal angular position vector $\delta(t) \in \mathbb{R}^{4}$, $\omega(t) \in \mathbb{R}^{3}$ denotes the angular velocity of the satellite body-fixed frame $\mathcal{F}$ with respect to $\mathcal{I}$ expressed in $\mathcal{F}$, $\tau_{cmg}(t) \in \mathbb{R}^{3}$ denotes the torque generated via a CMG cluster consisting of four single gimbal CMGs, the term $\dot{J}(t)\omega(t)$ represents the torque produced by the time variation of the satellite inertia matrix due to the motion of the CMGs, $\tau_{d}(t) \in \mathbb{R}^{3}$ denotes a general nonlinear disturbance (e.g., unmodeled effects), and the notation $\zeta^{\times}$ $\forall\zeta = [\zeta_1, \zeta_2, \zeta_3]^T$ denotes the skew-symmetric cross-product matrix. The torque generated from the CMG cluster can be modeled as

$$\tau_{cmg} = -\left(\dot{h}_{cmg} + \omega^{\times} h_{cmg}\right) - AF_d\dot{\delta} - AF_s sgn\dot{\delta} + AT_d, \tag{2}$$

where F_d, $F_s \in \mathbb{R}^{4\times4}$ are diagonal matrices whose elements are the unknown constant dynamic and static friction coefficients, respectively, of the four CMG gimbals, $h_{cmg}(t) \in \mathbb{R}^{3}$ represents the angular momentum of the CMG cluster, and $\dot{h}_{cmg}(t)$ is modeled as [9]

$$\dot{h}_{cmg} = hA\dot{\delta}, \tag{3}$$

where $h \in \mathbb{R}$ represents the constant angular momentum of each CMG expressed in the gimbal-fixed frame (i.e., h is the same for all four CMGs). In (2) and (3), $\dot{\delta}(t) \in \mathbb{R}^{4}$ denotes the CMG gimbal angular velocity control input, which is defined as

$$\dot{\delta} \triangleq \begin{bmatrix} \dot{\delta}_1 & \dot{\delta}_2 & \dot{\delta}_3 & \dot{\delta}_4 \end{bmatrix}^T, \tag{4}$$

where $\dot{\delta}_i(t) \in \mathbb{R}$ $\forall i = 1, 2, 3, 4$ denotes the angular velocity of the i^{th} CMG gimbal, $sgn\left(\dot{\delta}(t)\right) \in \mathbb{R}^{4}$ denotes a vector form of the standard $sgn(\cdot)$ function where the $sgn(\cdot)$ is applied to each element of $\dot{\delta}(t)$, and $A(\delta) \in \mathbb{R}^{3\times4}$ denotes a measurable Jacobian matrix. Also in (2), $T_d\left(\delta, \dot{\delta}\right) \in \mathbb{R}^{4}$ represents torques in the gimbal axes due to tachometer disturbances, defined explicitly as [6]

$$T_d \triangleq K_G E_d \dot{\delta}, \tag{5}$$

where $K_G \in \mathbb{R}^{4\times4}$ denotes a diagonal matrix of uncertain, constant forward loop gains for the four CMG gimbal loops, and $E_d(\delta) = diag\left\{ E_{d1}(\delta_1) \quad E_{d2}(\delta_2) \quad E_{d3}(\delta_3) \quad E_{d4}(\delta_4) \right\}$ is a matrix of disturbance voltages in the four gimbals, where $E_{di}(\delta_i)$ $\forall i = 1, 2, 3, 4$ are nonlinear functions of the i^{th} gimbal angle.

Property 1: The satellite inertia matrix in (1) can be lower and upper bounded as

$$\frac{1}{2}\lambda_{\min}\{J\}\|\xi\|^2 \le \xi^T J\xi \le \frac{1}{2}\lambda_{\max}\{J\}\|\xi\|^2 \quad \forall\xi \in \mathbb{R}^n, \tag{6}$$

where $\lambda_{\min}\{J\}$, $\lambda_{\max}\{J\} \in \mathbb{R}$ are the minimum and maximum eigenvalues of $J(\delta)$, respectively.

Property 2: Since the elements of the Jacobian matrix $A(\delta)$ are combinations of bounded trigonometric terms, the following inequality can be developed:

$$\|A(\delta)\|_{i\infty} \le \zeta_0, \tag{7}$$

where $\zeta_0 \in \mathbb{R}$ is a positive bounding constant, and $\|\cdot\|_{i\infty}$ denotes the induced infinity norm of a matrix.

Property 3: The static friction matrix F_s can be bounded as $\|F_s\|_{i\infty} < F_M$, where F_M is a known constant.

Property 4: The term $\tau_d(t) \in \mathbb{R}^3$ is a disturbance acting on the system due to the gravity-gradient. Similar to [10], $\tau_d(t)$ is assumed to be of the form $\tau_d = p(q)$, where $p(q) \in \mathbb{R}^3$ is an unknown nonlinear function of the quaternion $q(t) \triangleq \{q_0(t), q_v(t)\} \in \mathbb{R} \times \mathbb{R}^3$.

III. KINEMATIC MODEL

The rotational kinematics of the rigid-body satellite can be determined as [11]

$$\dot{q}_v = \frac{1}{2}\left(q_v^{\times}\omega + q_0\omega\right) \qquad \dot{q}_0 = -\frac{1}{2}q_v^T\omega. \tag{8}$$

In (8), $q(t)$ represents the unit quaternion [7] describing the orientation of the body-fixed frame $\mathcal{F}$ with respect to $\mathcal{I}$, subject to the constraint $\|q_v\|^2 + q_0^2 = 1$. The notation $q_d(t) \triangleq \{q_{0d}(t), q_{vd}(t)\} \in \mathbb{R}\times\mathbb{R}^3$ represents the desired unit quaternion that describes the orientation of the body-fixed frame $\mathcal{F}_d$ with respect to $\mathcal{I}$. Rotation matrices that bring $\mathcal{I}$ onto $\mathcal{F}$ and $\mathcal{I}$ onto $\mathcal{F}_d$, denoted by $R(q_v, q_0) \in SO(3)$ and $R_d(q_{vd}, q_{0d}) \in SO(3)$, respectively, can be defined as

$$R \triangleq \left(q_0^2 - q_v^T q_v\right) I_3 + 2q_v q_v^T - 2q_0 q_v^{\times} \tag{9}$$

$$R_d \triangleq \left(q_{0d}^2 - q_{vd}^T q_{vd}\right) I_3 + 2q_{vd} q_{vd}^T - 2q_{0d} q_{vd}^{\times}, \tag{10}$$

where I_3 denotes the 3×3 identity matrix. Based on (8), $\omega(t)$ can be expressed in terms of the quaternion as

$$\omega = 2\left(q_0\dot{q}_v - q_v\dot{q}_0\right) - 2q_v^{\times}\dot{q}_v. \tag{11}$$

The desired angular velocity body-fixed frame $\mathcal{F}_d$ with respect to $\mathcal{I}$ expressed in $\mathcal{F}_d$ can also be determined as

$$\omega_d = 2\left(q_{0d}\dot{q}_{vd} - q_{vd}\dot{q}_{0d}\right) - 2q_{vd}^{\times}\dot{q}_{vd}. \tag{12}$$

The subsequent analysis is based on the assumption that $q_{0d}(t)$, $q_{vd}(t)$, and their first three time derivatives are bounded for all time. This assumption ensures that $\omega_d(t)$ of (12) and its first two time derivatives are bounded for all time.

IV. CONTROL OBJECTIVE

The objective in this paper is to develop a gimbal velocity controller to enable the attitude of $\mathcal{F}$ to track the attitude of $\mathcal{F}_d$. To quantify the objective, an attitude tracking error denoted by $\tilde{R}(e_v, e_0) \in \mathbb{R}^{3\times 3}$ is defined that brings $\mathcal{F}_d$ onto $\mathcal{F}$ as

$$\tilde{R} \triangleq RR_d^T = \left(e_0^2 - e_v^T e_v\right) I_3 + 2e_v e_v^T - 2e_0 e_v^\times, \tag{13}$$

where $R(q_v, q_0)$ and $R_d(q_{vd}, q_{0d})$ were defined in (9) and (10), respectively, and the quaternion tracking error $e(t) \triangleq \{e_0(t), e_v(t)\} \in \mathbb{R} \times \mathbb{R}^3$ is defined as

$$e_0 \triangleq q_0 q_{0d} + q_v^T q_{vd} \qquad e_v \triangleq q_{0d} q_v - q_0 q_{vd} + q_v^\times q_{vd}. \tag{14}$$

Based on (13), the attitude control objective can be stated as

$$\tilde{R}\left(e_v(t), e_0(t)\right) \to I_3 \qquad \text{as} \qquad t \to \infty. \tag{15}$$

Based on the tracking error formulation, the angular velocity of $\mathcal{F}$ with respect to $\mathcal{F}_d$ expressed in $\mathcal{F}$, denoted by $\tilde{\omega}(t) \in \mathbb{R}^3$, is defined as

$$\tilde{\omega} \triangleq \omega - \tilde{R}\omega_d. \tag{16}$$

To facilitate the subsequent controller design, an auxiliary control signal, denoted by $r(t) \in \mathbb{R}^3$, is defined as [12]

$$r \triangleq \omega - \tilde{R}\omega_d + \alpha e_v, \tag{17}$$

where $\alpha \in \mathbb{R}^{3\times 3}$ is a constant, positive definite, diagonal control gain matrix. After substituting (17) into (16), the angular velocity tracking error can be expressed as

$$\tilde{\omega} = r - \alpha e_v. \tag{18}$$

Motivation for the design of $r(t)$ is obtained from the subsequent Lyapunov-based stability analysis and that fact that (11)-(14) can be used to determine the open-loop quaternion tracking error as

$$\dot{e}_v = \frac{1}{2}\left(e_v^\times + e_0 I\right)\tilde{\omega} \qquad \dot{e}_0 = -\frac{1}{2}e_v^T\tilde{\omega}. \tag{19}$$

From the definitions of the quaternion tracking error variables, the following constraint can be developed [11]:

$$e_v^T e_v + e_0^2 = 1, \tag{20}$$

where

$$0 \le \|e_v(t)\| \le 1 \qquad 0 \le |e_0(t)| \le 1, \tag{21}$$

where $\|\cdot\|$ represents the standard Euclidean norm. From (20),

$$\|e_v(t)\| \to 0 \Rightarrow |e_0(t)| \to 1, \tag{22}$$

and hence, (13) can be used to conclude that if (22) is satisfied, then the control objective in (15) will be achieved.

V. FEEDFORWARD NN ESTIMATION

NN-based estimation methods are well suited for dynamic models containing unstructured uncertainties and disturbances as in (1). The main feature that empowers NN-based controllers is the universal approximation property. Let $\mathbb{S}$ be a compact simply connected set of $\mathbb{R}^{N_1+1}$. Let $\mathbb{C}^n(\mathbb{S})$ be defined as the space where $f : \mathbb{S} \to \mathbb{R}^n$ is continuous. The universal approximation property states that there exist weights and thresholds such that some function $f(x) \in \mathbb{C}^n(\mathbb{S})$ can be represented by a three-layer NN as [13], [14]

$$f(x) = W^T\sigma\left(V^T x\right) + \varepsilon(x) \tag{23}$$

for some given input $x(t) \in \mathbb{R}^{N_1+1}$. In (23), $V \in \mathbb{R}^{(N_1+1)\times N_2}$ and $W \in \mathbb{R}^{(N_2+1)\times n}$ are bounded constant ideal weight matrices for the first-to-second and second-to-third layers, respectively, where N_1 is the number of neurons in the input layer, N_2 is the number of neurons in the hidden layer, and n is the number of neurons in the third layer. The activation function in (23) is denoted by $\sigma(\cdot) : \mathbb{R}^{N_1+1} \to \mathbb{R}^{N_2+1}$, and $\varepsilon(x) : \mathbb{R}^{N_1+1} \to \mathbb{R}^n$ is the functional reconstruction error. Based on (23), the typical three-layer NN approximation for $f(x)$ is given as [13], [14]

$$\hat{f}(x) = \hat{W}^T\sigma\left(\hat{V}^T x\right), \tag{24}$$

where $\hat{V}(t) \in \mathbb{R}^{(N_1+1)\times N_2}$ and $\hat{W}(t) \in \mathbb{R}^{(N_2+1)\times n}$ are subsequently designed estimates of the ideal weight matrices. The estimate mismatch for the ideal weight matrices, denoted by $\tilde{V}(t) \in \mathbb{R}^{(N_1+1)\times N_2}$ and $\tilde{W}(t) \in \mathbb{R}^{(N_2+1)\times n}$, are defined as

$$\tilde{V} \triangleq V - \hat{V} \qquad \tilde{W} \triangleq W - \hat{W}, \tag{25}$$

and the mismatch for the hidden layer output error for a given $x(t)$, denoted by $\tilde{\sigma}(x) \in \mathbb{R}^{N_2+1}$, is defined as

$$\tilde{\sigma} \triangleq \sigma - \hat{\sigma} = \sigma\left(V^T x\right) - \sigma\left(\hat{V}^T x\right). \tag{26}$$

The neural network estimate has several properties that facilitate the subsequent development. These properties are described as follows.

Property 5: (*Taylor Series Approximation*) The Taylor series expansion for $\sigma\left(V^T x\right)$ for a given x may be written as [13], [14]

$$\sigma\left(V^T x\right) = \sigma\left(\hat{V}^T x\right) + \sigma'\left(\hat{V}^T x\right)\tilde{V}^T x + O\left(\tilde{V}^T x\right)^2, \tag{27}$$

where $\sigma'\left(\hat{V}^T x\right) \equiv d\sigma\left(V^T x\right)/d\left(V^T x\right)|_{V^T x = \hat{V}^T x}$, and $O\left(\tilde{V}^T x\right)^2$ denotes the higher order terms. After substituting (27) into (26), the following expression can be obtained:

$$\tilde{\sigma} = \hat{\sigma}'\tilde{V}^T x + O\left(\tilde{V}^T x\right)^2, \tag{28}$$

where $\hat{\sigma}' \triangleq \sigma'\left(\hat{V}^T x\right)$.

Property 6: (*Boundedness of the Ideal Weights*) The ideal weights are assumed to exist and be bounded by known positive values so that

$$\|V\|_F^2 = tr\left(V^T V\right) \le \bar{V}_B \tag{29}$$

$$\|W\|_F^2 = tr\left(W^T W\right) \leq \bar{W}_B, \tag{30}$$

where $\|\cdot\|_F$ is the Frobenius norm of a matrix, and $tr(\cdot)$ is the trace of a matrix.

For notational convenience, let the matrix containing all NN weights be defined as follows:

$$Z \triangleq \begin{bmatrix} W & 0 \\ 0 & V \end{bmatrix}. \tag{31}$$

VI. Control Development

The contribution of this paper is control development that shows how the aforementioned standard NN feedforward estimation strategy can be combined with robust control methods as a means to achieve tracking control for satellite systems described by (1) and (2), which contain nonlinear disturbances and parametric uncertainty in addition to uncertainty caused by actuator dynamics.

A. Open-Loop Error System

The open-loop dynamics for $r(t)$ can be determined by taking the time derivative of (17) and premultiplying the resulting expression by $J(\delta)$ as

$$J\dot{r} = J\dot{\omega} + J\omega^{\times}\tilde{R}\omega_d - J\tilde{R}\dot{\omega}_d + J\alpha\dot{e}_v, \tag{32}$$

where the fact that

$$\dot{\tilde{R}} = -\omega^{\times}\tilde{R}$$

was utilized. After using (1), (2), (3), (5), (17), and (19), the expression in (32) can be expressed as

$$J\dot{r} = f - \Omega_1\dot{\delta} - hA\dot{\delta} - \omega^{\times}h_{cmg} - AF_s sgn\dot{\delta} - \frac{1}{2}\dot{J}r. \tag{33}$$

In (33), the uncertain function $f(r, q_v, q_0, e_v, e_0, \omega, \omega_d, \dot{\omega}_d, \delta, t) \in \mathbb{R}^3$ is defined as

$$f \triangleq -\omega^{\times}J\omega + J\omega^{\times}\tilde{R}\omega_d - J\tilde{R}\dot{\omega}_d + \frac{1}{2}J\alpha\left(e_v^{\times} + e_0 I\right)\tilde{\omega} + \tau_d, \tag{34}$$

and $\Omega_1(r, q_v, q_0, e_v, e_0, \omega_d, \dot{\omega}_d, \delta, t) \in \mathbb{R}^{3\times 4}$ denotes an uncertain auxiliary matrix, which is defined via the parameterization

$$\Omega_1\dot{\delta} = \left[\frac{\partial J}{\partial \delta}\dot{\delta}\right]\left(\frac{1}{2}r + \tilde{R}\omega_d - \alpha e_v\right) + AF_d\dot{\delta} + AK_G E_d\dot{\delta}. \tag{35}$$

The expression in (35) can be linearly parameterized in terms of a known regression matrix $Y_1(r, q_v, q_0, e_v, e_0, \omega, \omega_d, \dot{\omega}_d, \delta, \dot{\delta}, t) \in \mathbb{R}^{3\times p_1}$ and a vector of p_1 unknown constants $\theta_1 \in \mathbb{R}^{p_1}$ as

$$\Omega_1\dot{\delta} \triangleq Y_1\theta_1. \tag{36}$$

Some of the control design challenges for the open-loop system in (33) are that the control input $\dot{\delta}(t)$ is premultiplied by a nonsquare, uncertain time-varying matrix, and is embedded inside of a discontinuous nonlinearity (i.e., the $signum$ function). To address the fact that $\dot{\delta}(t)$ is premultiplied by a nonsquare unknown time-varying matrix, an estimate of the uncertainty in (36), denoted by $\hat{\Omega}_1(r, q_v, q_0, e_v, e_0, \omega_d, \dot{\omega}_d, \delta, t) \in \mathbb{R}^{3\times 4}$, is defined as

$$\hat{\Omega}_1\dot{\delta} \triangleq Y_1\hat{\theta}_1, \tag{37}$$

where $\hat{\theta}_1(t) \in \mathbb{R}^{p_1}$ is a subsequently designed estimate for the parametric uncertainty in $\Omega_1(r, q_v, q_0, e_v, e_0, \omega_d, \dot{\omega}_d, \delta, t)$. Based on (36) and(37), (33) can be rewritten as

$$J\dot{r} = f - B\dot{\delta} - \omega^{\times}h_{cmg} - \frac{1}{2}\dot{J}r - Y_1\tilde{\theta}_1 - AF_s sgn\dot{\delta}, \tag{38}$$

where $B(r, q_v, q_0, e_v, e_0, \omega_d, \dot{\omega}_d, \delta, t) \in \mathbb{R}^{3\times 4}$ is defined as

$$B = hA + \hat{\Omega}_1, \tag{39}$$

and the parameter estimate mismatch $\tilde{\theta}_1(t) \in \mathbb{R}^{p_1}$ is defined as

$$\tilde{\theta}_1 \triangleq \theta_1 - \hat{\theta}_1. \tag{40}$$

The auxiliary function in (34) can be represented by a three-layer NN as

$$f = W^T\sigma\left(V^T x\right) + \varepsilon(x). \tag{41}$$

In (41), the input $x(t) \in \mathbb{R}^{25}$ is defined as $x(t) \triangleq \begin{bmatrix} 1 & r(t) & q_v(t) & q_0(t) & e_v(t) & e_0(t) & \omega(t) & \omega_d(t) & \dot{\omega}_d(t) & \delta(t) \end{bmatrix}^T$ so that $N_1 = 24$, where N_1 was introduced in (23). Based on the assumption that the actual and desired trajectories are bounded, the following inequality holds:

$$\|\varepsilon(x)\| \leq \varepsilon_{b1}, \tag{42}$$

where $\varepsilon_{b1} \in \mathbb{R}$ is a known positive constant.

B. Closed-Loop Error System

Based on the open-loop dynamics in (38) and the subsequent stability analysis, the control input is designed as

$$\dot{\delta} = B^{+}\left[\hat{f} - \omega^{\times}h_{cmg} + K_v r - v + e_v\right], \tag{43}$$

where $K_v \in \mathbb{R}$ denotes a positive control gain, and $v(t) \in \mathbb{R}^3$ denotes a robustifying term, defined as [15]

$$v \triangleq -K_Z\left(\left\|\hat{Z}\right\|_F + Z_M\right)r - k_n r, \tag{44}$$

where $k_n \in \mathbb{R}$ denotes a positive control gain (i.e., nonlinear damping term), $\hat{Z} \in \mathbb{R}^{(N_1+N_2+2)\times(N_2+n)}$ is a subsequently designed estimate of Z, $Z_M \in \mathbb{R}$ satisfies the inequality

$$\|Z\|_F \leq Z_M, \tag{45}$$

and $K_Z \in \mathbb{R}$ is a control gain designed to satisfy the inequality

$$K_Z > c_2, \tag{46}$$

where c_2 is defined in (60). Also in (43), $B^{+}(r, q_v, q_0, e_v, e_0, \omega_d, \dot{\omega}_d, \delta, t) \in \mathbb{R}^{4\times 3}$ denotes the generalized inverse of $B(r, q_v, q_0, e_v, e_0, \omega_d, \dot{\omega}_d, \delta, t)$, which could be defined using the Moore-Penrose definition or the singularity robust pseudoinverse definition coined by Nakamura et al. as (e.g., see [16]–[18])

$$B^{+} = B^T\left(BB^T + \epsilon I_{3\times 3}\right)^{-1}. \tag{47}$$

In (47), $\epsilon(t) \in \mathbb{R}$ denotes a singularity avoidance parameter. For example, in [18] Nakamura et al. designed $\epsilon(t)$ as

$$\epsilon \triangleq \epsilon_0 \exp\left\{-\det\left(BB^T\right)\right\}, \tag{48}$$

so that $\epsilon(t)$ is negligible when $B(r, q_v, q_0, e_v, e_0, \omega_d, \dot{\omega}_d, \delta, t) \cdot B^T(r, q_v, q_0, e_v, e_0, \omega_d, \dot{\omega}_d, \delta, t)$ is nonsingular but increases to the constant parameter $\epsilon_0 \in \mathbb{R}$ as the singularity is approached. Also in (43), the feedforward NN component, denoted as $\hat{f}(t) \in \mathbb{R}^3$, is given by

$$\hat{f} \triangleq \hat{W}^T \sigma\left(\hat{V}^T x\right), \tag{49}$$

where the state vector $x(t) \in \mathbb{R}^{25}$ was defined in (41). The estimates of the NN weights in (49) are generated on-line (there is no off-line learning phase) as [15]

$$\dot{\hat{W}} \triangleq \Gamma_1(\hat{\sigma} r^T - \hat{\sigma}' \hat{V}^T x r^T - \kappa \|r\| \hat{W}) \tag{50}$$

$$\dot{\hat{V}} \triangleq \Gamma_2 x r^T \left(\hat{\sigma}'^T \hat{W}\right)^T - \kappa \Gamma_2 \|r\| \hat{V}, \tag{51}$$

where $\Gamma_1 \in \mathbb{R}^{(N_2+1)\times(N_2+1)}$, $\Gamma_2 \in \mathbb{R}^{(N_1+1)\times(N_1+1)}$ are constant, positive definite, symmetric control gain matrices, and $\kappa \in \mathbb{R}^+$ is a constant control gain.

Remark 1: The adaptive update laws given in (50) and (51) ensure that $\hat{W}(t)$ and $\hat{V}(t)$ remain bounded provided $x(t)$ remains bounded. This fact will be exploited in the subsequent stability analysis.

The closed-loop tracking error system can be developed by substituting (43) into (38) as

$$J\dot{r} = -\frac{1}{2}\dot{J}r + \tilde{f} - Y_1\tilde{\theta}_1 - K_v r + v - e_v - AF_s sgn\dot{\delta}, \tag{52}$$

where $\tilde{f}(x) \in R^3$ represents a function estimation error vector defined as

$$\tilde{f} \triangleq f - \hat{f}. \tag{53}$$

Based on (52) and the subsequent stability analysis, the parameter estimate $\hat{\theta}_1(t)$ is designed as

$$\dot{\hat{\theta}}_1 = proj(-\Gamma_3 Y_1^T r), \tag{54}$$

where $\Gamma_3 \in \mathbb{R}^{p_1 \times p_1}$ denotes a constant, positive-definite, diagonal adaptation gain matrix, and $proj(\cdot)$ denotes a projection algorithm utilized to guarantee that the i^{th} element of $\hat{\theta}_1(t)$ can be bounded as

$$\underline{\theta}_{1i} \leq \hat{\theta}_{1i} \leq \bar{\theta}_{1i}, \tag{55}$$

where $\underline{\theta}_{1i}, \bar{\theta}_{1i} \in \mathbb{R}$ denote known, constant lower and upper bounds for each element of $\hat{\theta}_1(t)$.

Remark 2: To determine $\hat{\theta}_1$, the adaptation law in (54) assumes the availablility of angular position and velocity measurements only.

Using (41), (49) and (53), the closed-loop error system in (52) can be expressed as

$$\begin{aligned} J\dot{r} &= -\frac{1}{2}\dot{J}r + W^T\sigma\left(V^T x\right) - \hat{W}^T\sigma\left(\hat{V}^T x\right) + v \\ &\quad -Y_1\tilde{\theta}_1 - K_v r + \varepsilon(x) - e_v - AF_s sgn\dot{\delta}. \end{aligned} \tag{56}$$

After adding and subtracting the terms $W^T\hat{\sigma}$ and $\hat{W}^T\tilde{\sigma}$ to (56), the following expression is obtained:

$$\begin{aligned} J\dot{r} &= -\frac{1}{2}\dot{J}r + \tilde{W}^T\hat{\sigma} + \hat{W}^T\tilde{\sigma} + \tilde{W}^T\tilde{\sigma} + \varepsilon(x) \\ &\quad -Y_1\tilde{\theta}_1 - K_v r + v - e_v - AF_s sgn\dot{\delta} \end{aligned} \tag{57}$$

where the notations $\hat{\sigma}$ and $\tilde{\sigma}$ were introduced in (26). The Taylor series approximation described in (27) and (28) can now be used to rewrite (57) as

$$\begin{aligned} J\dot{r} &= -\frac{1}{2}\dot{J}r + w - K_v r + v - e_v - Y_1\tilde{\theta}_1 \\ &\quad +\tilde{W}^T\left(\hat{\sigma} - \hat{\sigma}'\hat{V}^T x\right) + \hat{W}^T\hat{\sigma}'\tilde{V}^T x, \end{aligned} \tag{58}$$

where $w(t) \in R^3$ is defined as

$$\begin{aligned} w &= \tilde{W}^T\hat{\sigma}'V^T x + W^T O\left(\tilde{V}^T x\right)^2 \\ &\quad -AF_s sgn\dot{\delta} + \varepsilon(x). \end{aligned} \tag{59}$$

The NN reconstruction error $\varepsilon(x)$, the higher order terms in the Taylor series expansion of $f(x)$, and the static friction term $AF_s sgn\dot{\delta}(t)$ can be treated as disturbances in the error system. Moreover, these disturbances can be upper bounded as [15]

$$\|w(t)\| \leq c_0 + c_1\left\|\tilde{Z}\right\|_F + c_2\left\|\tilde{Z}\right\|_F\|r\|, \tag{60}$$

where $c_i \in R \; \forall i = 0, 1, 2$ are known positive constants, and c_0 is explicitly defined as

$$c_0 \triangleq \|A\|_{i\infty} F_M + \varepsilon_{b1} + c_3 Z_M, \tag{61}$$

and $c_3 \in R$ is a known positive constant.

C. Stability Analysis

Theorem 1: Given the closed-loop dynamics in (58), the adaptive controller of (43), (50), (51), and (54) ensures global uniformly ultimately bounded (GUUB) attitude tracking in the sense that

$$\|e_v(t)\| \to \varepsilon_0 \exp\{-\varepsilon_1 t\} + \varepsilon_2, \tag{62}$$

where $\varepsilon_0, \varepsilon_1, \varepsilon_2 \in \mathbb{R}$ denote positive bounding constants.

Proof: Let $V(e_0, e_v, r, t) \in \mathbb{R}$ be defined as the nonnegative function

$$\begin{aligned} V(t) &\triangleq e_v^T e_v + (1 - e_0)^2 + \frac{1}{2}r^T J r + \frac{1}{2}tr\left(\tilde{W}^T\Gamma_1^{-1}\tilde{W}\right) \\ &\quad +\frac{1}{2}tr\left(\tilde{V}^T\Gamma_2^{-1}\tilde{V}\right) + \frac{1}{2}\tilde{\theta}_1^T\Gamma_3^{-1}\tilde{\theta}_1. \end{aligned} \tag{63}$$

Based on (6), (21), (40), (50), (51), and (55), (63) can be upper and lower bounded as

$$\lambda_1\|y\|^2 + c_4 \leq V(t) \leq \lambda_2\|y\|^2 + c_5, \tag{64}$$

where $\lambda_1, \lambda_2, c_4, c_5 \in \mathbb{R}$ are known positive bounding constants, and $y(t) \in \mathbb{R}^6$ is defined as

$$y \triangleq \left[\begin{array}{cc} e_v^T & r^T \end{array}\right]^T. \tag{65}$$

After using (18), (19), (58), and exploiting the fact that

$$e_v^T e_v^\times \tilde{\omega} = 0,$$

the time derivative of $V(t)$ can be expressed as

$$\begin{aligned} \dot{V}(t) &= -\alpha e_v^T e_v + r^T\left(w - K_v r + v - Y_1\tilde{\theta}_1\right) \\ &\quad -tr\tilde{W}^T\left(\Gamma_1^{-1}\dot{\hat{W}}_1 - \hat{\sigma}r^T + \hat{\sigma}'\hat{V}^T x r^T\right) \\ &\quad -tr\tilde{V}^T\left(\Gamma_2^{-1}\dot{\hat{V}}_1 - x r^T\hat{W}^T\hat{\sigma}'\right) - \tilde{\theta}_1^T\Gamma_3^{-1}\dot{\hat{\theta}}_1. \end{aligned} \tag{66}$$

After substituting for the tuning rules given in (50), (51), and (54), (66) can be expressed as

$$\begin{aligned}\dot{V} &= -\alpha e_v^T e_v + r^T \left(w - K_v r + v\right) \\ &\quad + \kappa \left\|r\right\| tr \tilde{Z}^T \left(Z - \tilde{Z}\right). \end{aligned} \tag{67}$$

After substituting (44) and using the fact that $tr\tilde{Z}^T\left(Z-\tilde{Z}\right) = \langle \tilde{Z}, Z\rangle_F - \left\|\tilde{Z}\right\|_F^2 \leq \left\|\tilde{Z}\right\|_F \left\|Z\right\|_F - \left\|\tilde{Z}\right\|_F^2$, (67) can be upper bounded as follows [15]:

$$\begin{aligned}\dot{V}(t) &\leq -\alpha \left\|e_v\right\|^2 - K_{v\min}\left\|r\right\|^2 - k_n \left\|r\right\|^2 \\ &\quad + \left\|r\right\|\left\|w\right\| - K_Z\left(\left\|\hat{Z}\right\|_F + Z_M\right)\left\|r\right\|^2 \\ &\quad + \kappa\left\|r\right\|\left\|\tilde{Z}\right\|_F\left(Z_M - \left\|\tilde{Z}\right\|_F\right). \end{aligned} \tag{68}$$

After substituting the upper bound for $\|w\|$ given in (60) and utilizing inequality (46), $\dot{V}(t)$ can be bounded as

$$\dot{V}(t) \leq -\lambda_3 \left\|y\right\|^2 - k_n \left\|r\right\|^2 + \gamma\left\|r\right\|, \tag{69}$$

where $\lambda_3 \triangleq \min\{\alpha, K_{v\min}\}$, and $\gamma \triangleq c_0 + c_1\left\|\tilde{Z}\right\|_F + \kappa\left\|\tilde{Z}\right\|_F\left(Z_M - \left\|\tilde{Z}\right\|_F\right)$. Completing the squares in (69) yields

$$\dot{V}(t) \leq -\lambda_3\left\|y\right\|^2 + \frac{\gamma^2}{4k_n}. \tag{70}$$

Based on (64), (70) can be expressed as

$$\dot{V}(t) \leq -\frac{\lambda_3}{\lambda_2} V\left(t\right) + \varepsilon, \tag{71}$$

where $\varepsilon \in \mathbb{R}$ is a positive constant that is defined as

$$\varepsilon = \frac{\gamma^2}{4k_n} + \frac{\lambda_3 c_5}{\lambda_2}. \tag{72}$$

The linear differential inequality in (71) can be solved as

$$V(t) \leq \exp\left\{-\frac{\lambda_3}{\lambda_2}t\right\} V(0) + \varepsilon\frac{\lambda_2}{\lambda_3}\left(1 - \exp\left\{-\frac{\lambda_3}{\lambda_2}t\right\}\right). \tag{73}$$

The expressions in (63), (64), and (73) can be used to conclude that $r(t) \in \mathcal{L}_\infty$. Thus, from (18), (21), and (65), $\tilde{\omega}(t)$, $y(t) \in \mathcal{L}_\infty$, and (17) can be used to conclude that $\omega(t) \in \mathcal{L}_\infty$. Equation (19) then shows that $\dot{e}_v(t)$, $\dot{e}_0(t) \in \mathcal{L}_\infty$. Hence, (39), (43), (44), and (49)-(51) can be used to prove that the control input $\dot{\delta}(t) \in \mathcal{L}_\infty$. Standard signal chasing arguments can then be utilized to prove that all remaining signals remain bounded during closed-loop operation. The inequalities in (64) can now be used along with (72) and (73) to conclude that

$$\begin{aligned}\left\|y\right\|^2 &\leq \left(\frac{\lambda_2\left\|y(0)\right\|^2 + c_5}{\lambda_1}\right)\exp\left\{-\frac{\lambda_3}{\lambda_2}t\right\} \\ &\quad + \left(\frac{\lambda_2\gamma^2}{4k_n\lambda_3\lambda_1} + \frac{c_5 - c_4}{\lambda_1}\right). \end{aligned} \tag{74}$$

The result in (62) can now be directly obtained from (74). ■

VII. Conclusion

In this paper, a uniformly ultimately bounded NN attitude tracking controller for a rigid body satellite is presented. The controller adapts for time varying satellite inertia properties, parametric uncertainty in.the inertia matrix, unknown dynamic friction in the CMG gimbals, and input torque variations due to electromechanical disturbances in the gimbal loops. In addition, the NN controller compensates for unmodeled external disturbances and uncertainties in the input torque caused by unknown static CMG gimbal friction.

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