

# Model-Based Predictive Control of a Multi-Evaporator Vapor Compression Cooling Cycle

Matthew S. Elliott and Bryan P. Rasmussen

**Abstract**—This paper presents a decentralized control architecture for multiple evaporator vapor compression systems using model-based predictive control. Vapor compression systems are widely used for heating, air-conditioning and refrigeration, and constitute a major part of total US energy use. Advanced control strategies have the potential to significantly increase energy efficiency, while delivering the necessary amount of cooling capacity. This paper proposes a decentralized control approach based on a study of interacting dynamics, wherein the cooling capacity of each evaporator is controlled by a multi-input, multi-output MPC controller and standard PI controllers are used to regulate system pressures by modulating compressor speed and discharge valve opening. This is in contrast with traditional single-input, single-output control approaches, which can result in undesired dynamic behavior. The efficacy of the proposed control architecture is demonstrated on an experimental system.

## I. INTRODUCTION

Vapor compression cooling (VCC) cycles are the primary means of mechanical cooling today; they can be found in settings ranging from household refrigerators to office buildings. Control techniques for these cycles have traditionally consisted of simple electromechanical devices and on/off control strategies. Technological advances such as variable speed compressors, electronically controlled expansion valves, and improved computing speed now allow more precise and efficient control of these cycles. Since world energy demand continues to increase, and air conditioning is a major component of that demand, the adaptation of advanced control strategies to VCC cycles has the potential to make a serious impact on energy consumption. This paper presents a method of controlling a multiple evaporator system with a novel control architecture that uses a decentralized approach. The cooling capacity of each evaporator is regulated by its own multiple-input, multiple output (MIMO) controller, while the compressor and discharge valves use single-input, single-output (SISO) PI controllers to regulate system pressures. The MIMO loops use a model-based predictive controller that takes desired cooling capacity and evaporator superheat as the regulated outputs and the expansion valve opening and water flow valve opening as controllable inputs. This decentralized architecture reflects the spatially distributed

nature of the physical system, while ensuring efficiency and performance demands are met.

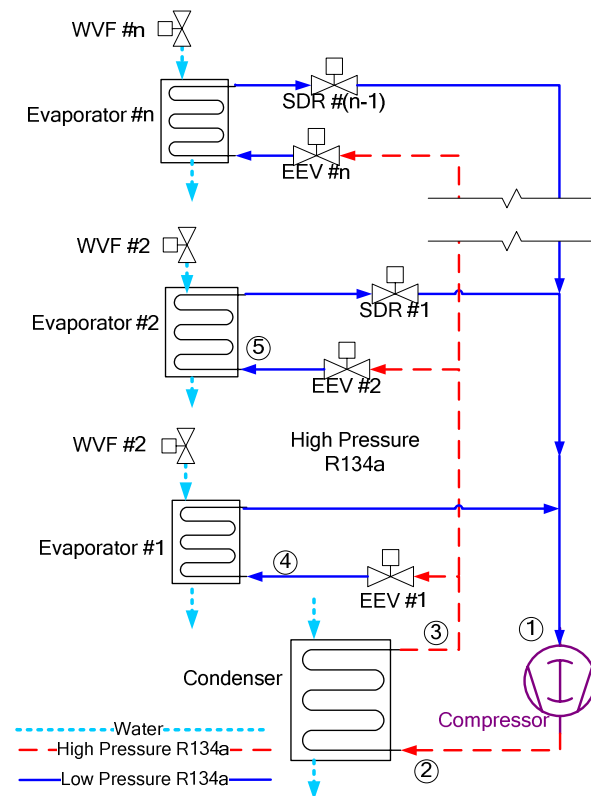


Fig. 1: General Multi-Evaporator System with designated states.

## II. BACKGROUND ON VAPOR COMPRESSION CYCLES

### A. General Multiple Evaporator System

In this paper we consider the general multi-evaporator vapor compression cycle shown in Figure 1 and adapted from the single evaporator system detailed in [4]. The first stage of the thermodynamic cycle is at the inlet of the compressor, where refrigerant is in a low pressure, gaseous state (denoted as state 1 in Figure 1). The compressor adds energy to the fluid by compressing it to a high pressure, high temperature gas (state 2). This gas passes into the condenser, where heat energy is rejected from the refrigerant to the secondary fluid (water or air). This causes the refrigerant to condense to a high pressure liquid. A receiver at the end of the condenser ensures that the refrigerant becomes a saturated liquid (state 3). This saturated liquid is fed into a set of expansion valves, which meter the

Matthew S. Elliott is a graduate researcher with the Texas A&M University Department of Mechanical Engineering, College Station, TX.

Bryan P. Rasmussen is an Assistant Professor of Mechanical Engineering at Texas A&M University, College Station, TX. (phone: 979-862-2776, fax: 979-845-3081, email: brasmussen@tamu.edu)

refrigerant flowing into the evaporators. The refrigerant is now a two-phase fluid (states 4 and 5). This two phase fluid absorbs heat from the water entering the evaporators, chilling the water and causing the refrigerant to evaporate. This low pressure gas exits the evaporators and returns to the compressor. The discharge valve (SDR) on the secondary evaporators creates a pressure differential between evaporators, thus allowing them to provide cooling at different saturation temperatures.

Multiple evaporator systems allow different amounts of cooling at different temperatures to be delivered to different regions in the same overall system, such as apartment units or large office buildings. These systems can also allow for storing perishables requiring different storage temperatures, such as in a supermarket case or in a refrigerated trailer truck. Since the evaporator temperature must be lower than the temperature of the fluid being cooled, pressure is an important operating condition of the system and must be controlled. However, from the standpoint of overall system performance, evaporator cooling is paramount. If the evaporators are not removing sufficient heat from the secondary fluid, the system is not meeting its performance requirements, regardless of the evaporator temperature. The final important element for control of the system is superheat, which occurs when the temperature of the refrigerant at the exit of the evaporator is higher than the saturation temperature at evaporator pressure. The existence of superheat is a guarantee that the evaporator is experiencing complete evaporation of the refrigerant; if superheat is lost, liquid refrigerant can pass into the compressor, which can damage it. However, excessive superheat means that the evaporator is operating inefficiently, since the amount of cooling occurring drops significantly once the fluid is completely evaporated. Superheat can be increased by increasing compressor speed, closing the expansion valve, or opening the discharge valve, since all of these actions lower the pressure, and therefore the saturation temperature, of the evaporator. Increasing the water flow across the valve also increases superheat, since it increases heat transfer from the refrigerant. Controlling these three conditions—cooling, pressure, and superheat—in each evaporator controls the operation of the system and therefore provides a means to deliver the required cooling in the most efficient way possible.

In traditional single evaporator systems, a mechanical expansion valve is used to regulate superheat. If the compressor is variable speed or two-stage, its speed is increased to meet increased cooling demand; since this increases superheat, the valve opens to reduce superheat to the desired level. In a multi-evaporator system, the dynamics of the two evaporators are very tightly coupled; changes in one expansion valve have a strong effect on the superheat and pressure of the other evaporators. Therefore, a completely SISO control approach will lead to oscillatory or dangerous limit cycle behavior. While this could potentially

be addressed with a large, centralized MIMO controller that runs the entire system, this sort of control is difficult to implement, given the spatially distributed nature of industrial systems and the complexity of the dynamics involved. Therefore, a more decentralized approach is preferred; the desired arrangement will give each evaporator its own controller that operates independently of the rest of the system but does not conflict with the objectives of the neighboring components.

### B. Dual Evaporator Experimental System

For the research detailed in this paper, a two-evaporator water chiller test apparatus was used. This test apparatus has variable control of the electronic expansion valves (EEVs), compressor, and water flow valves (WVVs). Furthermore, the second evaporator has a discharge valve (SDR) that allows the two evaporators to function at different pressures. Using multiple evaporators with EEVs allows different amounts of cooling to be delivered to different regions at different temperatures, such as different units in an apartment building. The refrigerant used is R134a, which is an HCFC widely used in automotive and industrial systems.

In order to measure system properties, including regulated variables, transducers are placed at salient points of the thermodynamic cycle. Pressure transducers are placed at the outlet of each evaporator and the condenser to measure the saturation pressures of the refrigerant. Thermocouples are immersed in the refrigerant flow at the inlet and outlet of each evaporator and the condenser. These temperature and pressure measurements allow computation of fluid properties such as enthalpy and density at the relevant points of the cycle. In order to measure mass flow of refrigerant through each evaporator, the calculated fluid densities are used in conjunction with turbine-type volumetric flow meters placed at the inlet of each EEV. These measurements allow on-line computation of cooling and superheat of each evaporator. Cooling is measured as:

$$\dot{Q}_n = \dot{m}_{ref,n} (h_{en,o} - h_{en,i}) \quad (1)$$

$\dot{Q}_n$   $\equiv$  Cooling at the  $n^{\text{th}}$  evaporator, kW

$\dot{m}_{ref,n}$   $\equiv$  Mass flow of R134a for  $n^{\text{th}}$  evaporator, kg/sec

$h_{en,o}$   $\equiv$  Enthalpy of R134a at  $n^{\text{th}}$  evaporator exit, kJ/kg

$h_{en,i}$   $\equiv$  Enthalpy of R134a at  $n^{\text{th}}$  evaporator inlet, kJ/kg

For this controller, superheat is calculated as:

$$SH_n = T_{en,ro} - T_{sat,en} \quad (2)$$

$SH_n$   $\equiv$  Superheat of the  $n^{\text{th}}$  evaporator,  $^{\circ}\text{C}$

$T_{en,ro}$   $\equiv$  Exit R134a temperature of  $n^{\text{th}}$  evaporator,  $^{\circ}\text{C}$

$T_{sat,en}$   $\equiv$  R134a saturation temperature at  $n^{\text{th}}$  evaporator pressure,  $^{\circ}\text{C}$

### III. DYNAMIC ANALYSIS

The first step towards adaptation of advanced control to this VCC cycle is developing an understanding of the relationships between control inputs (e.g., compressor speed, EEV opening) and outputs (e.g., evaporator pressure, cooling), as well as the interdependence between evaporators. In order to develop this understanding, a set of experimentally derived linear models were created at selected operating conditions [4]. The system was allowed to come to steady state operation, and then excited with a pseudo-random binary input. Because of the differing units and disparate scaling, the inputs and outputs were normalized before constructing empirical models, allowing the effects of each input into each output to be compared accurately [3]. The models are 1<sup>st</sup>, 2<sup>nd</sup>, and 3<sup>rd</sup> order prediction-error method (PEM) state-space models, cross-validated to ensure accuracy.

Figure 2 shows the frequency responses of the models derived by stepping the compressor speed. SISO models were developed for RPM to each output; the DC gain and bandwidth are shown. Clearly, compressor speed has a strong effect on evaporator pressures and superheats. This effect is stronger at lower speeds. However, varying the

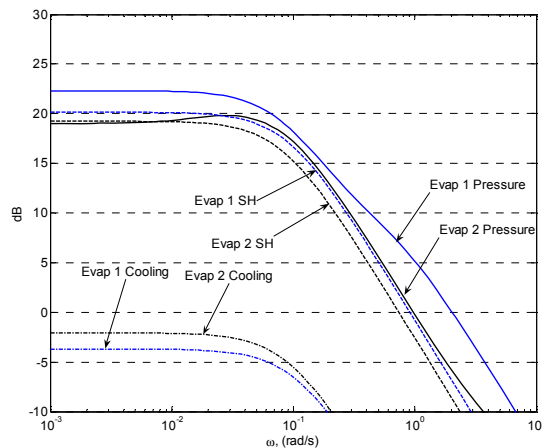


Fig 2: Normalized Frequency responses to step changes in RPM.

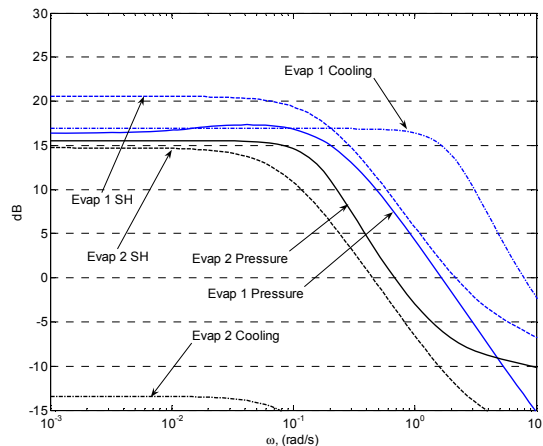


Fig 3: Normalized Frequency responses to step changes in EEV 1.

compressor speed alone has a weak effect on cooling; if the valve does not open to permit additional refrigerant mass flow, increasing compressor speed has the effect of increasing condenser pressure, and the additional energy input to the system by the compressor is rejected into the condenser. This suggests that the compressor can be used to deliver the energy into the system necessary to perform the cooling work without changing the cooling itself.

Figure 3 shows the effects of the first EEV. The EEV has a strong effect on the pressure and superheat of both evaporators at all operating conditions. The effect of the EEV on the first evaporator's cooling is also strong; however, the effect of the first EEV on the second evaporator's cooling is nonexistent.

Figure 4 details the responses to changes in the SDR. It has a strong effect on the pressure of the second evaporator, and therefore its superheat. It does not affect the first evaporator at steady state, which suggests that control of the first evaporator can be separated from the SDR.

Finally, Figure 5 details how the system responds to changes in water flow of the first evaporator via step changes in WFV #1. It has a strong effect on superheat and a smaller effect on cooling, and does not have any effects on the second evaporator.

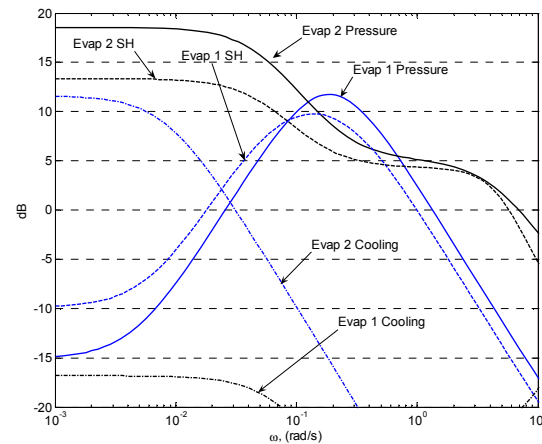


Fig. 4: Normalized Frequency responses to step changes in SDR.

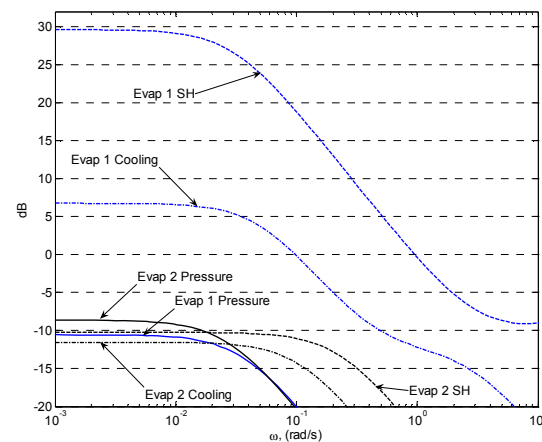


Fig. 5: Normalized Frequency responses to step changes in WFV1.

#### IV. PROPOSED CONTROL ARCHITECTURE

In the proposed control structure, the expansion valves are used primarily to regulate cooling, since they control how much refrigerant enters the evaporator, and cooling is strongly dependent on mass flow. Their actions have a strong input on superheat as well. The water flow valves will be used to help regulate superheat. These two actuators can be coupled to regulate cooling (the primary control objective) and superheat, resulting in a 2-input, 2-output plant for each evaporator. This is different from the industrially standard approach of using EEVs to control evaporator superheat. Figure 6 shows block diagrams of the proposed architecture.

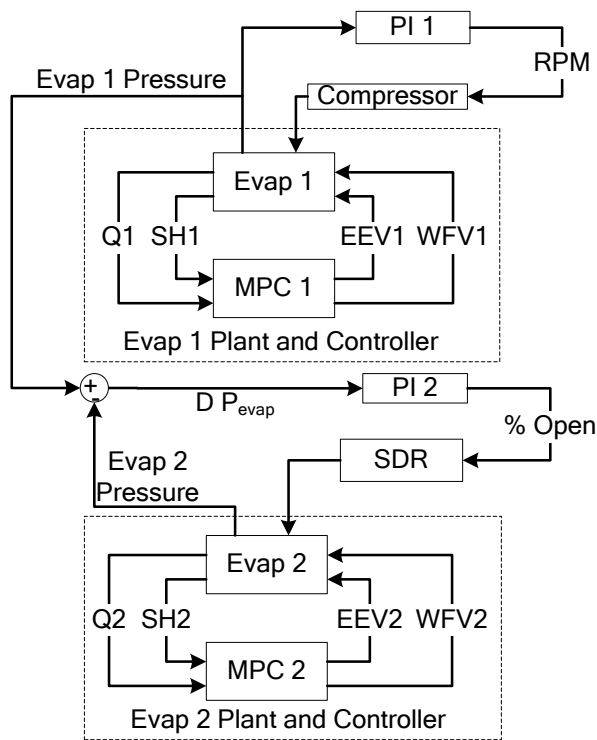


Figure 6: Proposed Control Architecture

When controlling superheat, the critical requirement is that superheat is present. Once this condition is met, the primary advantage of a close regulation of superheat is that as superheat is driven down to a minimum value, the evaporator operates with increasing efficiency. This regulation has traditionally been performed with the EEV. In the proposed architecture, cooling and evaporator pressure regulation can be performed regardless of the amount of superheat. From the perspective of cooling and pressure control, strict regulation of superheat to a setpoint is unnecessary; it is only necessary to ensure that it stays above a minimum value to protect the compressor, but within a reasonable band so that the evaporator does not operate inefficiently. Therefore, a control strategy is required that respects the physical limits of the actuators, will regulate cooling to a specific value, and will keep superheat within a

defined band of operating conditions without exerting unnecessary controller effort. This leads naturally to adopting a model predictive control approach.

The term model predictive control (MPC) refers to a suite of control strategies originally developed in industry during the 1970s. These approaches all use an explicit model of the physical system to derive the set of controller actions that minimize a cost function subject to a set of constraints. At each sampling instant, the controller calculates the cost over the prediction horizon, and selects the control actions over the control action horizon that minimizes the user-defined cost function. It applies the first of these control actions, and then at the next sampling instant repeats the process. In this way the prediction horizon recedes at each sampling instant; hence, MPC is also known as receding horizon control. One of the great advantages of MPC is its inherent ability to account for constraints. These constraints can be inherent to the actuators, e.g., a valve can not open past 100% open or close past 0% open. While classical control techniques like PID loops can be modified with a saturation to ensure that actuator limits are not exceeded, MPC has the advantage of being able to foresee and plan for these limitations, which can improve system performance over the long term [1]. MPC also has the advantage that additional constraints can be defined by the user to keep the system operating in a safe range, e.g., keeping evaporator superheat above a desired minimum. MPC has been adapted to HVAC systems as a system governor ([8], [9]), and used to control cooling of a single evaporator system [7].

Once the evaporator controller is implemented, the evaporators need a way to communicate to the compressor how much energy input is needed in order to achieve the total desired cooling. Since the compressor has a large impact on evaporator pressure, and changes in the expansion valve to meet cooling demand change the evaporator pressure, pressure of the first evaporator is chosen as the signal to communicate this need to the compressor. The first evaporator pressure is chosen because the pressure differential between the two evaporators can be independently regulated by the SDR valve. With this approach, the pressure of the two evaporators can be regulated using two SISO PI loops. For example, as one of the EEVs opens to allow more mass flow and achieve the set cooling capacity, the pressure of the evaporator will rise. The compressor will speed up to drop the pressure. Meanwhile, the pressure differential between the two evaporators is regulated by the SDR in the second evaporator. Therefore, 6 outputs—two evaporator pressures, two amounts of cooling, and two superheats—are regulated using two 2x2 MIMO plants and two SISO PI loops. One of the great advantages of this decentralized approach is that it is expandable to a large number of evaporators networked together over large physical distances, without the need for unreasonable increases in computing power.

## V. IMPLEMENTED CONTROLLER PERFORMANCE

The proposed architecture was implemented on the experimental system, and was successful in regulating cooling and pressures, and keeping superheat above a minimum level. The PI loop controlling the compressor to regulate the pressure of the first evaporator has proportional and integral gains of  $K_p=5.4$  and  $K_i=0.3$ . The PI using the SDR to regulate the pressure difference between evaporators has gains of  $K_p=0.2$  and  $K_i=0.15$ . These gains were developed using a Ziegler-Nichols tuning algorithm as detailed in [5].

The MPC controller was implemented using the MatLab MPC toolbox. The controller minimizes the following cost functions as detailed in [6]:

$$S_y(k) = \sum_{i=1}^P \sum_{j=1}^{n_y} \{w_j^y [r_j(k+i) - y_j(k+i)]\}^2 \quad (3)$$

$$S_{\Delta u}(k) = \sum_{i=1}^M \sum_{j=1}^{n_{mv}} \{w_j^{\Delta u} \Delta u_j(k+i-1)\}^2 \quad (4)$$

Equation (3) computes the weighted sum of squared deviations. This portion of the cost calculation penalizes deviation of the outputs from the setpoints.

- $k$   $\equiv$  Current sampling interval
- $k+i$   $\equiv$  Future sampling interval
- $P$   $\equiv$  Prediction horizon
- $n_y$   $\equiv$  Number of plant outputs
- $w_j^y$   $\equiv$  Weight of output  $j$
- $r_j(k+i) - y_j(k+i)$   $\equiv$  predicted deviation at instant  $k+i$

Equation (4) computes the weighted sum of controller adjustments. This equation penalizes large changes in the actuators, and makes the controller more robust.

- $M$   $\equiv$  Control horizon
- $k+i$   $\equiv$  Future sampling interval
- $P$   $\equiv$  Prediction horizon
- $n_{mv}$   $\equiv$  Number of manipulated variables (inputs)
- $w_j^{\Delta u}$   $\equiv$  Weight of change in input  $j$
- $\Delta u_j(k+i-1)$   $\equiv$  predicted adjustment of input  $u_j$  at future instant  $k+i-1$

In the MPC controllers, weights of 1000 and 0 (in equation (3)) were placed on the cooling and superheat, and rate weights of 1.0 and 0.1 (equation (4)) were placed on the EEV and WFV, respectively. A control interval of 2 seconds was used, with a control horizon of 3 intervals (6 seconds) and a prediction horizon of 35 intervals (70 seconds). The EEVs were constrained between 7% and 45%; the WFVs were constrained between 20% and 60%. An output constraint was placed on superheat bounding it between 3° C and 25° C. Since no weight was attached to

superheat, the controller will not take steps to regulate it unless the model predicts that superheat will exceed the specified constraints. The plant model used is an identified second order PEM state space model with EEV and WFV position as the inputs and superheat and cooling as the outputs. An experimental run to verify setpoint tracking of the individual local controllers was performed; figures 7-9 show the controller performance for this experimental run. These figures show that setpoint tracking for this control architecture is achieved without the controllers fighting each other, which indicates that if a global control law can be developed to determine the most energy efficient pressure and cooling setpoints, the local controllers will be able to meet the global law's requirements. Note that this test run consists of tracking of randomly selected setpoints and is not intended to achieve optimal energy efficiency. Figure 7 shows the cooling setpoints and actual cooling vs. time for an experimental run. Figure 8 shows that superheat is maintained above 10° C without going higher than 20° C. Superheat never actually approaches the lower bound in this run; this is because the combination of pressure and cooling setpoints is not the most energy efficient combination for the system conditions. A higher pressure at the cooling setpoint would allow the compressor to run at a lower speed for the same amount of cooling, resulting in higher efficiency.

As more cooling is requested of either valve, it opens to permit the extra mass flow needed. Since this has the effect of increasing evaporator pressures, the compressor speed increases to keep pressure at the required setpoint; the net effect is that as more cooling is needed, the compressor increases the energy input to the system in order to achieve the desired cooling. For a decrease in cooling requirements, the reverse process occurs. Figure 9 shows the evaporator pressures during the same test run. During the large change in cooling setpoints at approximately 575 and 725 seconds, the pressure rises, and the compressor speed increases to drive it back towards its setpoint. The compressor response is seen in Figure 10.

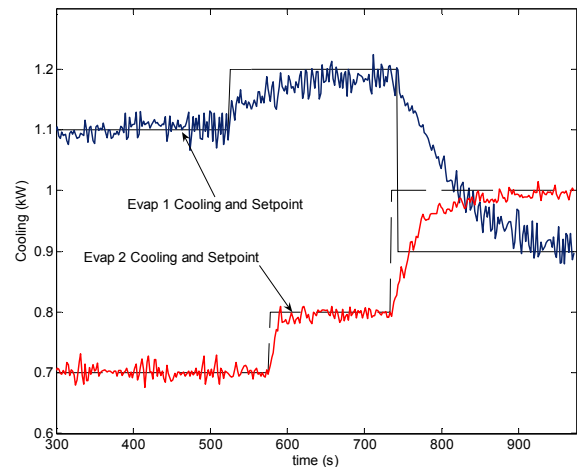


Fig. 7: Setpoint tracking of Cooling

## VI. FUTURE WORK

The next step in this effort is to use the successful component controllers in coordination with a global controller that maximizes the energy efficiency of the water chiller system. Since the ultimate goal of this research is to open avenues for minimizing energy consumption, this global controller would act in a supervisory role to regulate the chilled water temperature in the most efficient manner possible, setting the desired pressures and cooling of each of the components, and driving superheat to a minimum constraint. The ability of MPC to explicitly account for constraints allows MPC-controlled processes to operate near those constraints; in this case, the evaporators will be driven to operate much more closely to a minimum level of superheat than would normally be advisable [1]. This approach can help establish a level of efficiency greater than that achieved by simply altering the setpoints. The MPC framework also allows for demand shedding, where an energy consumption constraint can be imposed and cooling will not meet its setpoint unless the constraint is met. Another important front will be to account for nonlinearities in the system; as currently implemented, the controllers are detuned such that they function over a wide range of conditions at a sacrifice of performance. Using models that more accurately predict the behavior at current conditions will allow the controller to be more aggressive in setpoint tracking.

## REFERENCES

- [1] E.F. Camacho and C. Bordons, *Model Predictive Control*. London: Springer, 1999.
- [2] S. Skogestad and I. Postlethwaite, *Multiple Feedback Control: Analysis and Design*, 2<sup>nd</sup> ed. West Sussex, England: John Wiley & Sons, Ltd., 2005.
- [3] Lennart Ljung, *System Identification: Theory for the User*, 2<sup>nd</sup> ed. Upper Saddle River, New Jersey: Prentice Hall PTR, 1999.
- [4] M. J. Moran and H. N. Shapiro, *Fundamentals of Engineering Thermodynamics*, 3<sup>rd</sup> ed. New York: John Wiley & Sons, 1996.
- [5] K. Ogata, *Modern Control Engineering*, 4<sup>th</sup> ed. Upper Saddle River, New Jersey: Prentice Hall Inc., 2002.
- [6] *MATLAB Model Predictive Control Toolbox* Documentation: <http://www.mathworks.com/access/helpdesk/help/toolbox/mpc/>
- [7] D. Leducq, J. Guilpart, G. Trystam, "Non-linear predictive control of a vapour compression cycle," *International Journal of Refrigeration*, vol. 29, pp. 761-772, August 2006.
- [8] M. Xu, S. Li, W. Cai, L. Lu. "Effects of a GPC-PID control strategy with hierarchical structure for a cooling coil unit," *Energy Conversion and Management*, vol. 46, pp. 132-145, January 2006.
- [9] J. MacArthur. "Receding Horizon Control: A model-based policy for HVAC applications," *Proceedings of the 1993 Winter Meeting of ASHRAE Transactions*, January 1993.

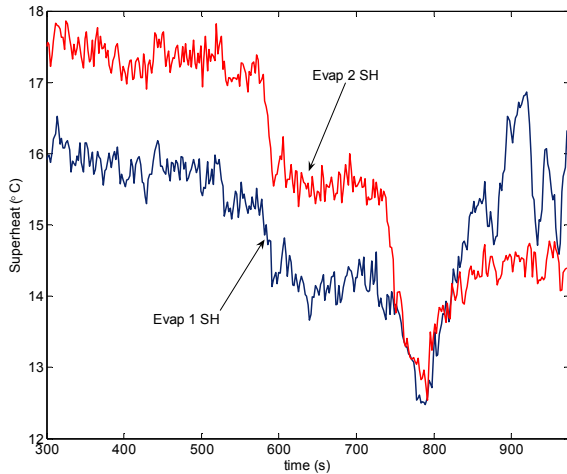


Fig. 8: Regulation of Evaporator Superheat for MIMO MPC

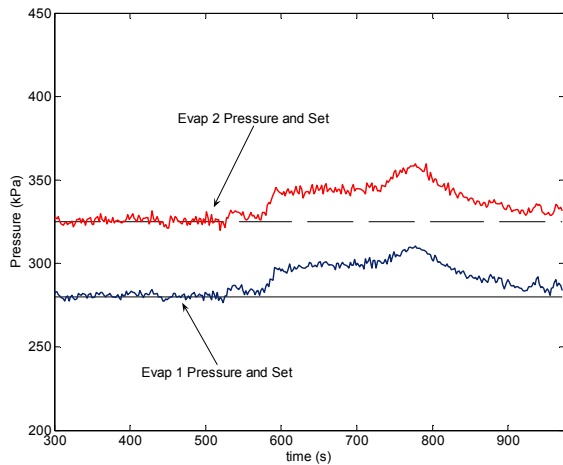


Fig. 9: Refrigerant Pressures

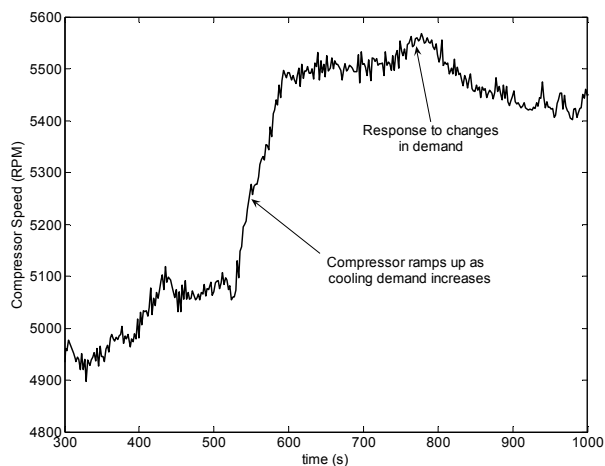


Fig. 10: Compressor Speed Regulating Evaporator Pressure