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Closed-loop System Identification of Ankle Dynamics Using A Subspace Method with Reference Input as Instrumental Variable

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Abstract—**Joint stiffness, the dynamic relationship between the angular position of a joint and the torque acting about it, can be used to describe the dynamic behavior of the human ankle during posture and movement. Joint stiffness can be separated into intrinsic stiffness and reflex stiffness, which are modeled as linear and LNL systems, respectively. For most functional tasks, the ankle interacts with a compliant load. The joint stiffness can be viewed as being operated in closed-loop because the torque is fed back to change the position of the ankle. Consequently, standard open loop identification methods will give biased results. In this paper, we present a new method to estimate intrinsic and reflex stiffness from the total torque measured in closed-loop. A MOESP (Multivariable Output-Error State-Space) subspace system identification method is used to estimate the dynamics of each pathway directly from measured data. The past reference input is used as an instrumental variable to eliminate noise fed back via the controller loop. Simulation and experimental studies demonstrate that the method produces accurate results.**

Keywords—**ankle dynamics, closed-loop system identification, subspace method, MOESP algorithm**

I. Introduction

The concept of dynamic joint stiffness is used to study the mechanical behavior of the mechanisms acting about the ankle. Joint stiffness is defined as the dynamic relationship between the angular position of a joint and the torque acting about it. Joint stiffness plays an important role in control of the posture since it is the joint stiffness that produces the resistance to an external perturbation.

Joint stiffness can be separated into two components: intrinsic and reflex stiffness. The intrinsic component is due to the mechanical properties of the joint, passive tissue, and active muscle fibers; the reflex component is due to muscle activation in response to the activation of stretch receptors in the muscle. Ref. [1] found that the parallel cascade model, shown in Figure 1, could describe joint dynamic stiffness well.

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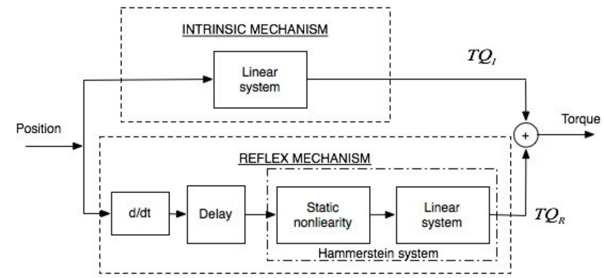

Figure 1 Parallel-cascade structure of ankle dynamics. TQ_I and TQ_R denote intrinsic torque and reflex torque. The position signal is the input signal to the parallel-cascade method, while the torque signal, the sum of the intrinsic and reflex torques, is the measured output. Intrinsic stiffness is modeled as a linear system. Reflex stiffness is modeled by a series connection of a differentiator, a delay, a static nonlinearity and a linear low-pass system.

For perturbations about an operating point, intrinsic stiffness can be modeled well by a second-order, quasi-linear system with transfer function [1]:

$$H_{IS}(s)=\frac{TQ_I(s)}{P(s)}=Is^2+Bs+K \tag{1}$$

where TQ_I is the intrinsic torque,

P is the position,

I, B, and K are the position-dependent inertial, viscous, and elastic parameters, respectively [1].

The reflex component is due to muscle activation in response to the activation of stretch receptors in the muscle. Reflex stiffness has a LNL structure, a series connection of differentiator and a static non-linearity followed by a 2nd or 3rd order low-pass system in series with a delay [1]. Equation 2 shows the transfer function for the 3rd order low-pass filter for reflex stiffness.

$$H_{RS}(s)=\frac{TQ_R(s)}{V_R(s)}=\frac{G_R\omega_n^{\ 2}p}{(s^2+2\zeta\omega_n s+\omega_n^{\ 2})(s+p)}e^{-s\tau} \tag{2}$$

where TQ_R is reflex torque,

$V_R(s)$ is half-wave rectified joint angular velocity,

G_R is reflex gain,

ω_n is 2nd order natural frequency,

ξ is damping parameter,

p is 1st order cut-off frequency,

τ is reflex delay.

The intrinsic and reflex torque cannot be distinguished experimentally; only their sum can be measured. Thus, a direct estimation of $H_{IS}(s)$ and $H_{RS}(s)$ is not feasible.

A number of tools have been developed to separate the intrinsic and reflex components of the net torque. A parallel cascade method [1] takes advantage of the delay in reflex pathway, and estimates the intrinsic and reflex torques iteratively. A real-time algorithm [2] uses a specially designed input sequence to eliminate the correlation between the reflex torque and intrinsic torque. Recently, we developed a subspace-based method to estimate the intrinsic and reflex torque directly from the measured net torque in a one-step procedure. A state space model for overall ankle dynamics [3][4] is estimated directly from measured data. The intrinsic and reflex torques are then estimated by simulating the estimated state space model with appropriate inputs.

Although these methods can separate the intrinsic and reflex torque from the net torque, they require the experimental data be collected in an open loop experiment in which the net torque, TQ, doesn't change the joint position, θ. This requirement is usually satisfied when the experiment is operated under position control mode, where the actuator operates as a position servo and drives the subject's ankle position to follow the command input.

To study ankle dynamics when the subject makes voluntary movements, an impedance controller is used to allow the subject to move the ankle against a simulated compliant load, subject to disturbance torque. The torque is fed back by the impedance controller to change the position. Therefore, under impedance control the system forms a closed-loop system, as Figure 2 [5] shows. Thus identifying the ankle dynamics becomes a closed-loop system identification problem.

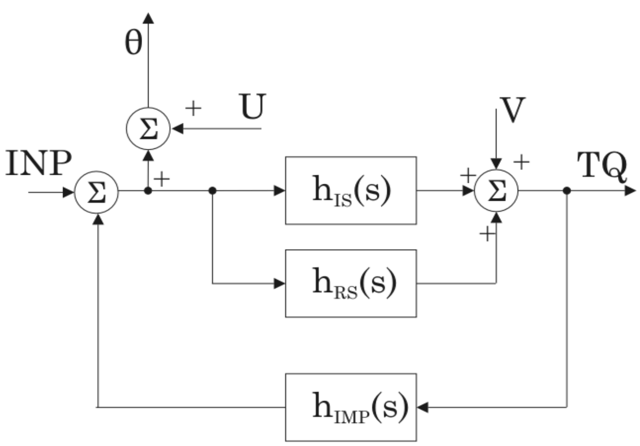

Figure 2. Simplified block diagram of the closed-loop ankle-actuator system. $H_{imp}(s)$ represents dynamics of the impedance controller. $H_{IS}(s)$ and $H_{RS}(s)$ represent transfer functions for intrinsic and reflex stiffness. INP, TQ and θ represent input perturbation, torque and position signals. U and V represent torque and position error signals.

A direct application of the open loop identification methods to data from closed-loop system will produce biased results [6]. Specifically, to identify the ankle dynamics, the parallel cascade method and the real-time algorithm require the measurement noise to be uncorrelated with the input signal. However, in closed-loop experiments, the input signal, θ, will contain feedback measurement noise that is correlated the noise in the measured torque. Thus, the parallel cascade method and the real-time algorithm will produce biased estimates.

The subspace method, however, can estimate the ankle dynamics in closed-loop. We showed previously that a subspace method, EIV-MOESP [7] could separate the intrinsic and reflex torque from the net torque when the experiment was conducted under impedance control mode [8]. Past input and past output were used as instrumental variables. Although this method provides unbiased estimates for ankle dynamics in closed-loop, it is not efficient in computation since a large number of instrumental variables are constructed [8].

In this paper, we present a more efficient way to separate the torque from the intrinsic stiffness and that from the reflex stiffness with compliant loads. The past reference signal is used as an instrumental variable to eliminate noise feedback. The MOESP structure is used to estimate the state space model of the ankle dynamics. The intrinsic and reflex torque is obtained by simulating the estimated state space model for the ankle dynamic with proper inputs.

This paper is structured as follows. Section II presents the state space model for ankle dynamics. Section III describes the past reference input method of MOESP family. Sections IV and V provide simulation examples and experimental results to test and validate the algorithm. Section VI summarizes the contribution of this paper.

II. State Space Model for Ankle Dynamic

A. State Space Model for Intrinsic Stiffness

For perturbations about an operating point, intrinsic stiffness can be described as a linear relationship between position and torque, which is modeled well by a second-order quasi-linear system with the transfer function of Equation 1. Alternatively, the intrinsic stiffness can be described by Equation 3 using the inputs $\begin{bmatrix} P & \dot{P} & \ddot{P} \end{bmatrix}$, where P is measured position, $\dot{P}$ is the differentiated position and $\ddot{P}$ is second-order differentiated position.

$$\frac{TQ_I}{P} = \begin{bmatrix} P & \dot{P} & \ddot{P} \end{bmatrix} \begin{bmatrix} K \\ B \\ I \end{bmatrix} \tag{3}$$

Thus, a state space model relating the inputs $\begin{bmatrix} P & \dot{P} & \ddot{P} \end{bmatrix}$ to the output TQ_I is,

$$\begin{aligned} X^I_{k+1} &= A_I X^I_k + B_I U_{ik} \\ TQ_{Ik} &= C_I X^I_k + D_I U_{ik} \end{aligned} \tag{4}$$

where TQ_I is the intrinsic torque,

U_{ik} represents the constructed inputs $\begin{bmatrix} P & \dot{P} & \ddot{P} \end{bmatrix}$

X^I_k is the state vector for this state space model,

A_I, B_I and C_I matrices will be zero.

B. State Space Model for Reflex Stiffness

Reflex stiffness has a LNL structure comprising a series connection of a differentiator, a static non-linearity and a linear dynamic system. If velocity is used as the input, the reflex stiffness can be described by a Hammerstein system. The Hammerstein system can be identified using an extended subspace method [9] as follows. Assume the static nonlinearity $n_{RS}(\cdot)$ can be approximated by a basis expansion $g(\cdot)$, so that:

$$
\begin{aligned}
z_k = g(u_k, \tau) &= \sum_{i=1}^{r} \tau_i g_i(u_k) \\
&= \begin{bmatrix} \tau_1 & \cdots & \tau_r \end{bmatrix} \begin{bmatrix} g_1(u_k) \\ \vdots \\ g_r(u_k) \end{bmatrix}
\end{aligned} \tag{5}
$$

where $g_i(u_k)$ are the terms of the basis function

τ_i are the scale factors for each basis function term,

u_k and z_k denote the input to the nonlinearity and the output from the nonlinearity.

A state space model can be used for $H_{RS}(s)$, the linear part of the reflex stiffness:

$$
\begin{aligned}
X^r{}_{k+1} &= A_r X^r{}_k + B_r z_k \\
TQ_{Rk} &= C_r X^r{}_k + D_r z_k
\end{aligned} \tag{6}
$$

where TQ_R is the reflex torque,

z_k is the output from the static nonlinearity,

$X^r{}_k$ is the state vector for linear part of the reflex Stiffness

A_r, B_r, C_r, D_r are the system matrices for $H_{RS}(s)$.

From Equation 5, the output from the nonlinearity is the product of a row vector containing nonlinear parameters and a column vector containing the kernel of the basis function. If we define:

$$
\begin{aligned}
\tilde{B} &= \begin{bmatrix} B\tau_1, & \cdots & B\tau_r \end{bmatrix} \\
\tilde{D} &= \begin{bmatrix} D\tau_1, & \cdots & D\tau_r \end{bmatrix} \\
U_{rk} &= \begin{bmatrix} g_1(u_k), & \cdots & g_r(u_k) \end{bmatrix}^T
\end{aligned} \tag{7}
$$

the Hammerstein system can be rewritten as

$$
\begin{aligned}
X^h{}_{k+1} &= A_h X^h{}_k + \tilde{B} U_{rk} \\
TQ_{Rk} &= C_h X^h{}_k + \tilde{D} U_{rk}
\end{aligned} \tag{8}
$$

where TQ_R is the torque from the reflex stiffness

X_k^h is the internal state vector,

A_h, C_h, represent the system matrices for reflex stiffness.

Thus, once a basis function has been chosen, the static nonlinearity and the SISO linear system of the Hammerstein system can be described by a MISO linear state space model using $U_k = [g_1(u_k), \;\; \cdots \;\; g_r(u_k)]^T$.

To describe the nonlinearity in the reflex stiffness, we opted to use a Chebyshev polynomial [10] to avoid the conditioning problems associated with the high order components in regular polynomials. The constructed input matrix becomes:

$$
U_{rk} = \begin{bmatrix} T_1(x) & T_2(x) & \cdots & T_{n_N}(x) \end{bmatrix}^T \tag{9}
$$

where the Chebyshev polynomials are given by:

$$
\begin{aligned}
T_1(x) &= 1 \\
T_2(x) &= v_k \\
T_n(x) &= 2 \cdot v_k \cdot T_{n-1}(x) - T_{n-2}(x)
\end{aligned} \tag{10}
$$

C. State space model for joint stiffness

A direct estimate of separate state space models for intrinsic and reflex is not possible. However, a state space model for the overall parallel cascade model of ankle dynamics can be estimated because the measured torque TQ is the sum of the torques from the intrinsic and reflex stiffness (i.e. $TQ = TQ_I + TQ_R$). Specifically, Equation 4 and Equation 7 can be combined to give:

$$
\begin{aligned}
X_{k+1} &= A_h X_k + \begin{bmatrix} 0 & \tilde{B} \end{bmatrix} \begin{bmatrix} U_{ik} \\ U_{rk} \end{bmatrix} \\
TQ &= C_h X_k + \begin{bmatrix} D_I & \tilde{D} \end{bmatrix} \begin{bmatrix} U_{ik} \\ U_{rk} \end{bmatrix}
\end{aligned} \tag{11}
$$

where TQ is the measured net torque,

A_h, $\tilde{B}$, C_h, $\tilde{D}$ are the system matrices for reflex stiffness in Equation 7,

D_I is the system matrices for intrinsic stiffness in Equation 4,

U_{ik} is the constructed input to intrinsic stiffness

U_{rk} is the constructed input to reflex stiffness.

For this state space model, the input $\begin{bmatrix} U_{ik} \\ U_{rk} \end{bmatrix}$ can be constructed from the measured input, and the output TQ is obtained directly from the measured data. The state space model of Equation 8, can be estimated from input $\begin{bmatrix} U_{ik} \\ U_{rk} \end{bmatrix}$ to output TQ.

III. Identification

This section presents the algorithm to estimate the state space for ankle dynamics when the data is collected from a closed-loop system.

The difficulty with closed-loop system identification is feedback noise. Instrumental variables can be used to eliminate these noise terms. An instrumental variable should be uncorrelated with the noise, but correlated with the states. Thus, the noise term will be eliminated using the instrumental variable, while the information of the internal states will be preserved. The requirements for the instrumental variables, Θ, are as follows:

$$\lim_{N\to\infty}\frac{1}{N}v_{i,j,N}\Theta=0 \quad (12)$$

$$\lim_{N\to\infty}\frac{1}{N}w_{i,j,N}\Theta=0 \quad (13)$$

$$\operatorname{rank}(\lim_{N\to\infty}\frac{1}{N}X_{i,N}\Theta)=n \quad (14)$$

where $X_{i,N}$ is the state vector,

$w_{i,j,N}$ and $v_{i,j,N}$ are the Hankel matrices for the process noise and measurement noise respectively. The Hankel matrix for a signal r has the form.

$$R_{i,j,n} = \begin{bmatrix} r(i) & r(i+1) & \cdots & r(i+n-1) \\ r(i+1) & r(i+2) & & r(i+n) \\ \vdots & & & \vdots \\ r(i+j-1) & r(i+j) & \cdots & r(i+j+n-2) \end{bmatrix} \quad (15)$$

where i is the left upper entry of the Hankel matrix,

j is the number of the rows,

n is the number of columns.

The past reference input is an excellent candidate as the instrumental variable to identify the ankle dynamics in closed-loop. Equations 12 and 13 are easily satisfied since the $w_{i,j,N}$ and $v_{i,j,N}$ are white noise and voluntary movement. The voluntary movement is generated by the subject, and it is uncorrelated with the external perturbation. Finally, if the reference input is chosen to be persistently exciting [6], Equation 14 will be satisfied.

Therefore, a member of the MOESP family of algorithms, namely PR-MOESP [11] (MOSEP using Past Reference input as instrumental variables), is well suited for closed-loop system identification. The past reference input is used as instrumental variables to eliminate the correlation between the input signal and noise terms in the output signals. Using PR-MEOSP, a state space method can be estimated for the joint stiffness from the constructed input $\begin{bmatrix} U_{ik} \\ U_{rk} \end{bmatrix}$ to the measured torque TQ.

Identifying the state space model of Equation 11 does not estimate the intrinsic stiffness and the reflex stiffness directly. However, simulating the estimated system with the appropriate inputs permits the torque from the intrinsic and reflex stiffness to be estimated. Specifically, the output from the simulation with the input signal $\begin{bmatrix} U_{ik} \\ 0 \end{bmatrix}$ is an estimate of the intrinsic torque TQ_I. Similarly, the response to the input $\begin{bmatrix} 0 \\ U_{rk} \end{bmatrix}$ will estimate the torque from the reflex stiffness

IV. SIMULATION STUDY

To test and validate the algorithm, simulated data were generated using Matlab's Simulink. Intrinsic stiffness was modeled as a second-order, quasi linear system with transfer function:

$$\frac{TQ_I(s)}{\theta(s)}=0.015s^2+0.8s+150 \quad (16)$$

where θ is joint angle,

TQ_I is torque from the intrinsic stiffness.

Reflex stiffness was modeled by a series connection of a differentiator, a delay for 40ms [1], a half-wave rectifier and a second order low-pass filter as

$$\frac{TQ_R(s)}{V_R(s)}=\frac{3200}{s^2+80s+1600} \quad (17)$$

where TQ_R is reflex torque,

V_R is half-wave rectified joint angular velocity.

A Pseudorandom Binary Sequence Signal (PRBS) was used as the external position input. These perturbations were similar to those used previously to identify ankle dynamics [1]; they moved rapidly between two values at random multiples of the switching interval. The perturbation signal had peak-to-peak amplitude of 0.04 rad and a switching interval of 100 ms. Gaussian white noise with SNR as 10 dB was used to as the measurement noise. Another Gaussian noise, filtered by a second order Bessel low pass filter with cut-off frequency of 2 Hz, was used to simulate the voluntary torque. The net torque was fed back via an impedance controller [8].

The percentage Variance Accounted For (%VAF) was used to measure how well the identified torque predicted the true torque. The VAF between the true and identified torque was calculated as:

$$\%\text{VAF}=\left(1-\frac{\text{variance}(y-y_{est})}{\text{variance}(y)}\right)\times 100\% \quad (18)$$

The PR-MOESP was implemented by modifying the Matlab SMI 2.0 Toolbox [12].

The parallel-cascade method [1], EIV-MOESP [5] and PR-MOESP were used to estimate the intrinsic and reflex torque. The results are shown in Table 1. The first column lists the torque being compared, while the first row lists the method that was used to provide estimate. This simulation study shows that the new algorithm provides good estimates for the intrinsic torque, reflex torque and net torque.

	PR-MOESP	EIV-MOESP	Parallel-cascade
Net torque	92%	90%	90%
Intrinsic torque	98%	97%	56%
Reflex torque	97%	97%	66%

Table 1. Comparison of three identification methods using simulated data

Figure 3 shows the estimated torques and the measured torques from the PR-MOESP and parallel-cascade method. Clearly, the parallel-cascade method provided biased estimate for intrinsic and reflex torques. The PR-MOESP estimated the intrinsic and reflex torques accurately.

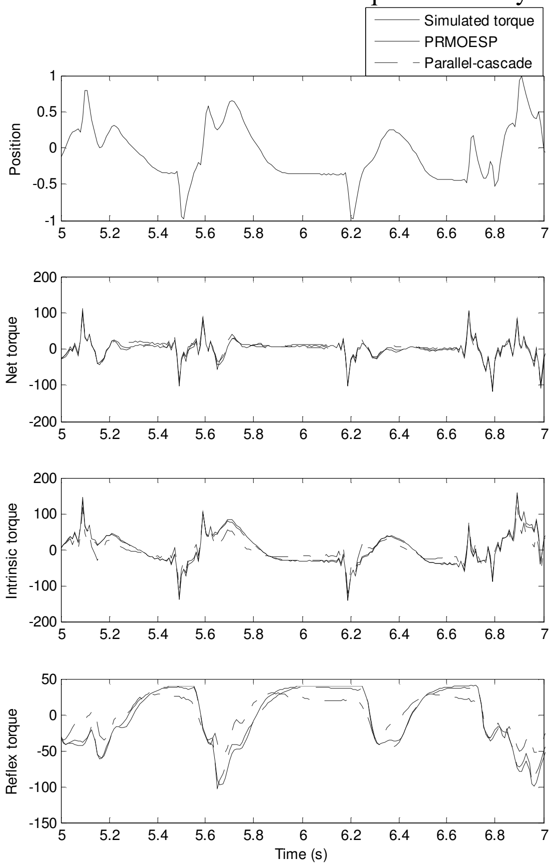

Figure 3 Comparison of estimated and observed torques

Next, we investigated the performance of PR-MOESP and parallel-cascade method with different reflex gain. No noise was added to the simulation. 20 Monte-Carlo simulations were conducted, each with different reflex gain from 10 to 30 with an increment of 1. Each Monte-Carlo simulation contained 100 trials, each lasting for 50 seconds. VAFs between the simulated and predicted intrinsic, reflex and net torque were computed. Figure 4 shows the mean of the VAFs as a function of reflex gain. It is evident that the parallel cascade method provided biased result, especially when the reflex gain was high. Indeed, the parallel-cascade method is a correlation-based identification method. The reflex torque is viewed as noise when estimating the intrinsic torque. In closed-loop, the reflex torque is fed back by the controller. Thus, the position is correlated with the reflex torque. Therefore, the estimate the intrinsic stiffness is biased. The PR-MOESP used the instrumental variable to eliminate the effects from the noise. Clearly, the PR-MOESP estimates the intrinsic, reflex and net torques regardless the value of the reflex gain. The estimate the intrinsic stiffness depends the correlation between the As the reflex gain increased, the performance of the parallel cascade method decreased.

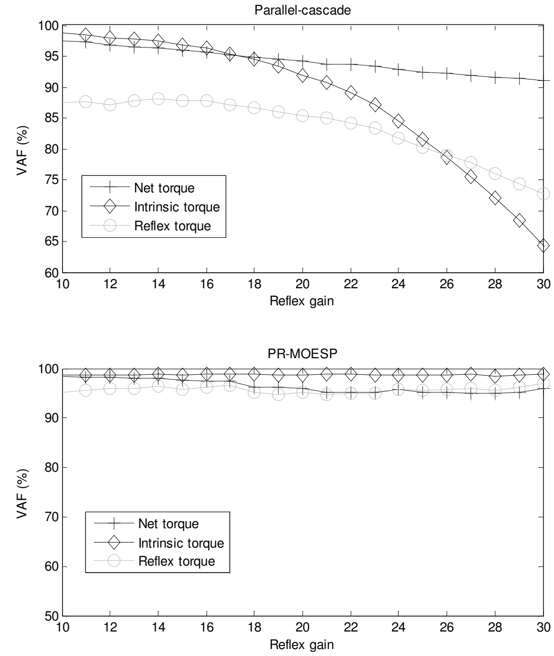

Figure 4 Mean VAF between simulated and predicted torque as a function of reflex gain.

V. Experimental Studies

Experimental data was used to test and validate the algorithm. The subject was a 25 year-old-female with no history of muscular disease. The experiment was done under impedance control mode, where the subject moved the ankle against a simulated compliant load. The experiment trial lasted for 60 seconds. The input and output data were recorded at a sampling rate of 1KHz and then decimated to 100 Hz. Details of the experiment are available in [1]. Using the algorithm, we estimated the net torque, intrinsic torque and reflex torque respectively. The results are shown in Table 2.

Figure 5 shows the measured torque and the estimated torques. The estimated net torque fit the measured torque with VAF of 85%. The estimated intrinsic torque contributed 49% in VAF to the net torque, while the estimated reflex torque contributed 43% in VAF to the net torque. The parallel cascade method was also applied to the same data. The estimated net torque fit the true measured torque with VAF of 66%. The estimated reflex torque contributed 52% to the net torque and the estimated intrinsic torque contributed 20% for the total torque. This result shows that the new subspace method provides better estimate of the net torque, intrinsic torque and reflex torque than the parallel cascade method.

	PR-MOESP	EIV-MOESP	Parallel-cascade
Net torque	85%	83%	66%
Intrinsic torque	49%	50%	52%
Reflex torque	43%	38%	20%

Table 2. Comparison of three identification methods from the experimental data

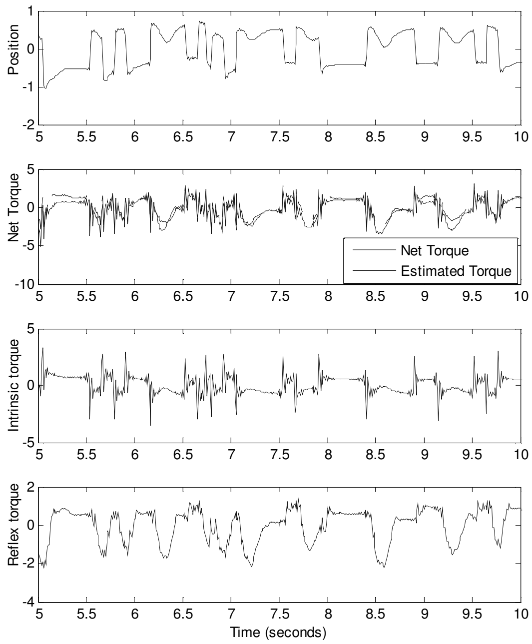

Figure 5 Comparison of estimated and observed torque

VI. CONCLUSION AND DISCUSSION

In this paper, we presented a new subspace method to estimate ankle dynamics with compliant loads. The parallel cascade model of ankle dynamics is described by a MISO state space model with constructed inputs from the measured position signal and terms of the basis function. Using the past reference input as an instrumental variable, the MOESP algorithm was modified to estimate the MISO state space model for ankle dynamic. Simulating the estimated model with appropriate inputs allows the intrinsic and reflex torques to be estimated.

In the simulation study, we compared the result of the PR-MOESP and the results from EIV-MOESP and parallel-cascade method. It is not surprising that parallel cascade method provided biased estimates for intrinsic and reflex torque though the estimate of the net torque was good. That is because the parallel cascade method uses the estimate of the net torque as the only criteria to monitor the iteration of the algorithm. The algorithm stopped when the algorithm couldn't improve the estimate for the net torque. If the data was collected in open-loop, the estimate of intrinsic and reflex torque improves when the estimate of net torque improves. However, when the data is collected from closed-loop system, the estimate of the intrinsic and reflex torque by parallel cascade method is highly biased even if the algorithm improves the estimate of net torque.

The results from the PR-MOESP and EIV-MOESP were similar. However, PR-MOESP has two advantages over EIV-MOESP. First, PR-MOESP is more economic. EIV-MOESP uses past input and past output as instrumental variables. For ankle dynamic, three inputs are needed for intrinsic stiffness, and five inputs are needed for reflex stiffness if a fifth order basis function is chosen. Thus, nine columns of instrumental variables were required for EIV-MOESP. PR-MOESP uses the past reference input as the instrumental variable. The external position perturbation is the reference input to the experiment. Only one column of instrumental variable is required.

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