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A Feedforward Control Synthesis Approach for LPV Systems

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***Abstract*— It is well-known that tracking and regulation control objectives can be achieved independently. This paper focuses on the synthesis of the feedforward part of a two-degree-of-freedom LPV/LFT controller. Here, the feedforward filter synthesis, which includes a constraint on the control signal, is cast as an $\mathcal{L}_2$ full-information minimization problem. The effectiveness of the proposed approach is demonstrated on the design of an LPV/LFT missile autopilot.**

***Index Terms*— LPV and LFT systems, full-information control law, $\mathcal{L}_2$ gain, tracking, two-degree-of-freedom LPV controllers, LPV feedforward filter synthesis, missile control.**

I. INTRODUCTION

This paper addresses the design of the tracking part of a two-degree-of freedom controller for LPV systems. This is the so-called feedforward filter part of a two-degree-of-freedom controller which aims at improving the tracking and/or the measurable disturbance rejection performance of the feedback loop. The design of LTI feedforward controllers in the presence of uncertainties was considered in e.g. [4]. As shown in [12] in the case of LTI and polytopic systems, an important property of a two-degree-of-freedom controller synthesis, which is exploited here, is that tracking controller and output feedback regulator syntheses are two independent control problems. In addition, the former reduces to a simple static state-feedback or full-information controller synthesis. This paper extends the feedforward filter synthesis ideas proposed in [12] to a class of parameter-dependent systems, namely, the class of LPV systems which can be represented as the feedback interconnection of a linear system and a matrix with measurable time-varying entries [13], [10], [9]. Here, the feedforward synthesis is formulated as a full-information $\mathcal{L}_2$ control law synthesis. From a practical point of view, the approach of this paper presents the following interesting properties:

- the feedforward filter is designed to improve the tracking performance of an existing feedback loop; the feedforward filter synthesis algorithm does not requires the explicit knowledge of the feedback regulator.
- the feedforward filter synthesis reduces to synthesize a simple (i.e. a static) state-feedback or a full-information control law.

The paper is structured as follows. Section II reviews some of the basics of $\mathcal{L}_2$ performance for parameter-dependent systems. Sections III and IV present and detail the synthesis of the feedforward filter for a parameter-dependent system. The effectiveness of the gain-scheduling feedforward synthesis is illustrated in Section V on a non-linear longitudinal axis missile control problem. Conclusions are given in Section VI.

The notation used is standard: $\mathbf{R}^{m\times n}$ denotes the set of real $m \times n$ matrices, I is the identity matrix and I_n is the $n \times n$ identity matrix. $0_{n\times p}$ is the $n \times p$ zero matrix. A diagonal matrix with entry d_i, $i = 1, \dots, n$, on its diagonal is denoted $\mathrm{diag}(d_1, \dots, d_n)$. A^T is the transpose of matrix A. For a square matrix A, $\det(A)$ is the determinant of A. For a symmetric matrix, $A = A^T$, $A > 0$ means that A is a positive definite matrix. $\mathbf{L}_2^l[0,\infty)$ is the space of $\mathbf{R}^l$-valued signals $w(t) : [0,\infty) \to \mathbf{R}^l$ of finite energy $\|w\|^2 = \int_0^\infty w(t)^T w(t)dt$. The $\mathcal{L}_2$ gain of an operator H is given by $\|H\| = \sup\{\|H(w)\|/\|w\| : \quad w \in \mathbf{L}_2^l[0,\infty), w \neq 0\}$. $\mathcal{T}_{zw}$ is the mapping $w \mapsto z$ relating the two real-valued signals z and w.

II. PRELIMINARIES

This section introduces some definitions and a technical result. Let us consider a parameter-dependent system $\mathcal{P}(\Delta)$ given by

$$\begin{aligned} \dot{x} &= \mathbf{A}(\Delta(t))x + \mathbf{B}(\Delta(t))w \\ z &= \mathbf{C}(\Delta(t))x + \mathbf{D}(\Delta(t))w \end{aligned} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state vector, $w(t) \in \mathbf{R}^{n_w}$ is the system input disturbance and $z(t) \in \mathbf{R}^{n_z}$ is the output error. $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ are real-valued continuous functions of $\Delta(t) \in \mathbf{\Delta}$ where $\mathbf{\Delta}$ is assumed to be a polytope of $n_q \times n_p$ matrices defined by its vertices $\{\Delta_1, \dots, \Delta_r\}$. The assumptions made here are quite general since Δ is not supposed to be square and diagonal and the polytope is not restricted to be a hyper-rectangle.

A. Linear Fractional Representations

Throughout this paper, we make the following assumptions:

A1. The plant $\mathcal{P}(\Delta)$ can be expressed as the following Linear Fractional Transformation representation (LFT)

$$\mathcal{P}(\Delta) : \left\{ \begin{array}{l} \dot{x} = Ax + B_q q + B_w w \\ p = C_p x + D_{pq} q + D_{pw} w \\ z = C_z x + D_{zq} q + D_{zw} w \\ q = \Delta(t)p, \quad \Delta(t) \in \mathbf{\Delta} \end{array} \right. \tag{2}$$

where $p \in \mathbf{R}^{n_p}$, $q \in \mathbf{R}^{n_q}$ with the assumptions that $\det(I + D_{pq}\Delta(t)) \neq 0$ for all $\Delta(t) \in \mathbf{\Delta}$ (well-posedness) and $\Delta(t)$ is available in real-time.

A2. The polytope $\mathbf{\Delta}$ contains the origin.

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The well-posedness of (2) guarantees the equivalence between the system representations (1) and (2).

B. $\mathcal{L}_2$ Performance for LPV/LFT Systems

Theorem 1: (Dual quadratic $\mathcal{L}_2$ performance characterization). If there exist multipliers

$$\begin{aligned}
\Pi_P &= \begin{bmatrix} 0 & P \\ P & 0 \end{bmatrix}, P \in \mathbf{R}^{n\times n}, P > 0, \\
\Pi_\gamma &= \begin{bmatrix} \gamma^2 I_{n_z} & 0 \\ 0 & -I_{n_w} \end{bmatrix}, \\
\Pi &= \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} = \Pi^T, \Pi \in \mathbf{R}^{(n_p+n_q)\times(n_p+n_q)}
\end{aligned}$$

such that

$$V^T \mathrm{diag}(\Pi_P, \Pi_\gamma, \Pi) V > 0, \qquad (3)$$

$$\begin{bmatrix} -\Delta_i^T \\ I \end{bmatrix}^T \Pi \begin{bmatrix} -\Delta_i^T \\ I \end{bmatrix} < 0, \quad i = 1, \ldots, r, \qquad (4)$$

$$Q > 0,$$

where

$$V = \begin{bmatrix} S_x^T & E_x^T & E_z^T & S_w^T & E_1^T & S_1^T \end{bmatrix}^T,$$

with

$$\left[\begin{array}{c} S_x \\ \hline S_1 \\ \hline S_w \end{array}\right] = - \left[\begin{array}{ccc} A^T & C_z^T & C_p^T \\ \hline B_q^T & D_{zq}^T & D_{pq}^T \\ \hline B_w^T & D_{zw}^T & D_{pw}^T \end{array}\right],$$

and

$$\left[\begin{array}{c} E_x \\ \hline E_z \\ \hline E_1 \end{array}\right] = \left[\begin{array}{ccc} I_n & 0 & 0 \\ \hline 0 & I_{n_z} & 0 \\ \hline 0 & 0 & I_{n_p} \end{array}\right],$$

then the $\mathcal{L}_2$ gain of $\mathcal{T}_{zw}$ is less than or equal to γ, for all values of the parameter $\Delta(t) \in \mathbf{\Delta}$.

A proof for the primal version of Theorem 1, i.e. obtained with a Lyapunov function of the form $V(x) = x^T P x$, $P > 0$, can be found in [13]. The dual form given here is obtained by applying the dualization Lemma [13] to the set of primal inequalities. The approach taken here is more general and less conservative than the approach given in [15] where it is assumed that Δ is square and the multiplier positive definite (see [15] for details). This result can also be demonstrated using the concepts of implicit systems and quadratic separator analysis given in [5]. A relatively simple proof of Theorem 1, can be derived using the S-procedure [1] within the IQC analysis framework [7]. An alternative analysis result, in line with the work of [15], is given in [11].

Parameter-dependent Lyapunov functions allow to consider time-varying parameters with bounded rates of variations and so, they have the potential to yield less conservative results than those obtained with a single quadratic Lyapunov function. Sufficient conditions using parameter-dependent Lyapunov functions could be used to tackle the $\mathcal{L}_2$ performance analysis problem of this paper e.g. [5], [15] and [16]. The simplest choice would be to use affine parameter-dependent Lyapunov functions [3], [15]. However, if we adopt the approach of [15] then, it can be shown that, when the parameter set and the set of the parameter rates of variations are symmetric with respect to the origin (this is common in practice) then affine parameter-dependent Lyapunov functions do not offer any improvements over single quadratic Lyapunov functions [15]. More general parameter-dependent Lyapunov functions, represented by LFTs, have been used in e.g. [5] and [16]. Unfortunately, no general procedure exists for selecting the Lyapunov function structure (i.e. its LFT parameters). Hence, the use of parameter-dependent Lyapunov functions, for the general class of LPV/LFT systems, remains, to this date, an open question.

III. TRACKING FOR LFT SYSTEMS

The feedforward filter synthesis is illustrated in the interconnection diagram of Figure 1. G_p is the nominal model of the parameter dependent plant. T_M is a reference model which may be an LTI or a parameter dependent. T_M represents the desired closed-loop transfer matrix between the set point r and the actual output Ey_m where, the matrix E selects the plant model outputs to be controlled. W_1 may be constant, LTI or a parameter dependent weight. W_1 is used to penalize the error between the reference model output and the plant output subset. W_2 is used to penalize the control action required to achieve the tracking objectives. In addition, a measurable disturbance, represented by d in Figure 1 can be rejected by a suitable choice of W_1. This can achieved by replicating in W_1 the frequency content of the disturbance (Internal Model Principle). Disturbance rejection

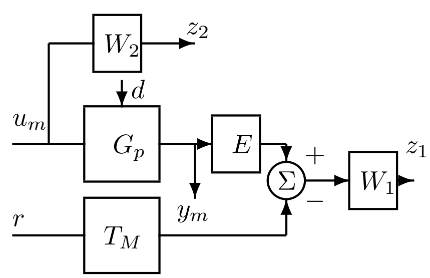

Fig. 1. Interconnection structure for the feedforward filter design: the reference tracking and disturbance rejection problem

(of the measurable disturbance d) and tracking objectives can be cast as a standard $\mathcal{L}_2$ minimization problem by defining $w^T = [r^T, d^T]$ and $z^T = [z_1^T, z_2^T]$ in the setting of Figure 1. Now, we suppose that the interconnection diagram of Figure 1 is a parameter dependent system with state-space representation:

$$FW(\Delta): \begin{cases} \dot{x} = Ax + B_q q + B_w w + B_u u_m, \\ p = C_p x + D_{pq} q + D_{pw} w + D_{pu} u_m, \\ z = C_z x + D_{zq} q + D_{zw} w + D_{zu} u_m, \\ u_m = F_x x + F_w w, \\ y_m = C_y x + D_{yq} q + D_{yw} w + D_{yu} u_m, \\ q = \Delta(t) p, \quad \Delta(t) \in \mathbf{\Delta}. \end{cases} \qquad (5)$$

The plant matrices A, B_q etc., in the above state-space representation can be obtained from the parameter dependent state-space realizations of T_M, W_1, W_2 and G_p. These state-space matrices are given in the Appendix for the tracking problem. They could also be computed with the Maltab LFR Toolbox [6]. The synthesis problem consists of computing a control law such that the $\mathcal{L}_2$ gain between w and z is less than or equal to a positive number γ (typically, but not necessarily, the weights are selected so that the objectives are met if γ is less than 1), for all Δ in $\boldsymbol{\Delta}$. Because all the systems in Figure 1 are simulation models the state vectors of T_M, the weights and G_p are available for feedback. Hence, the feedforward filter can be obtained with a full-information control law which minimizes the $\mathcal{L}_2$ gain of $\mathcal{T}_{zw}$ over the polytope of matrices $\boldsymbol{\Delta}$.

IV. FEEDFORWARD FILTER SYNTHESIS

A. $\mathcal{L}_2$ Full-Information Synthesis

More precisely, the feedforward filter synthesis consists of computing the full-information controller such that the $\mathcal{L}_2$ gain of $\mathcal{T}_{zw}$ in (5) is less than or equal to γ over the whole parameter trajectory set.

Corollary 1: (Feedforward synthesis) Consider the system (5). If there exist a matrix $P > 0$, a positive scalar γ, a matrix $Y \in \mathbf{R}^{n_u \times n}$, a matrix $F_w \in \mathbf{R}^{n_u \times n_w}$ and a symmetric multiplier

$$\Pi = \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \in \mathbf{R}^{(n_p+n_q)\times(n_p+n_q)},$$

with $Q < 0$, such that

$$L(P, Y, F_w, \gamma) := \begin{bmatrix} L_{11} & L_{12} & L_{13} & L_{14} \\ \star & L_{22} & L_{23} & L_{24} \\ \star & \star & L_{33} & L_{34} \\ \star & \star & \star & L_{44} \end{bmatrix} < 0$$

$$R + S^T\Delta_i^T + \Delta_i S + \Delta_i Q \Delta_i^T > 0, \quad i = 1, \ldots, r$$

with $L_{11} = AP + PA^T + B_uY + Y^TB_u^T + B_qRB_q^T$, $L_{12} = PC_z^T + Y^TD_{zu}^T + B_qRD_{zq}^T$, $L_{13} = PC_p^T + Y^TD_{pu}^T + B_qS^T + B_qRD_{pq}^T$, $L_{14} = B_w + B_uF_w$, $L_{22} = -\gamma I_{n_z} + D_{zq}RD_{zq}^T$, $L_{23} = D_{zq}S^T + D_{zq}RD_{pq}^T$, $L_{24} = D_{zw} + D_{zu}F_w$, $L_{33} = Q + SD_{pq}^T + D_{pq}S^T + D_{pq}RD_{pq}^T$, $L_{34} = D_{pw} + D_{pu}F_w$, $L_{44} = -\gamma I_{n_w}$, then the full-information control law $u_m = F_xx + F_ww$, with $F_x = YP^{-1}$, is such that the $\mathcal{L}_2$ gain of $\mathcal{T}_{zw}$ is less than or equal to γ, for all values of the parameter $\Delta(t) \in \boldsymbol{\Delta}$.

Proof: Corollary 1 follows from Theorem 1 if one applies the later to the system $FW(\Delta)$ given in (5). The 4-by-4-block synthesis inequality $L(P, Y, F_w, \gamma)$ is then obtained by applying the Schur complement formula to inequality (3) divided by γ. ■

The control signal u_m and the controlled output y_m are the outputs of the feedforward filter while r and d will consist of its inputs. u_m is the 'ideal' control signal which if applied to the plant, in the presence of signals r or d, will produce the 'ideal' control output y_m. That is a guarantee of an attenuation of the effect of d on Ey_m, and the guarantee that Ey_m follows the reference signal r.

The full-information control law is static, but note that the feedforward filter in Figure 1 is a parameter-dependent system. The feedforward filter is never used alone. It must be implemented on an existing feedback control loop as follows; Let y_c and u_c denote, respectively, the input and the output of the feedback regulator K (which may be an LTI or a parameter-dependent feedback regulator). The two-degree-of-freedom controller (made of the association of K and of the feedforward filter) is obtained by adding the feedforward control input signal u_m to u_c, and y_c is obtained by subtracting y_m to the plant output measurement y. In the presence of uncertainty, $y \approx y_m$, $u \approx u_m$ providing that the regulator can cancel the error between the ideal reference signal y_m and the output measurement y (see [12] for more details).

V. DESIGN EXAMPLE

A. Missile Model

We consider the longitudinal model taken from [8]. The system state-space representation is given by

$$\begin{aligned} \dot{\alpha} &= f_1(\alpha, q, \delta, M) \\ &\equiv K_\alpha M C_n(\alpha, \delta, M)\cos(\alpha) + q, \quad (6) \\ \dot{q} &= f_2(\alpha, \delta, M) \equiv K_q M^2 C_m(\alpha, \delta, M), \quad (7) \\ \eta &= h_1(\alpha, \delta, M) \equiv K_\eta M^2 C_n(\alpha, \delta, M), \quad (8) \end{aligned}$$

where the aerodynamics coefficients are, for positive values of the angle of attack α, given by

$$C_n(\alpha, \delta, M) = a_n\alpha^3 + b_n\alpha^2 + c_n(2 - M/3)\alpha + d_n\delta,$$
$$C_m(\alpha, \delta, M) = a_m\alpha^3 + b_m\alpha^2 + c_m(-7 + 8M/3)\alpha + d_m\delta.$$

Plant variables and numerical values are given in table I. The variables η and q are measured variables available for feedback and the input to the plant is the tail deflection δ. The tail deflection actuator is modelled as a second order system with a damping factor of 0.7 and a natural frequency of 150 rad/s. In addition, we will suppose that the Mach number (M) and angle of attack (α) are estimated on-line. α and M will be used for scheduling purposes. The missile autopilot performance requirements are as follows:

- The autopilot must ensure stability and performance over the operating range $0 < \alpha \leq 35^o$ and $2 \leq M \leq 4$.
- Track step demands in vertical acceleration with time constants no greater than 0.35s. Overshoot no greater that 10%. Steady-state error less than 1%.

It can be shown that plant linearizations at constant operating conditions exhibit non-minimum phase zeros and badly damped modes [8], [2]. This is typical of a tail controlled missile; acceleration has unstable zero dynamics with respect to the control deflection input. The non-minimum phase characteristics of a missile lead to control difficulties. Unstable zero dynamics impose bandwidth limitations and prevent the direct use of non-linear dynamic inversion control strategies.

Variables	Description
α	Angle of attack [rad].
q	Pitch rate [rad/s].
M	Mach number.
δ	Commanded Tail deflection [rad].
η	Normal acceleration [g].
$g = 32.2$	Gravity acceleration [ft.s^{-2}]
$K_\alpha = 0.0207$	Normal force coefficient
$K_q = 1.2320$	Pitching moment coefficient
$K_\eta = 0.6659$	Load factor coefficient
$a_n = 19.34$ [rad^{-3}]	$a_m = 40.485$ [rad^{-3}]
$b_n = -31.008$ [rad^{-2}]	$b_m = -64.166$ [rad^{-2}]
$c_n = -9.7174$ [rad^{-1}]	$c_m = 2.922$ [rad^{-1}]
$d_n = -1.9481$ [rad^{-1}]	$d_m = -11.803$ [rad^{-1}]

TABLE I

VARIABLE DESCRIPTION AND NUMERICAL VALUES

B. LFR Missile Model

This section briefly explains how a linear fractional representation of the longitudinal missile axis model is obtained. We follow the ideas already proposed in the LFR Matlab toolbox. The LFR Toolbox [6] provides a collection of Matlab functions to simplify the construction and the analysis of linear fractional models.

Define $x = [\alpha, q]^T$, $f = [f_1, f_2]^T$, $h = [h_1, q]^T$. The missile state-space matrices resulting from linearization are given by

$$\mathbf{A}(\alpha, M, \delta) = \frac{\partial f}{\partial x}, \quad \mathbf{B}(\alpha, M) = \frac{\partial f}{\partial \delta}, \tag{9}$$

$$\mathbf{C}(\alpha, M) = \frac{\partial h}{\partial x}, \quad \mathbf{D}(M) = \frac{\partial h}{\partial \delta}. \tag{10}$$

Now, if one considers the equilibrium surface corresponding to a zero angular acceleration about the pitch axis, i.e. $\dot{q} = f_2(\alpha, M, \delta) = 0$, one gets

$$\delta = -[a_m\alpha^3 + b_m\alpha^2 + c_m(8M/3 - 7)\alpha]/d_m. \tag{11}$$

The parameter-dependent missile state-space matrices are obtained by substituting the value of δ given in (11) into the evolution matrix given in (9). The trigonometric function $\cos(\alpha)$ in (6) is approximated by a 2^{nd} order Taylor series at the mid-range value $\alpha_0 = 17.18°$, i.e.:

$$\cos(\alpha) \approx \cos(\alpha_0) - \sin(\alpha_0)(\alpha - \alpha_0) - \frac{1}{2}\cos(\alpha_0)(\alpha - \alpha_0)^2.$$

With this approximation, the state-space matrices entries are multivariate polynomials in M and α and hence they have linear fractional representations. In this model, the mid-value point for Mach number is taken as $M_0 = 3$ and M is such that $M = M_0 + \bar{M}$, $|\bar{M}| \le 1$. Similarly, the mid-value point for the angle of attack is selected as $\alpha_0 = 17.18°$ and α is such that $\alpha = \alpha_0 + \bar{\alpha}$, $|\bar{\alpha}| \le 17.18°$. In this representation, the parameter vector is $\theta = [\bar{\alpha}, \bar{M}]^T$. With the actuator dynamics added, the linear fractional missile representation $G_p(\theta)$ has 4 states, $\Delta(\theta) = \mathrm{diag}(\bar{\alpha}I_4, \bar{M}I_7)$ and the polypope of matrices $\boldsymbol{\Delta}$, which contains all the possible values of Δ, has 4 vertices corresponding to the 4 combinations of the extreme values of $\bar{M}$ and $\bar{\alpha}$. For more detail on how such LFR model is numerically constructed the reader is referred to the LFR Toolbox documentation [6].

C. Feedforward Synthesis

A second order response is required for the closed-loop transfer $\mathcal{T}_{\eta r}$. To meet the tracking requirements, the reference model was chosen as

$$T_M = \frac{-0.05s + 1}{s^2/\omega_{n_a}^2 + 2s\xi_a/\omega_{n_a} + 1}$$

where $\xi_a = 1$ and $\omega_{n_a} = 14$ rad/s. Note that an instable zero was added to T_M. This is because the vertical acceleration channel presents a non minimum phase characteristic. If the plant is non minimum phase, adding an instable zero in the reference model turns out to be useful in preventing unnecessary control activity about the frequency of the unstable zero. The weights were chosen as:

$$W_1 = \frac{120}{s + 0.01}, \quad W_2 = \frac{0.15s + 0.006}{0.00625s + 2}.$$

The tuning of the weights is similar to that used in a standard mixed-sensitivity $\mathcal{H}_\infty$ controller synthesis, see e.g. [14] for more information on the selection and the tuning of frequency performance weights. W_1 was selected to ensure a small steady-state error and to enforce tracking performance in the frequency range $[0, 50]$ rad/s approximately and W_2 is used to penalize the fin deflection. Because the tracking requirement is on the vertical acceleration, $E = [1, \quad 0]$.

With the weighting functions defined above, the LMI optimisation synthesis algorithm of Section IV gives an $\mathcal{L}_2$ performance of about 5.32 and the corresponding full-information control law is:

$$\begin{aligned} F_x &= 1000 \times [-1.4770, -5.3286, 0.2587, 9.2841, \ldots \\ &\quad -61.2416, -2.0663, -2.8142, -9.4081], \\ F_w &\approx 0. \end{aligned}$$

D. LPV Feedforward Time Responses

The time responses of the feedforward filter, for 10 values of Mach numbers in the interval $[2, 4]$ and for a $-12g$ step demand in the vertical acceleration are shown in Figure 2. We can see that the time responses meet the requirements for Mach numbers higher than 2.2. The responses for Mach numbers less than 2.2 are just slightly slower than required. Observe that angle of attack and control deflection increase as Mach number decreases. This is because the control surfaces, at low mach number, are less effective than they are at high Mach numbers. The responses of Figure 2 compare extremely well with the responses given in [6], [2], [8]. As seen in Figure 2, the feedforward filter responses wary non-linearly with the parameters. They reflect the physical non-linear changes in the controlled missile dynamics. Therefore, it is expected that such a parameter-dependent feedforward filter will provide superior tracking performance than it would normally be the case with a simple low-pass LTI filtering strategy of the acceleration demand.

E. Full Non-linear Missile Time Responses with a Two-degree-of-freedom LPV/LFT Controller

An LPV/LFT output feedback controller $K(\Delta)$ was designed according to the LPV synthesis given in [11]. The

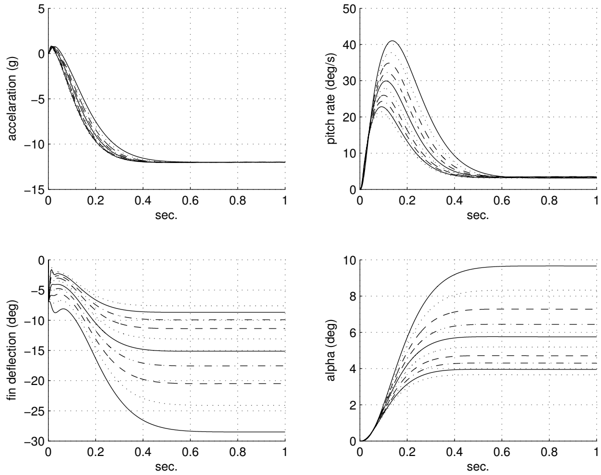

Fig. 2. Feedforward LFV/LFT filter responses to a step demand of -12g; Acceleration [g] and corresponding pitch rate $[\deg.\mathrm{s}^{-1}]$, fin deflection [deg] and angle of attack [deg], for 10 values of Mach number in the interval $[2,4]$.

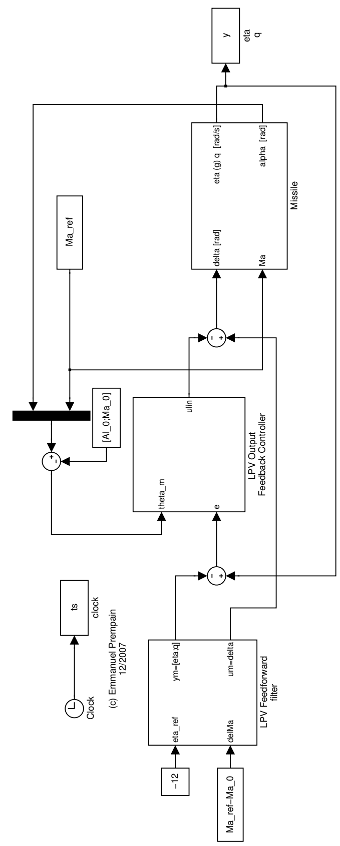

Fig. 3. Full non-linear Simulink simulation model. The 2dof LPV controller is made of the association of the LPV/LFT feedforward filter designed in this Section and of an LPV/LFT output feedback regulator designed according to the synthesis algorithms given [11]. The missile block implements the non-linear missile equations given at the beginning of Section V. In addition, the missile block includes actuator and sensors models.

LPV synthesis in [11] extends the loop shaping design procedure of McFarlane and Glover to LPV/LFT plants, takes advantage of the full knowledge of the parameter polytope's description and makes use of unstructured multipliers allowing the least possible conservatism if arbitrarily fast parameter variation is allowed. The LPV output feedback regulator was obtained with the pre and post open-loop compensators $W_{1s} = \frac{150}{s+150}$ and $W_{2s} = \mathrm{diag}(\frac{5}{s}, 0.6)$, see [11] for details. The one degree-of-freedom controller $K(\Delta)$ provides fast responses across the full flight envelop. However, the closed-loop responses provided by $K(\Delta)$ suffer from large overshoots, especially in the acceleration channel. To overcome this problem, we are going to use the feedforward filter of this section. Figure 3 shows the Simulink structure of the whole control system based on the two-degree-of-freedom (2dof) LPV/LFT controller made of $K(\Delta)$ and $FW(\Delta)$. Note that the feedforward filter is outside the feedback loop and so does not require the missile output measurements. Actually, $FW(\Delta)$ uses its internal states, in this case $\bar{\alpha}$, and the external parameter $\bar{M}$ to update (schedule) its state-space matrices.

The non-linear missile responses obtained with the 2dof LPV/LFT controller of Figure 3 are given in Figure 4. The non-linear responses are similar to those obtained with the LPV feedforward filter of Figure 2. But, the responses obtained with the two-degree-of-freedom LPV controller are considerably better than those obtained with the LPV output feedback regulator alone (those are not shown here due to a lack a space). Because the feedforward filter is outside the feedback loop (Figure 3), the tracking performance improvement, due to $FW(\Delta)$, does not affect the closed-loop stability margins.

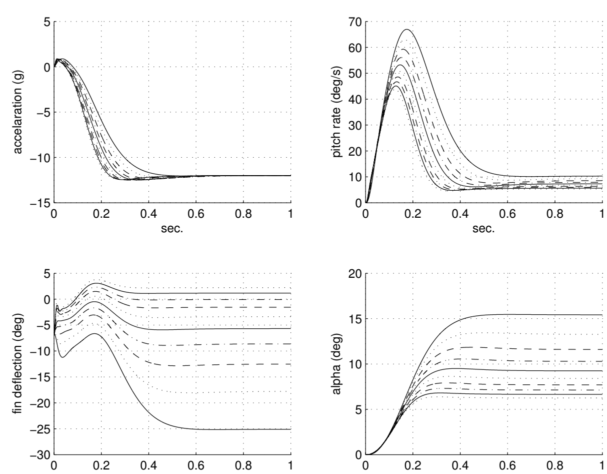

Fig. 4. Non-linear missile responses to a step demand of -12g obtained with the 2dof LPV/LFT controller of Figure 3, for 10 values of Mach number in the interval $[2,4]$.

VI. CONCLUSIONS

In this paper, a simple synthesis of the feedforward filter of two-degree-of-freedom LPV/LFT controller was presented.

The proposed synthesis is also extremely simple to implement and has the following merits:

- The feedforward filter synthesis requires only the plant model. No information on the feedback controller is required. This is particularly useful if one considers the usual complexity associated with most output feedback gain-scheduled controllers.
- The feedforward filter synthesis reduces to solving an $\mathcal{L}_2$ full-information control problem. This makes the feedforward filter synthesis especially attractive from a numerical point of view.
- Because the LPV/LFT feedforward filter dynamics vary with the parameters, superior tracking performance can be achieved over more conventional LTI filtering strategies.

In the missile autopilot example, almost all the autopilot objectives were achieved by adding an LPV/LFT feedforward filter to the LPV feedback loop, however, we believe that all of the objectives could be met with parameter-dependent weighting functions.

VII. ACKNOWLEDGMENTS

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VIII. APPENDIX

A. State-Space Matrices for Feedforward Tracking Synthesis

Suppose that W_1, W_2, G_p and T_M are parameter-dependent systems with linear factional representations S_1, S_2, S_3 and S_4 respectively, where the S_j's, for $j = 1, \ldots, 4$, are defined as:

$$S_j : \begin{cases} \dot{x}_j = A_j x_j + B_{qj} q_j + B_{uj} u_j, \\ p_j = C_{pj} x_j + D_{pqj} q_j + D_{puj} u_j, \\ y_j = C_{yj} x_j + D_{yqj} q_j + D_{yuj} u_j, \\ q_j = \Delta_j(t) p_j, \end{cases} \tag{12}$$

Define $x^T = [x_1^T, x_2^T, x_3^T, x_4^T]$, $p^T = [p_1^T, p_2^T, p_3^T, p_4^T]$, $q^T = [q_1^T, q_2^T, q_3^T, q_4^T]$, $z^T = [z_1^T, z_2^T]$, $w = r$ and $\Delta(t) = \mathrm{diag}(\Delta_1(t), \ldots, \Delta_4(t))$. Using the annotations of Figure 1, with $d = 0$, we have $u_2 = u_3$, $u_3 = u_m$, $u_4 = r$, $z_1 = y_1$, $z_2 = y_2$, $y_m = y_3$, $u_1 = E(y_3 - y_4)$. It is easy to show that the state-space matrices of the open-loop full-information interconnection are given by:

$$A = \begin{bmatrix} A_1 & 0 & B_{u1}EC_{y3} & -B_{u1}C_{y4} \\ 0 & A_2 & 0 & 0 \\ 0 & 0 & A_3 & 0 \\ 0 & 0 & 0 & A_4 \end{bmatrix},$$

$$B_q = \begin{bmatrix} B_{q1} & 0 & B_{u1}ED_{yq3} & -B_{u1}D_{yq4} \\ 0 & B_{q2} & 0 & 0 \\ 0 & 0 & B_{q3} & 0 \\ 0 & 0 & 0 & B_{q4} \end{bmatrix},$$

$$B_w = \begin{bmatrix} -B_{u1}D_{yu4} \\ 0 \\ 0 \\ B_{u4} \end{bmatrix}, \quad B_u = \begin{bmatrix} B_{u1}ED_{yu3} \\ B_{u2} \\ B_{u3} \\ 0 \end{bmatrix},$$

$$C_p = \begin{bmatrix} C_{p1} & 0 & D_{pu1}EC_{y3} & -D_{pu1}C_{y4} \\ 0 & C_{p2} & 0 & 0 \\ 0 & 0 & C_{p3} & 0 \\ 0 & 0 & 0 & C_{p4} \end{bmatrix},$$

$$D_{pq} = \begin{bmatrix} D_{pq1} & 0 & D_{pu1}ED_{yq3} & -D_{pu1}D_{yq4} \\ 0 & D_{pq2} & 0 & 0 \\ 0 & 0 & D_{pq3} & 0 \\ 0 & 0 & 0 & D_{pq4} \end{bmatrix},$$

$$D_{pw} = \begin{bmatrix} -D_{pu1}D_{yu4} \\ 0 \\ 0 \\ D_{pu4} \end{bmatrix}, \quad D_{pu} = \begin{bmatrix} D_{pu1}ED_{yu3} \\ D_{pu2} \\ D_{pu3} \\ 0 \end{bmatrix},$$

$$C_z = \begin{bmatrix} C_{y1} & 0 & D_{yu1}EC_{y3} & -D_{yu1}C_{y4} \\ 0 & C_{y2} & 0 & 0 \end{bmatrix},$$

$$D_{zq} = \begin{bmatrix} D_{yq1} & 0 & D_{yu1}ED_{yq3} & -D_{yu1}D_{yq4} \\ 0 & D_{yq2} & 0 & 0 \end{bmatrix},$$

$$D_{zw} = \begin{bmatrix} -D_{yu1}D_{yu4} \\ 0 \end{bmatrix}, \quad D_{zu} = \begin{bmatrix} D_{yu1}ED_{yu3} \\ D_{yu2} \end{bmatrix}.$$

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