# An Initial Study of a Combined Robust and Adaptive Control Approach to Guaranteed Performance Flight Control

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Abstract-In this paper an initial study is presented regarding a new combined robust and adaptive control approach to fault-tolerant flight control. The approach is based on designing a suitable tracking-error feedback (TEF) term such that the plant is stabilized over the entire uncertainty set. If such a term can be found, the adaptive control part depends only on the reference model state and the reference input. Hence, instead of dealing with a nonlinear time-varying system arising in the context of standard adaptive control, the designer needs to analyze a stable linear time-varying system. This is an important step toward the development of effective analysis tools for performance of adaptive systems. In the case when the tracking error feedback term stabilizes the plant only over a subset of the uncertainty set, the TEF term can still be used to minimize the tracking error so that a linear time-varying approximation is close to the original system. The proposed approach is illustrated on an example system through analytic development and simulations.

## I. INTRODUCTION

Over the past two decades, there has been increased interest in the applications of adaptive control to aerospace systems to enhance their performance and safety. This is primarily due to the fact that it was long recognized that adaptive control has a potential to solve difficult problems associated with control design to handle modeling uncertainties resulting from imprecise knowledge of plant parameters in different flight regimes, modeling simplifications, unanticipated events, faults, failures, and changes due to structural damage.

Adaptive systems are inherently nonlinear, time-varying, and difficult to tune, and often need to be retuned with each change of the operating regime. Such systems exhibit bursting, and also *inconsistent transient performance* in the sense that different changes of a single uncertain plant parameter can cause very different transient response of the overall adaptive control system.

Due to their nonlinear nature and complex transient performance, stability and performance guarantees similar to those for linear feedback systems are not available in the context of adaptive systems. For these reasons, it is very difficult to expect flight control systems to be flight certified if the existing adaptive control strategies are used. *Hence there is a great need for new adaptive control designs that achieve guaranteed transient performance and are flight certifiable.*  In order to improve flight safety, adaptive systems with guaranteed performance need to be designed for situations characterized by the presence of large uncertainties such as those arising due to severe flight-critical failures, faults, and aircraft structural damage.

**State-of-the-Art:** In the last several years there has been increased interest in the use of adaptive control techniques in the context of fault-tolerant and reconfigurable flight control in the presence of faults and failures of aircraft subsystems and components [1], [20], [5], [19], [9], [10]. Most of the techniques are derived from standard direct or indirect adaptive control (see e.g. [11]). However, in all cases very little can be said about transient performance of any of the proposed techniques even in the ideal case (i.e. in the case without noise, external disturbances and/or unmodeled dynamics). Hence flight control systems employing the existing techniques cannot be expected to be flight certified.

One of the features of standard adaptive control systems is that their stability analysis results in guaranteed (uniform) stability and asymptotic convergence of the tracking error to zero. In addition, if the signals in the system are Persistently Exciting (P.E.), asymptotic convergence of the parametric errors to zero is also guaranteed. However, in both cases nonconservative error bounds are highly difficult to calculate. As a result, at present very little is known about the transient performance of adaptive systems.

It is well known that both transient and steady-state performance of adaptive control systems depend critically on many factors including:

• Command signals and their level of excitation.

• Size of parametric uncertainty, and generally unknown initial conditions of the parameter estimates.

• Choice of the adaptive algorithm for parameter adjustment (for instance, response obtained using the "pure" gradient algorithm can be very different from that obtained using the gradient algorithm with projection; response obtained using Recursive Least Squares (RLS) is different than that using gradient adaptation, etc.).

• Tuning parameters such as adaptive gains, estimator gains in the case of indirect adaptive control, or Lyapunov matrix

- P in the case of direct adaptive control.
- Control objective (as, for instance, specified by a suitably chosen reference model)
- Control input constraints, including position and rate limits, and actuator dynamics.
- Signal delay, and
- External disturbances, unmodeled dynamics and noise.

One of the well known features of adaptive control systems is that, in a few cases, tuning procedures can be made more sophisticated than those based on trial and error. This is due to the fact that the response of adaptive control systems is very difficult to calculate, and the effect of tuning parameters, initial conditions and signal excitation cannot be predicted in advance.

Adaptive Control Metrics: In the adaptive control field, there are virtually no results related to the performance of general adaptive systems, and to the related metrics. One of the reasons is that, from the Lyapunov analysis, mainly highly conservative bounds can be derived assuming that the parametric error is maximum at all time. In addition, in the case of P.E. signals, exponential convergence is assured [11], however non-conservative bounds and rate of convergence are extremely difficult to calculate. This is even more pronounced in the case when the P.E. signals are a part of the closed-loop. Even when the signals are P.E., it is well known that the convergence of the parametric errors to zero is generally very slow due to the stiffness of the corresponding set of differential equations [11].

A relevant recent reference that deals with stability margins for adaptive controllers is [14]. In that paper, it is proposed that the analysis be carried out on a Reduced Linear Asymptotic System (RLAS) in cases when the reference input is constant. The approach from [14] has two major shortcomings: (i) It assumes that the reference input is constant, which makes RLAS linear and time-invariant. However, it is not clear at all as to how to extend the approach to the cases when the reference input is time-varying. In such a case the meaning of the stability margin calculated for the linear time-invariant RLAS in the context of the true nonlinear time-varying adaptive system becomes obscure; (ii) The RLAS does not, in general, capture main features of the transient behavior of the adaptive system. Since the adaptive system will converge to RLAS asymptotically, the latter is a valid approximation only at the steady state (for constant reference inputs), while its transient response may be very far from that of the original adaptive system. Keeping in mind that it is the transient response of the adaptive system that could lead to excitation of unmodeled dynamics or control input saturation, only using the steady-state analysis fails to accurately predict transient performance. Hence alternate methods are needed to arrive at meaningful performance metrics for adaptive systems.

Flight control systems have several favorable features that could potentially be used in gaining better understanding of the transitent performance of adaptive systems:

- System states are measurable so that simpler adaptive control techniques for such systems can be used;
- Sometimes accelerations are also measurable. A question that arises in this context is as to how could this be used in improving the transient performance and arriving at the related metrics.

• Flight control commands belong to a relatively small set (ramped step, ramped pulse, doublet, etc.).

• Linear models of aircraft dynamics in fixed flight regimes are fairly well known, and can be scheduled with the flight regime.

Regarding the latter, if there is fault/failure/damage, the models may be no longer valid. In addition, unmodeled dynamics may become non-negligible (e.g. aeroservoelastic modes can be excited). Damaged aircraft also exhibits loss of symmetry, and coupling between the longitudinal and lateral dynamics. This may result in the loss of control leading to a crash. *Hence nonlinear adaptive control strategies are of great interest in practice.* 

**Proposed Approach:** The proposed approach is shown in a flowchart in Figure 1 and described below:



Fig. 1. Flowchart of the Proposed Approach

• The proposed approach combines robust control with adaptive control. The latter can be direct, indirect, or combined direct and indirect. The resulting robust adaptive control system should achieve good tracking performance and result in a small tracking error. • Based on the small tracking error, the adaptive system can now be approximated by a linear time-varying (LTV) system. If a robust controller can be found that stabilizes the plant over the entire parametric set, the LTV system is the exact model of the adaptive control system.

• The linear time-varying system can now be tuned over a set of reference inputs and initial conditions based on its analytic response using a suitable technique such as Genetic Algorithms or Monte-Carlo simulations. The tuning continues until a performance criterion, based on the Adaptive Control Metrics specifications, is minimized.

As a result, a Guaranteed Performance Adaptive Control (GPAC) system is obtained that satisfies performance specifications.

In the paper we carry out an initial study of the combined robust and adaptive control approach as described below.

## **II. PROBLEM STATEMENT**

As it is well known, major problems related to the performance of adaptive systems are due to the fact that the resulting closed-loop system is nonlinear and time-varying. To alleviate these problems, a new adaptive control structure is proposed that results in a linear time-varying closed-loop system. The main idea of the approach is described below.

Let the model of the plant dynamics be of the form:

$$\dot{x} = A(p)x + BK(p)u,\tag{1}$$

where  $x \in \mathbb{R}^n$  and  $u \in \mathbb{R}^m$  denote respectively the system state and control input vectors, A and B are matrices of appropriate dimensions,  $K = \text{diag}[k_1 \ k_2 \ ... \ k_m], \ k_i > 0, \ i = 1, 2, ..., m, p \in \mathbb{R}^l$  is a vector of uncertain parameters such that  $p \in \mathcal{S}_p = \{p : (p_i)_{min} \le p_i \le (p_i)_{max}, \ i = 1, 2, ..., l\}$ , where  $(p_i)_{min}$  and  $(p_i)_{max}$  are known. Last m elements of p are elements of the diagonal matrix K. In the above equation, matrices A and K are uncertain while B is assumed known.

The objective is to design a control input u(t) so that the output of the uncertain plant follows asymptotically the output of the following reference model:

$$\dot{x}_m = A_m x_m + B_m r,\tag{2}$$

where  $A_m$  is an a.s. matrix, and r is a vector of bounded piecewise continuous reference input.

#### **III. COMBINED ROBUST AND ADAPTIVE CONTROL**

The approach proposed in this paper is based on specifying u as:

$$u = u_a + u_{tef},\tag{3}$$

where  $u_a$  is the adaptive control input of the form:

$$u_a = \Theta x_m + K_c r, \tag{4}$$

while  $u_{tef}$  denotes the tracking error feedback. The latter is chosen as:

$$u_{tef} = \Lambda(x - x_m),\tag{5}$$

where  $\Lambda$  is an a.s. matrix. From (1), (2), and (3)-(5) it follows that:

$$\dot{x} = (A(p) + BK(p)\Lambda)e + (A(p) + BK(p)\Theta)x_m + BK(p)K_cr,$$

where  $e = x - x_m$  denotes the tracking error.

The adaptive control design proceeds by defining  $\Theta^*$  and  $K_c^*$  such that:

$$A + BK\Theta^* = A_m, \ BKK_c^* = B_m.$$
(6)

Let  $\Phi = [(\Theta - \Theta^*)^T (K_c - K_c^*)^T]^T$  and  $\omega(x_m, r) = [x_m^T r^T]^T$ . Then the *Error Equation* for the above system is of the form:

$$\dot{e} = \bar{A}e + BK\Phi\omega(x_m, r)$$

where  $\bar{A} = A(p) + BK(p)\Lambda$ .

We note that, in contrast to the error equations in standard adaptive control that are nonlinear and time-varying [11], the latter equation is *linear and time-varying* since  $\omega$  depends on  $x_m(t)$  and r(t).

However, unlike the standard adaptive control case [11], matrix  $\bar{A}(p) = A(p) + BK(p)\Lambda$  is unknown. This is an important issue in direct adaptive control since: (i) This matrix needs to be stable in order to prove the overall system stability; and (ii) Lyapunov matrix P related to  $\bar{A}(p)$  is needed to implement the adaptive control laws.

To address this problem, we propose the following:

1. Choose the matrix  $\Lambda$  such that  $\bar{A}(p)$  is a.s. for all  $p \in \bar{S}_p \subseteq S_p$ , and some performance specifications are met (e.g.  $H^{\infty}$  or  $H^2$ );

2. Use the tools from Robust Stability to arrive at a matrix P such that the Lyapunov Matrix Inequality is satisfied over the parametric set, i.e.

$$\bar{A}^T(p)P + P\bar{A}(p) \le -Q, \ \forall p \in \bar{\mathcal{S}}_p,\tag{7}$$

3. If  $\bar{S}_p = S_p$ , express the resulting adaptive control system as a linear time-varying system.

4. If  $\bar{S}_p \subset S_p$ , approximate the resulting adaptive control system by a linear time-varying system.

In this paper we will consider two cases:

**Case 1 (Exact LTV Solution):** There exists a matrix  $\Lambda$  such that  $A(p) + B(p)\Lambda$  is asymptotically stable for all  $p \in S_p$ ; and

**Case 2 (Approximate LTV Solution):**  $\Lambda$  stabilizes  $A(p) + B(p)\Lambda$  over a subset of  $S_p$ , i.e. when  $p \in \overline{S}_p \subset S_p$ .

We will illustrate the proposed approach on an example of a second-order plant.

### A. Example of Exact LTV Solution

Let the plant be of the form:

$$\dot{x}_1 = x_2 \tag{8}$$

$$\dot{x}_2 = a_1 x_1 + a_2 x_2 + bu \tag{9}$$

where  $x = [x_1 \ x_2]^T$  denotes the state, u is the control input, and  $p = [a_1 \ a_2 \ b]^T \in S_p$  where

$$S_p = \{(a_1, a_2, b) : -2 \le a_i \le 2, \ i = 1, 2; \ 0.2 \le b \le 1\}.$$

In this case the reference model is of the form:

$$\dot{x}_{m1} = x_{m2} \tag{10}$$

$$\dot{x}_{m2} = -k_1 x_{m1} - k_2 x_{m2} + k_1 r, \qquad (11)$$

where  $k_i > 0$ , while the control law is of the form:

$$u = \theta_1 x_{m1} + \theta_2 x_{m2} + k_c r - \lambda^T (x - x_m),$$
(12)

where  $\lambda = [\lambda_1 \ \lambda_2]^T$  and  $\lambda_i > 0$ .

Upon substituting the control law into the plant equation and subtracting the reference model one obtains:

$$\dot{x}_1 = x_2 \tag{13}$$

$$\dot{x}_2 = (a_1 - b\lambda_1)e_1 + (a_2 - b\lambda_2)e_2 + (a_1 + b\theta_1)x_{m1} + (a_2 + b\theta_2)x_{m2} + bk_c r, \quad (14)$$

where  $e_i = x_i - x_{mi}$ , i = 1, 2.

By noting that  $\theta_i^*$  and  $k_c^*$  exist such that

$$a_1 + b\theta_1^* = -k_1, \quad a_2 + b\theta_2^* = -k_2, \quad bk_c^* = k_1,$$

and defining  $\phi = [\theta_1 - \theta_1^* \ \theta_2 - \theta_2^* \ k_c - k_c^*]$  and  $\omega = [x_{m1} \ x_{m2} \ r]^T$ , the above equation can be expressed as:

$$\dot{e} = \begin{bmatrix} 0 & 1\\ a_1 - b\lambda_1 & a_2 - b\lambda_2 \end{bmatrix} e + \begin{bmatrix} 0\\ b \end{bmatrix} \phi^T \omega(x_m, r).$$

Choice of matrix  $\Lambda$ : We note that the closed-loop system will be stable for all  $\lambda_1 \geq (a_1)_{max}/b_{min} = 10$  and all  $\lambda_2 \geq (a_2)_{max}/b_{min} = 10$ .

Let us assume that an additional performance specifications are that: (i)  $\lambda_1 b - a_1 \ge 1$ , and (ii)  $\lambda_2 b - a_2 \ge 1.4$ . Then:

$$\lambda_1 \ge \frac{a_1 + 1}{b} \ge 15, \quad \lambda_2 \ge \frac{a_2 + 1.4}{b} \ge 17.$$

We now choose  $\lambda_2 = 25$ , and  $\lambda_1 = 15$ .

Finding matrix P: We next express  $\overline{A}$  as:

$$\bar{A}(p) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} a_1 + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} a_2 + \begin{bmatrix} 0 & 0 \\ -\lambda_1 & -\lambda_2 \end{bmatrix} b$$

and use the function quadstab from the Matlab LMI Toolbox [3] to obtain:

$$P = \begin{bmatrix} 567.1302 & 154.0955\\ 154.0955 & 278.3873 \end{bmatrix}.$$
 (15)

This is the matrix P that will be used in the adaptive laws.

<u>Adaptive laws</u>: Let  $\overline{P} = P[0 \ 1]^T$ , and  $\theta = [\theta_1 \ \theta_2 \ k_c]^T$ . Adaptive laws are chosen in the form:

$$\dot{\theta} = \dot{\phi} = -\operatorname{sign}(b)\Gamma\omega\bar{P}^{T}e,$$
 (16)

where  $\Gamma = \Gamma^T > 0$ .

**Stability Analysis:** The following tentative Lyapunov function is chosen:

$$V(e,\phi) = \frac{1}{2} [e^T P(p)e + |b|\phi^T \Gamma^{-1}\phi].$$
 (17)

Its first derivative along the solutions of (15), (16) yields:

$$\dot{V}(e,\phi) \le -\frac{1}{2}q_{min}(Q)||e||^2 \le 0,$$

where  $Q = P^{-1}$ , and  $q_{min}(Q)$  is the minimum eigenvalue of Q. It follows that e and  $\phi$  are bounded. Since  $x_m$  is bounded, this implies that x is bounded as well, and that u is also bounded. Upon integrating  $\dot{V}$  from 0 to  $\infty$  one obtains:

$$V(0) - V(\infty) \ge \frac{1}{2} q_{min}(Q) \int_0^\infty \|e(\tau)\|^2 d\tau.$$

Hence  $e \in \mathcal{L}^2$ . Since  $\dot{e}$  can be readily shown to be bounded, it follows that  $\lim_{t\to\infty} e(t) = 0$ .

Let 
$$z = [e^T \ \phi^T]^T$$
. Then the origin of  $\dot{z} = M(t)z,$ 

is uniformly stable, and  $\lim_{t\to\infty} z_1(t) = 0$ . Let also  $\bar{b} = [0 \ b]^T$ . Matrix M(t) is defined as:

$$M(t) = \begin{bmatrix} \bar{A} & \bar{b}\omega^T(x_m(t), r(t)) \\ -\operatorname{sign}(b)\Gamma\omega(x_m(t), r(t))\bar{P}^T & 0 \end{bmatrix}$$

Hence the adaptive control system is expressed as a *linear time-varying system*. This parameterization is important for arriving at the perfromance metrics for adaptive control systems.

<u>Simulation</u>: We next simulated the above adaptive system for the reference input of the form of a doublet. The following initial conditions are chosen:  $\theta(0) = [0 \ 0 \ 0]^T$ , and  $\Gamma = \text{diag}([4 \ 4 \ 0.4])$ . The simulation result for  $a_1 = 2, a_2 = 2$  and b = 0.2 is shown in Figure 2.

It is seen that all the signals are bounded, tracking error tends to zero asymptotically, and the overall response is excellent despite the uncertainty that makes the open-loop plant highly unstable and weakly controllable. The response of the controller parameters indicates that there is not enough P.E. in the reference input. However, despite the fact that the controller parameters do not converge to their true values, the



Fig. 2. Response of the Combined Adaptive Control Systems from the Example

system is stable and the tracking performance is excellent.

## B. Example of an Approximate Solution

We next note that, in Case 2, the plant can be stabilized only over a subset of the parametric set. In that case standard adaptive control needs to be used. However, it would still be beneficial to somehow keep the Tracking Error Feedback (TEF) term. The main motivation is *to assure that, by joint work of the adaptive controller and the TEF, the tracking error will be small over the entire time interval*. In such a case, a linear time-varying (LTV) approximation of the adaptive system may be adequate.

However, the issue here is that the TEF term will be multiplied by unknown vector b and will, therefore be unknown. For this reason we present an approach that assures that the TEF term is known by adjusting an additional parameter. Now that term can be chosen as a tradeoff between tracking accuracy and undesired high-gain effects.

The main idea is to modify the control law as follows:

$$u = \theta^T x + l\lambda^T e + k_c r, \tag{18}$$

i.e. a term containing a product of an adjustable parameter and TEF is added to the control law.

The resulting closed-loop system is now of the form:

$$\dot{x} = \left(A + \bar{b}\theta^{*T} + \bar{b}l^*\lambda^T\right)e + \bar{b}\phi^T\omega + \left(A + \bar{b}\theta^{*T}\right)x_m + \bar{b}k_c^*r,$$

resulting in the following matching conditions:

$$A + \bar{b}\theta^{*T} = A_m, \ \bar{b}k_c^* = b_m, \ \begin{bmatrix} 0\\1 \end{bmatrix} = \bar{b}l^*$$
(19)

Upon defining  $\phi = [\theta - \theta^* \ l - l^* \ k_c - k_c^*]^T$ , we obtain the following error equation:

$$\dot{e} = \bar{A}e + \bar{b}\phi^T\omega,\tag{20}$$

where  $\omega = [x_1 \ x_2 \ \lambda^T e \ r]^T$ ,  $\phi = [\phi_\theta \ \phi_l \ \phi_{k_c}]^T$ , and

$$\bar{A} = A_m + \begin{bmatrix} 0\\1 \end{bmatrix} \lambda^T.$$
(21)

The adaptive law is now of the exact same from as (16). The stability of the overall system can be demonstrated along the same lines as in the previous section.

We next show that, in the simple example below, the LTV approximation captures dominant response during transients, while a linear time-invariant approximation results in large transient errors.

We use the same plant from the previous section and implement the above adaptive control procedure resulting in:

$$\dot{z} = \begin{bmatrix} \bar{A} & \bar{b}\omega^T(x,r) \\ -\operatorname{sign}(b)\Gamma\omega(x,r)\bar{P}^T & 0 \end{bmatrix} z.$$
(22)

The response of the system is shown in Figure 3, and that of the tracking error from the nonlinear adaptive system is shown in Figure 4 in blue. Linear time-varying approxima-



Fig. 3. Response of the Adaptive Control Systems from the Example

tion is obtained when x is replaced by  $x_m$  in the  $\omega$  term above. The resulting response of the tracking error is shown in the same figure in red. We also simulated a linear system obtained when fixed controller parameters are used based on values to which the adaptive controller parameters have converged. It is seen that the LTI approximation completely fails to to capture main transient response properties. We also note that it was found that, in most cases, the LTV system accurately predicts the values to which the controller parameters converge, and achieves response close to that of the original system.

Hence, even in the case when we cannot find a matrix  $\Lambda$  to stabilize all plants from a given set, using the TEF compensator can reduce the tracking error and assure that the LTV approximation is close to the original adaptive system.



Fig. 4. Response of the tracking error of the original adaptive system and its LTV and LTI approximations

## **IV. CONCLUSIONS AND FUTURE WORK**

In this paper an initial study is presented regarding a new combined robust and adaptive control approach to faulttolerant flight control. The approach is based on designing a suitable tracking-error feedback (TEF) term such that the plant is stabilized over the entire uncertainty set. If such a term can be found, the adaptive control part depends only on the reference model state and the reference input. Hence, instead of having to deal with a nonlinear time-varying system arising in the context of standard adaptive control, the designer needs to analyze a stable linear time-varying system. This is an important step toward the development of effective analysis tools for performance of adaptive systems. In the case when the tracking error feedback term stabilizes the plant only over a subset of the uncertainty set, the TEF term can still be used to minimize the tracking error so that a linear time-varying approximation is close to the original system. The proposed approach is illustrated on an example system through analytic development and simulations.

While in the paper the proposed approach is illustrated on a second-order example, future work will include the extensions to MIMO uncertain plants, and actual study of the (exact or approximate) linear time-varying systems arising from the combined approach, to find the performance bounds and the relationship between the system response and free-design parameters, level of persistent excitation, and reference inputs.

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