# Pricing Stock Options in Mergers and Acquisitions with Jump-diffusion Model 

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Abstract - We develop a jump-diffusion model to price options on the stocks involved in mergers and acquisitions. The test results indicate that our model performs well in explaining observed option prices. The model can be used by risk arbitrageurs to control risks associated with merger deals using options.

Key words: mergers and acquisitions, option pricing, jump diffusion, risk arbitrage

## I. INTRODUCTION

There are extensive researches on the behavior of implied volatilities around merger or acquisition announcements ([5], [11], [12]), empirical studies of excess return of risk arbitrage in merger and acquisitions ([1], [7], [10]), and investigations on the informational content of speculation spread ([2], [6]).

There has been little work done so far in valuing options on the stocks of firms involved in a merger deal. BaroneAdesi, Brown, and Harlow [3] used option prices of the bidder and especially the target to predict the market's expectation of merger deal's consummation. However, their objective was not to build a pricing model. Subramanian [14] developed a theoretical framework in valuing options on the stocks of firms involved in a merger deal after the merger announcement and before the deal either goes through or is called off. In his model the jump size is a monotonic function of time, which is inconsistent with the market observations.

In this paper, a jump-diffusion model to determine the option prices when a merger or acquisition deal is pending is developed in section II. Section III shows the results of numerical implementation. Section IV concludes this paper.

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## II. A JUMP DIFFUSION MODEL

## A. Model Setup

We consider a probability space $(\Omega, F, P)$ equipped with a complete, right continuous filtration $\left\{F_{t}\right\}$. We assume that the probability space is large enough to accommodate four $F_{t}$-adapted Wiener processes $\mathrm{W}_{1}, \mathrm{~W}_{2}, \mathrm{~W}_{3}, \mathrm{~W}_{4}$, and two $F_{t}$ adapted single jump processes $\mathrm{Q}_{1}, \mathrm{Q}_{2}$. The jump processes $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$ are independent of the Brownian motions and have identical parameter $\lambda$. The Brownian Motions $\mathrm{W}_{1}, \mathrm{~W}_{2}$ are independent of each other and the Brownian Motions $W_{3}, W_{4}$. However, $W_{3}$ and $W_{4}$ may have a nonzero correlation.

For simplicity, we assume the stocks pay no dividend. Let $\mathrm{S}_{1}$ and $\mathrm{S}_{2}$ be the underlying asset prices of the acquirer and target company. Before the deal is announced and after the deal is called off, the stock prices are assumed to satisfy the Black-Scholes model:

$$
\begin{align*}
& d S_{1}=\mu_{1}^{\prime} S_{1} d t+\sigma_{1}^{\prime \prime} S_{1} d W_{3}  \tag{1}\\
& d S_{2}=\mu_{2}^{\prime} S_{2} d t+\sigma_{2}^{\prime} S_{2} d W_{4}  \tag{2}\\
& d W_{3} d W_{4}=\rho d t \tag{3}
\end{align*}
$$

Where $\mu_{1}^{\prime}$ and $\mu_{2}^{\prime}$ are the instantaneous expected return on stock of the acquirer and target company; $\sigma_{1}^{\prime \prime}$ and $\sigma_{2}^{\prime \prime}$ are the instantaneous variance of the return; $\rho$ is the correlation between $\mathrm{W}_{3}$ and $\mathrm{W}_{4} ; \mu_{1}^{\prime}, \mu_{2}^{\prime}, \sigma_{1}^{\prime \prime}, \sigma_{2}^{\prime \prime}, \rho$ are assumed to be constant and $-1<\rho<1$.
We assume that a merger or acquisition announcement is made at time $\mathrm{t}=0$ and the deal is expected to go through at the time horizon T. However, there is a nonzero probability that the deal could be called off at some time in the interval $[0, \mathrm{~T})$ and the effects of this breakdown process are the jump processes $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$. The jump sizes are random variables $Y_{1}$ and $Y_{2}$. The risk-neutral probability that the deal is called off during a time interval $[t, t+d t] \subset[0, T)$ provided it has not yet been called off is $\lambda d t$. Therefore, the risk-neutral probability that the deal will be called off at some time before the deal date is $1-\exp (-\lambda T)$. The risk-neutral probability that there is no jump in $[0, T)$, or alternatively the risk-neutral probability that the deal is successful, is $\exp (-\lambda T)$.
We consider the scenario where the terms of the merger deal are such that each shareholder of the target firm will receive $\alpha$ alpha share of the stock of the acquirer firm. If the deal goes through, we have

$$
\begin{equation*}
S_{2}(T)=\alpha S_{1}(T) \tag{4}
\end{equation*}
$$

We also assume that the terms of the merger deal allow an option with maturity $\tau$ and strike K of the target company to be exchanged for $\alpha$ option with maturity $\tau$ and strike $\mathrm{K} / \alpha$ of the acquirer company when the deal goes through.

The two stock price processes when the merger has been announced and has not yet been called off are assumed to satisfy:

$$
\begin{align*}
& d S_{1}=\mu_{1} S_{1} d t+\sigma_{1} S_{1} d W_{1}+d Q_{1}  \tag{5}\\
& d S_{2}=\mu_{2} S_{2} d t+\sigma_{2} S_{2} d W_{1}+\sigma_{2}^{\prime} S_{2} d W_{2}+d Q_{2} \tag{6}
\end{align*}
$$

Where $\mu_{1}$ and $\mu_{2}$ are the instantaneous expected return; $\sigma_{1}, \sigma_{2}$ and $\sigma_{2}^{\prime}$ are the instantaneous variance of the return; $\mu_{1}, \mu_{2}, \sigma_{1}, \sigma_{2}, \sigma_{2}^{\prime}$ are assumed to be constant; $d Q_{1}$ and $d Q_{2}$ are single jump processes with identical parameter $\lambda$.

Under the risk-neutral measure, we must have

$$
\begin{aligned}
& \mu_{1}^{\prime}=\mu_{2}^{\prime}=r \\
& \mu_{1}=r-\lambda k_{1} \\
& \mu_{2}=r-\lambda k_{2}
\end{aligned}
$$

where r is the risk-free interest rate; $k_{i} \equiv E\left(Y_{i}-1\right)$ where $\left(\mathrm{Y}_{\mathrm{i}}-1\right)$ is the percentage change in the stock price if the jump process occurs, $\mathrm{i}=1,2$; and E is the expectation operator over the random variables $\mathrm{Y}_{1}$ and $\mathrm{Y}_{2}$. As in Subramanian [14], it is straightforward to show that $\sigma_{1}=\sigma_{2}$ and $\sigma_{2}^{\prime}=0$ (see appendix A for proof).

In summary, $S_{1}$ and $S_{2}$ satisfy the following stochastic differential equations under the risk-neutral probability for $0 \leq t<T$,

$$
\begin{align*}
d S_{1} & =1_{\{N(t)=0\}}\left[\left(r-\lambda k_{1}\right) S_{1} d t+\sigma_{1} S_{1} d W_{1}\right] \\
& +1_{\{N(t)=1\}}\left[r S_{1} d t+\sigma_{1}^{\prime \prime} S_{1} d W_{3}\right]+d Q_{1}  \tag{7}\\
d S_{2} & =1_{\{N(t)=0\}}\left[\left(r-\lambda k_{2}\right) S_{2} d t+\sigma_{1} S_{2} d W_{1}\right] \\
& +1_{\{N(t)=1\}}\left[r S_{2} d t+\sigma_{2}^{\prime \prime} S_{2} d W_{4}\right]+d Q_{2} \tag{8}
\end{align*}
$$

where $N(t)=0$ represents the scenario when the jump has not occurred and $N(t)=1$ represents the scenario when the jump has occurred.

## B. Pricing of European Option

First we consider the pricing of European call options that expires before the expected merger deal closing date (i.e., maturity time $\tau<T$ ) only. The pricing of longer maturities options (i.e., $\tau>T$ ) could be determined by similar analysis.

Suppose that both option price $\mathrm{H}_{1}$ of the acquirer company and option price $\mathrm{H}_{2}$ of the target company can be written as a twice-continuously differentiable function of the stock price and time. Using Ito's Lemma for the continuous part and an analogous lemma for the jump part (see, for example, [13 chapter 6]), then option prices where the deal has not been called off (i.e., the jump has not occurred) satisfies the following dynamics:

$$
\begin{align*}
& d H_{1}\left(S_{1}, t\right)=\left(H_{1 t}\left(S_{1}, t\right)+H_{1 S}\left(S_{1}, t\right)\left(r-\lambda k_{1}\right) S_{1}+\frac{1}{2} \sigma_{1}^{2} S_{1}^{2} H_{1 S S}\left(S_{1}, t\right)\right. \\
& \left.+\lambda E\left(H_{1}\left(S_{1} Y_{1}, t\right)-H_{1}\left(S_{1}, t\right)\right)\right) d t+H_{1 S}\left(S_{1}, t\right) \sigma_{1} S_{1} d W_{1}+d Q_{H 1} \\
& d H_{2}\left(S_{2}, t\right)=\left(H_{2 t}\left(S_{2}, t\right)+H_{2 S}\left(S_{2}, t\right)\left(r-\lambda k_{2}\right) S_{2}+\frac{1}{2} \sigma_{1}^{2} S_{2}^{2} H_{2 S S}\left(S_{2}, t\right)\right.  \tag{9}\\
& \left.+\lambda E\left(H_{2}\left(S_{2} Y_{2}, t\right)-H_{2}\left(S_{2}, t\right)\right)\right) d t+H_{2 S}\left(S_{2}, t\right) \sigma_{1} S_{2} d W_{1}+d Q_{H 2} \tag{10}
\end{align*}
$$

We form a portfolio $P_{1}$ with stock and option of the target company as $H_{1}+\varpi_{1} S_{1}$. The portfolio $\mathrm{P}_{1}$ has the following dynamics

$$
\begin{align*}
& d P_{1}=\left(H_{1 t}+\left(H_{1 S}+\varpi_{1}\right) r S_{1}-\lambda k_{1} S_{1} H_{1 S}+\frac{1}{2} \sigma_{1}^{2} S_{1}^{2} H_{1 S S}\right. \\
& \left.+\lambda E\left(H_{1}\left(S_{1} Y_{1}, t\right)-H_{1}\left(S_{1}, t\right)\right)\right) d t+\left(H_{1 S}+\varpi_{1}\right) \sigma_{1} S_{1} d W_{1} \\
& +d H_{1 j u m p}+\varpi_{1} d Q_{1} \tag{11}
\end{align*}
$$

In our model setup, the source of the jumps is mergerrelated information, which represents 'non-systematic' risk. Setting $\varpi_{1}=-H_{1 S}$ will eliminate the systematic risk from the portfolio and thus the beta of the portfolio will be zero. As suggested in Merton [9], if the Capital Asset Pricing model holds, then the expected return on all zero-beta securities must equal the risk-free rate. Thus, we have

$$
\begin{align*}
H_{1 t}+ & \frac{1}{2} \sigma_{1}^{2} S_{1}^{2} H_{1 S S}-r H_{1}+\left(r-\lambda k_{1}\right) S_{1} H_{1 S} \\
& +\lambda E\left(H_{1}\left(S_{1} Y_{1}, t\right)-H_{1}\left(S_{1}, t\right)\right)=0 \tag{12}
\end{align*}
$$

subject to the boundary conditions

$$
\begin{align*}
& H_{1}(0, t)=0  \tag{13}\\
& H_{1}\left(S_{1}, \tau\right)=\max \left[0, S_{1}-K\right]
\end{align*}
$$

where K is the strike of the option.
Define $F\left(S, \tau ; K, r, \sigma^{2}\right)$ to be the Black-Scholes option
pricing formula [4]. F can be written as

$$
F\left(S, \tau ; K, r, \sigma^{2}\right)=S N\left(d_{1}\right)-K \exp (-r \tau) N\left(d_{2}\right)
$$

where

$$
N(y) \equiv \frac{1}{\sqrt{2 \pi}} \int_{-\infty}^{y} \exp \left(-s^{2} / 2\right) d s
$$

the cumulative normal distribution function,

$$
d_{1}=\frac{\log (S / K)+\left(r+\sigma^{2} / 2\right) \tau}{\sigma \sqrt{\tau}}
$$

and

$$
d_{2}=d_{1}-\sigma \sqrt{\tau}
$$

Note that there is either exactly one jump or no jump at all. Therefore the value of $H_{1}\left(S_{1}, \tau\right)$ has two components.
The first component represents the possibility that the deal is not called off. The value of the first component is $\exp (-\lambda \tau) F\left(S_{1}, \tau ; K, r-\lambda k_{1}, \sigma_{1}^{2}\right)$. The second component represents the possibility that the deal is called off.

Recall that the risk neutral probability that the deal is called off at $0<s<\tau$ is $\lambda \exp (-\lambda s)$. Therefore,
$S_{1}(s-)=S_{1}(0) \exp \left[\int_{0}^{s}\left(r-\lambda k_{1}-(1 / 2) \sigma_{1}^{2}\right) d t+\sigma_{1} \int_{0}^{s} d W_{1}(t)\right]$

Since $S_{1}(s)=Y_{1} S_{1}(s-)$, we have

$$
\begin{align*}
& S_{1}(\tau)=S_{1}(s-) Y_{1} \exp \left[\left(r-(1 / 2) \sigma_{1}^{\prime 2}\right)(\tau-s)+\sigma_{1}^{\prime \prime} W_{3}(\tau-s)\right] \\
& =S_{1}(0) \exp \left[\left(r-\lambda k_{1}-(1 / 2) \sigma_{1}^{2}\right) s+\sigma_{1} W_{1}(s)\right] Y_{1} \cdot \\
& \quad \exp \left[\left(r-(1 / 2) \sigma_{1}^{\prime 2}\right)(\tau-s)+\sigma_{1}^{\prime \prime} W_{3}(\tau-s)\right] \tag{15}
\end{align*}
$$

Thus, value of second component is

$$
\begin{align*}
& \exp (-r \tau) \int_{0}^{\tau} d s \lambda \exp (-\lambda s) E\left[\left(S _ { 1 } ( 0 ) Y _ { 1 } \operatorname { e x p } \left[\left(r-\lambda k_{1}-(1 / 2) \sigma_{1}^{2}\right) s\right.\right.\right. \\
& \left.\left.\left.+\sigma_{1} W_{1}(s)+\left(r-(1 / 2) \sigma_{1}^{\prime \prime 2}\right)(\tau-s)+\sigma_{1}^{\prime \prime} W_{3}(\tau-s)\right]-K\right)^{+}\right] \tag{16}
\end{align*}
$$

Assume $\mathrm{Y}_{1}$ follows log-normal distribution, specifically $\log \left(\mathrm{Y}_{1}\right)$ follows $N\left(\log \left(k_{1}+1\right)+\delta_{1}^{2} / 2, \delta_{1}^{2}\right)$. We replace $\mathrm{W}_{3}$ with $\mathrm{W}_{1}$ as they are independent. Also note that the jump processes are independent with $\mathrm{W}_{1}$. Then (16) becomes

$$
\begin{align*}
& \exp (-r \tau) \int_{0}^{\tau} d s \lambda \exp (-\lambda s) E\left[\left(S _ { 1 } ( 0 ) \operatorname { e x p } \left[\left(r-\lambda k_{1}-(1 / 2) \sigma_{1}^{2}\right) s\right.\right.\right. \\
& \quad+\left(r-(1 / 2) \sigma_{1}^{\prime 2}\right)(\tau-s)+\log \left(1+k_{1}\right) \\
& \left.\left.\left.\quad+\sigma_{1} W_{1}(s)+\sigma_{1}^{\prime \prime} W_{1}(\tau-s)+W_{1}\left(\delta_{1}^{2}\right)\right]-K\right)^{+}\right] \tag{17}
\end{align*}
$$

After taking integral (see Appendix B), finally we have

$$
\begin{equation*}
S_{1}(0) \int_{0}^{\tau} \lambda \exp (-\lambda s) N\left(\alpha_{1}\right) d s-\exp (-r \tau) K \int_{0}^{\tau} \lambda \exp (-\lambda s) N\left(\alpha_{1}^{\prime}\right) d s \tag{18}
\end{equation*}
$$

where
$\alpha_{1}=\frac{\log \left(S_{1}(0) / K\right)+r \tau-\lambda k_{1} s+\log \left(1+k_{1}\right)+(1 / 2)\left(\sigma_{1}^{2} s+\sigma_{1}^{\prime 2}(\tau-s)\right)}{\sqrt{\left(\left(\sigma_{1}^{2} s+\sigma_{1}^{\prime \prime 2}(\tau-s)+\delta_{1}^{2}\right)\right)}}$
$\alpha_{1}^{\prime}=\frac{\log \left(S_{1}(0) / K\right)+r \tau-\lambda k_{1} s+\log \left(1+k_{1}\right)-(1 / 2)\left(\sigma_{1}^{2} s+\sigma_{1}^{\prime 2}(\tau-s)\right)}{\sqrt{\left(\left(\sigma_{1}^{2} s+\sigma_{1}^{\prime 2}(\tau-s)+\delta_{1}^{2}\right)\right)}}$
In most merger deals, the equity size of the acquirer company is significantly larger than that of the target company. It is reasonable to assume that even if the merger is called off, it will not change the variance of stock process, which indicates $\sigma_{1}^{\prime \prime}=\sigma_{1}$. Then $\alpha_{1}$ and $\alpha_{1}^{\prime}$ could be simplified to

$$
\begin{aligned}
& \alpha_{1}=\frac{\log \left(S_{1}(0) / K\right)+r \tau-\lambda k_{1} s+\log \left(1+k_{1}\right)+(1 / 2) \sigma_{1}^{2} \tau}{\sqrt{\left(\left(\sigma_{1}^{2} \tau+\delta_{1}^{2}\right)\right)}} \\
& \alpha_{1}^{\prime}=\frac{\log \left(S_{1}(0) / K\right)+r \tau-\lambda k_{1} s+\log \left(1+k_{1}\right)-(1 / 2) \sigma_{1}^{2} \tau}{\sqrt{\left(\left(\sigma_{1}^{2} \tau+\delta_{1}^{2}\right)\right)}}
\end{aligned}
$$

Combining first component with second component, we have $H_{1}\left(S_{1}, \tau ; K, r, \sigma_{1}^{2}, \lambda, k_{1}, \delta^{2}\right)$

$$
\begin{aligned}
& =\exp (-\lambda \tau) F\left(S_{1}, \tau ; K, r-\lambda k_{1}, \sigma_{1}^{2}\right) \\
& +S_{1}(0) \int_{0}^{\tau} \lambda \exp (-\lambda s) N\left(\alpha_{1}\right) d s-\exp (-r \tau) K \int_{0}^{\tau} \lambda \exp (-\lambda s) N\left(\alpha_{1}^{\prime}\right) d s
\end{aligned}
$$

Usually $\sigma_{2}^{\prime \prime}=\sigma_{1}$ does not hold for the target company.
However, using similar analysis we would still be able to derive the solution for $H_{2}\left(S_{2}, \tau ; K, r, \sigma_{1}^{2}, \sigma_{2}^{\prime \prime}, \lambda, k_{2}, \delta_{2}^{2}\right)$.

## III. NUMERICAL IMPLEMENTATION AND RESULTS

## A. Data Description

The data set consisting daily stock and option price data was obtained from TBSP Inc. For each deal and for each
day, we collect the closing price for the stocks, closing bid and ask prices for the options. The options we use have a maturity date earlier than the expected deal completion date. We pick call and put options with strike prices at or near the strike price, and strike prices immediately above and below this strike price.

The data on the daily Treasure-bill yield are obtained from Department of Treasure (www.treas.gov). For convenience, the Treasure-bill yield whose maturity date is closest to the option maturity date is used in the computation.

## B. Implementation and Methodology

The model has eight basic parameters that are described below.
$\sigma_{1}^{\prime \prime}$ : Volatility of stock 1 after the deal has been called off
$\sigma_{2}^{\prime \prime}$ : Volatility of stock 2 after the deal has been called off
$\sigma_{1}$ : Volatility of both stocks when the deal is pending
$\lambda$ : Parameter of jump process
$k_{1}, k_{2}$ : Mean of the percentage change in the stock price $\gamma_{1}, \gamma_{2}$ : Standard deviation of the logarithm of the jump size
The parameters $\sigma_{1}^{\prime \prime}$ and $\sigma_{2}^{\prime \prime}$ is chosen to be the at-themoney implied volatilities 3 months before the deal was announced.

The other parameters of the model are calibrated to the market prices of the multiple options on both the stocks of varying strikes and maturities. We calculate the model parameters to minimize the sum of the magnitudes of the differences between the actual option prices and those predicted by the model. Specifically, we find
$\left(\sigma_{1}, \lambda, k_{1}, k_{2}, \gamma_{1}, \gamma_{2}\right)$ that minimizes

$$
\begin{equation*}
\sum_{n=1}^{N} \frac{\left|H_{n}\left(t, \tau_{n}, K_{n}\right)-\hat{H}_{n}\left(t, \tau_{n}, K_{n}\right)\right|}{\hat{H}_{n}\left(t, \tau_{n}, K_{n}\right)} \tag{19}
\end{equation*}
$$

where N is the total number of options on both stocks, $H_{n}\left(t, \tau_{n}, K_{n}\right)$ is the theoretical price predicted by the model for $i$ th option, $\hat{H}_{n}\left(t, \tau_{n}, K_{n}\right)$ is the observed price for $i$ th option.

The calibrated parameters are then used to predict theoretical prices for the options. This would also enable us to examine the model's out-of-sample cross-sectional pricing performance. For this purpose, we rely on previous day's option prices to back out the parameter values and then use them as input to compute current day's modelbased option prices. Equation (19) is used to compute percentage pricing error.

Following Merton [8], the dividends are treated as a continuous yield in the implementation. Since equity options in the U.S. market are American and not European, we implement our model numerically using the standard binomial tree-based methodology to derive theoretical American option prices.
C. Results

| Target | Acquirer | In- <br> Sample <br> Mean <br> Model <br> Error | In- <br> Sample <br> Stdev <br> Model <br> Error | Out-of- <br> Sample <br> Mean <br> Model <br> Error | Out-of- <br> Sample <br> Stdev <br> Model <br> Error |
| :--- | :--- | :--- | :--- | :--- | :--- |
| MCDT | BRCD | $1.21 \%$ | $0.14 \%$ | $18.14 \%$ | $3.54 \%$ |
| VRTS | SYMC | $0.94 \%$ | $0.11 \%$ | $14.94 \%$ | $2.63 \%$ |
| PIXR | DIS | $0.70 \%$ | $0.08 \%$ | $10.10 \%$ | $1.89 \%$ |
| MXO | STX | $0.84 \%$ | $0.10 \%$ | $12.29 \%$ | $2.47 \%$ |
| SILI | VSH | $0.75 \%$ | $0.10 \%$ | $17.21 \%$ | $3.31 \%$ |
| STCR | QTWW | $1.17 \%$ | $0.12 \%$ | $16.78 \%$ | $2.91 \%$ |
| RDN | MTG | $1.02 \%$ | $0.15 \%$ | $17.82 \%$ | $4.24 \%$ |

Table 1. Summarized Results
Table 1 presents the summarized results of the tests. For each deal, we display both in-sample and out-of-sample mean percentage error. Although our model performs very well for in-sample fitting, it is plausible that it is a consequence of having a large number of structural parameters. Considering that the bid-ask spreads in the prices of most options are very close to $10 \%$, the results of out-of-sample pricing are acceptable. For out-of-sample pricing, the presence of more parameters may actually cause over-fitting and have the model penalized if the extra parameters do not improve its structural fitting.

## IV. CONCLUSIONS

We develop a jump-diffusion model to price options on the stocks involved in mergers and acquisitions. We assume parameter $\lambda$ to be constant and the jump size follows lognormal distribution. Although empirical studies ([2], [6]) indicate that both possibility of deal success and jump size are correlated with speculation spread and it is not perfectly consistent with our model assumptions, our test results show that our model performs well in explaining observed option prices. The model can be used by risk arbitrageurs to control risks associated with merger deals using options.

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## APPENDIX

## A. Proof of $\sigma_{1}=\sigma_{2}$ and $\sigma_{2}^{\prime}=0$.

Let $\beta(t)=S_{2}(t) / S_{1}(t)$. We consider the situation when the jump does not occur. From equation (5), (6),
$\mu_{1}=r-\lambda k_{1}$ and $\mu_{2}=r-\lambda k_{2}$, we must have

$$
\begin{gathered}
\beta(t)=\beta(0) \exp \left[\left(\lambda k_{1}-\lambda k_{2}+\frac{1}{2}\left(\sigma_{2}-\sigma_{1}\right)^{2}-\frac{1}{2}\left(\sigma_{2}^{2}-\sigma_{1}^{2}\right)-\frac{1}{2} \sigma_{2}^{\prime 2}\right) t\right. \\
\left.+\left(\sigma_{2}^{\prime}-\sigma_{1}^{\prime}\right) W_{1}(t)+\sigma_{2}^{\prime \prime 2} W_{2}(t)\right]
\end{gathered}
$$

When the deal goes through, we also have $\beta(T)=\alpha$ where
$\alpha$ is the stock for stock exchange ratio of the merger deal. Therefore

$$
\begin{aligned}
\log (\alpha / \beta(0))=( & \left.\lambda k_{1}-\lambda k_{2}+\frac{1}{2}\left(\sigma_{2}-\sigma_{1}\right)^{2}-\frac{1}{2}\left(\sigma_{2}^{2}-\sigma_{1}^{2}\right)-\frac{1}{2} \sigma_{2}^{\prime 2}\right) T \\
& +\left(\sigma_{2}-\sigma_{1}\right) W_{1}(T)+\sigma_{2}^{\prime 2} W_{2}(T)
\end{aligned}
$$

Since $k_{1}, k_{2}, \sigma_{1}, \sigma_{2}, \sigma_{2}^{\prime}, \alpha, \beta(0)$ are constants, thus $\left(\sigma_{2}-\sigma_{1}\right) W_{1}(T)+\sigma_{2}^{\prime \prime 2} W_{2}(T)$ is bounded. Therefore, we must have $\sigma_{1}=\sigma_{2}$ and $\sigma_{2}^{\prime}=0$ as $\mathrm{W}_{1}$ and $\mathrm{W}_{2}$ are independent Wiener processes.
Q.E.D
B. Derivation of equation (18)

$$
\begin{aligned}
& \exp (-r \tau) \int_{0}^{\tau} d s \lambda \exp (-\lambda s) E\left[\left(S _ { 1 } ( 0 ) \operatorname { e x p } \left[\left(r-\lambda k_{1}-(1 / 2) \sigma_{1}^{2}\right) s\right.\right.\right. \\
& +\left(r-(1 / 2) \sigma_{1}^{\prime 2}\right)(\tau-s)+\log \left(1+k_{1}\right) \\
& \left.\left.\left.+\sigma_{1} W_{1}(s)+\sigma_{1}^{\prime \prime} W_{1}(\tau-s)+W_{1}\left(\delta_{1}^{2}\right)\right]-K\right)^{+}\right] \\
& =\exp (-r \tau) \int_{0}^{\tau} d s \lambda \exp (-\lambda s) E\left[\left(S _ { 1 } ( 0 ) \operatorname { e x p } \left[r \tau-\lambda k_{1} s-(1 / 2) \sigma_{1}^{2} s\right.\right.\right. \\
& -(1 / 2) \sigma_{1}^{\prime 2}(\tau-s)+\log \left(1+k_{1}\right) \\
& \left.\left.\left.+W_{1}\left(\sigma_{1}^{2} s+\sigma_{1}^{\prime \prime 2}(\tau-s)+\delta_{1}^{2}\right)\right]-K\right)^{+}\right]
\end{aligned}
$$

Setting $P=S_{1}(0) \exp \left[r \tau-\lambda k_{1} s-(1 / 2) \sigma_{1}^{2} s-(1 / 2) \sigma_{1}^{\prime 2}(\tau-s)\right.$ $\left.+\log \left(1+k_{1}\right)+W_{1}\left(\sigma_{1}^{2} s+\sigma_{1}^{\prime 2}(\tau-s)+\delta_{1}^{2}\right)\right]$
Above

$$
\begin{aligned}
& =\exp (-r \tau) \int_{0}^{\tau} d s \lambda \exp (-\lambda s) \int_{K}^{\infty} \frac{1}{\sqrt{2 \pi\left(\sigma_{1}^{2} s+\sigma_{1}^{\prime \prime 2}(\tau-s)+\delta_{1}^{2}\right)}}(P-K) \\
& \exp \left(-\frac{\left(\log \left(P / S_{1}(0)\right)-r \tau-\lambda k_{1} s-\log \left(1+k_{1}\right)+(1 / 2) \sigma_{1}^{2} s+(1 / 2) \sigma_{1}^{\prime \prime 2}(\tau-s)\right)^{2}}{2\left(\left(\sigma_{1}^{2} s+\sigma_{1}^{\prime \prime 2}(\tau-s)+\delta_{1}^{2}\right)\right)}\right) \frac{d P}{P} \\
& =\exp (-r \tau) \int_{0}^{\tau} d s \lambda \exp (-\lambda s) \int_{K}^{\infty} \frac{1}{\sqrt{2 \pi\left(\sigma_{1}^{2} s+\sigma_{1}^{\prime \prime 2}(\tau-s)+\delta_{1}^{2}\right)}} \\
& \exp \left(-\frac{\left(\log \left(P / S_{1}(0)\right)-r \tau-\lambda k_{1} s-\log \left(1+k_{1}\right)+(1 / 2) \sigma_{1}^{2} s+(1 / 2) \sigma_{1}^{\prime \prime 2}(\tau-s)\right)^{2}}{2\left(\left(\sigma_{1}^{2} s+\sigma_{1}^{\prime 2}(\tau-s)+\delta_{1}^{2}\right)\right)} d P\right. \\
& -K \exp (-r \tau) \int_{0}^{\tau} d s \lambda \exp (-\lambda s) \int_{K}^{\infty} \frac{1}{\sqrt{2 \pi\left(\sigma_{1}^{2} s+\sigma_{1}^{\prime \prime 2}(\tau-s)+\delta_{1}^{2}\right)}} \\
& \exp \left(-\frac{\left(\log \left(P / S_{1}(0)\right)-r \tau-\lambda k_{1} s-\log \left(1+k_{1}\right)+(1 / 2) \sigma_{1}^{2} s+(1 / 2) \sigma_{1}^{\prime \prime 2}(\tau-s)\right)^{2}}{2\left(\left(\sigma_{1}^{2} s+\sigma_{1}^{\prime 2}(\tau-s)+\delta_{1}^{2}\right)\right)}\right) \frac{d P}{P} \\
& =S_{1}(0) \int_{0}^{\tau} \lambda \exp (-\lambda s) N\left(\alpha_{1}\right) d s-\exp (-r \tau) K \int_{0}^{\tau} \lambda \exp (-\lambda s) N\left(\alpha_{1}^{\prime}\right) d s
\end{aligned}
$$

where

$$
\begin{aligned}
& \alpha_{1}=\frac{\log \left(S_{1}(0) / K\right)+r \tau-\lambda k_{1} s+\log \left(1+k_{1}\right)+(1 / 2)\left(\sigma_{1}^{2} s+\sigma_{1}^{\prime 2}(\tau-s)\right)}{\sqrt{\left(\left(\sigma_{1}^{2} s+\sigma_{1}^{\prime 2}(\tau-s)+\delta_{1}^{2}\right)\right)}} \\
& \alpha_{1}^{\prime}=\frac{\log \left(S_{1}(0) / K\right)+r \tau-\lambda k_{1} s+\log \left(1+k_{1}\right)-(1 / 2)\left(\sigma_{1}^{2} s+\sigma_{1}^{\prime 2}(\tau-s)\right)}{\sqrt{\left(\left(\sigma_{1}^{2} s+\sigma_{1}^{\prime 2}(\tau-s)+\delta_{1}^{2}\right)\right)}}
\end{aligned}
$$

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