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Reconfigurable Control Allocation Applied to an Aircraft Benchmark Model

Youmin Zhang, Camille A. Rabbath and Chun-Yi Su

Abstract— **In this paper, a study of reconfigurable control allocation applied to a realistic and nonlinear aircraft model is presented. A pseudo-inverse based method for reconfigurable control allocation has been investigated in an ADMIRE (Aero-Data Model In Research Environment) aircraft benchmark model. Partial loss of control effectiveness type faults have been implemented. Simulation results show satisfactory performance for accommodating the partial loss of control effectiveness in control effectors.**

I. INTRODUCTION

Due to the increased requirements on the reliability, maneuverability and survivability of modern and future aircraft, more control effectors/surfaces are being introduced. This requires the control allocation function, together with baseline flight control laws, to be effectively implemented with the overall flight control systems. In particular, in the case of control effector (actuator) failures or control surface damages, an effective re-distribution (or re-allocation) of the control surface deflections with the remaining healthy control effectors is needed to achieve acceptable performance.

In a conventional aircraft, there are three major control effectors: aileron, elevator and rudder. Aircraft flight control systems are usually designed utilizing one control effector (actuator) for each rotational degree of freedom. Essentially, the aileron is used differentially to produce a rolling moment, the elevator generates a pitching moment, and the rudder controls the yawing moment of the aircraft. The control allocation problem is defined as the determination of the positions/deflections of control effectors that generate a given set of desired moments specified by a flight control law which transfers pilot's command given by a control stick. With three desired moments and three independent control effectors to generate these moments, a unique solution can be found or in this case the control allocation function can be merged into the baseline flight control law. However, due to the increased requirements on the reliability, maneuverability and survivability of modern and future aircraft, control effectors are no longer limited to these three conventional control effectors and new control effectors have been introduced. As an example, there are 11 individual control effectors in an innovative control effectors tailless aircraft [1] and also 11 control effectors in the ADMIRE aircraft used in this study [2]. With an increase in the number of redundant control effectors, the problem of allocating these controls to achieve the desired moments becomes non-unique and far more complex. Such redundancy thus calls for effective control allocation schemes to distribute the required control moments over the effector suite, which cannot be handled by the baseline flight control law. In particular, in the case of effector failures or control surface damages, an effective re-distribution (or re-allocation) of the control surface deflections with the remaining healthy control actuators is needed to maintain acceptable performance even in the presence of control effector failures. This requirement asks for so-called reconfigurable control allocation or control re-allocation technique, which is an important and necessary part of the reconfigurable (or fault-tolerant) flight control systems.

The control allocation problem without consideration to system failures has been intensively studied following the work of Durham [3], [4]. A review of existing methods can be found in [5] and a comparison of different control allocation methods is documented in [6]. However, the control re-allocation problem has not been well investigated except a few notable works presented in [7], [8], [9].

In this paper, in view of its simplicity and satisfactory performance, we propose a reconfigurable control allocation scheme based on a pseudo-inverse method [5], which is evaluated in a realistic and nonlinear ADMIRE aircraft model. The paper is organized as follows: The control re-allocation issue and problem formulation are presented in Section II. A simple, yet effective, solution based on the pseudo-inverse method is presented in Section III. Modeling of flight control effector faults as loss in control effectiveness is introduced in Section IV. Brief description of the ADMIRE benchmark aircraft model, which is implemented in the Matlab/Simulink environment, together with the simulation evaluation results are presented in Section V. Finally, conclusions and future work are given in Section VI.

II. CONTROL RE-ALLOCATION PROBLEM FORMULATION

To demonstrate the function and placement of control re-allocation in an overall active fault-tolerant flight control system (AFTFCS), the general structure of a typical AFTFCS is shown in Fig. 1. As can be seen from Fig. 1, there are typically four sub-systems: 1) a reconfigurable controller, which includes two parts of a flight control law module and a control (re-)allocation module; 2) a Fault Detection and Diagnosis (FDD) module; 3) a control reconfiguration mechanism module; and 4) an autopilot which plays a role of command generator/governor.

Y. M. Zhang is with the Department of Mechanical and Industrial Engineering, Concordia University, Montreal, Quebec H3G 1M8, Canada, ymzhang@encs.concordia.ca.

C. A. Rabbath is with the Defence Research and Development Canada, Valcartier, Quebec G3J 1X5, Canada, Camille-Alain.Rabbath@drdc-rddc.gc.ca.

C-Y. Su is with the Department of Mechanical and Industrial Engineering, Concordia University, Montreal, Quebec H3G 1M8, Canada, cysu@alcor.concordia.ca.

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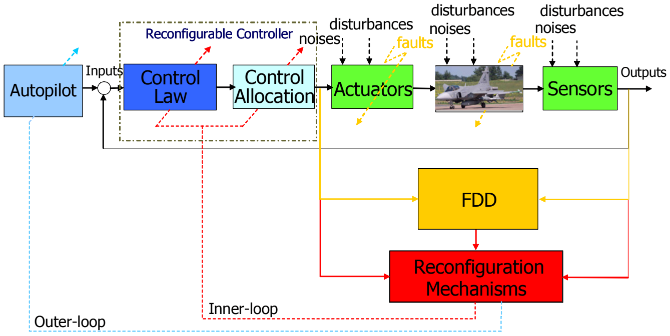

Fig. 1. General structure of AFTFCS.

Ideally, in the FDD module, any fault in the aircraft control effectors/surfaces should be detected and isolated as quickly as possible. Furthermore, fault parameters, aircraft state and output variables, and post-fault aircraft model need to be estimated on-line in real-time. Based on the on-line information on the post-fault aircraft model, the reconfigurable controller should be designed automatically to maintain the stability and the desired transient and steady-state performance. To avoid potential control actuator saturation and to take into consideration the degraded performance after fault occurrence, in addition to a reconfigurable controller, command input issued by the autopilot may need to be re-adjusted automatically to provide new command input or reference trajectory. Depending on the type, the severity of faults, and the methodology used for implementing the above-mentioned four functional modules in an AFTCS, different methods have been developed for fault-tolerant (reconfigurable) flight control systems [10], [11]. Up to now, most research focuses on the development of the reconfigurable flight control law [12], [13], [14], [15], [16], [17]. However, in view of overactuated control actuation available in modern aircraft, a simple strategy for accommodating the fault effects by reconfiguring the control allocation block, placed between flight control law and flight control actuators, is proposed, based on the use of the existing baseline flight control law.

Then, the problem of control re-allocation is that once one or more control surfaces get partially lost or get stuck during the flight, the control re-allocation scheme should be able to use the redundancy of operable control surfaces to cancel the effects of the damaged control surfaces and still provide the same (or almost the same) desired control input synthesized by the flight control law. The idea here is that instead of designing a reconfigurable flight control law in the presence of control actuator failures or control surface damages, the control allocation module is reconfigured with the use of the original baseline flight control law. A necessary condition for using control re-allocation techniques for reconfiguration (also for control allocation under normal flight conditions) is the existence of control actuator redundancy, which is the case for most modern civil and commercial airplanes. The advantages of this reconfiguration strategy are twofold: 1) the complex baseline flight control law does not need to be changed in the presence of actuator failures, then the inherent stability of the aircraft can be maintained to certain degrees with a certain amount of time after the occurrence of a control effector failure; 2) actuator position and rate limits can readily be taken into account in the control re-allocation schemes, in comparison with flight control law reconfiguration. Handling actuator physical constrains in the control re-allocation schemes is the result of using constrained optimization techniques for the solution of the control allocation problem.

Let the linearized dynamics of the normal aircraft at a trim condition be given by

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \tag{1}$$

where $\mathbf{x} \in \Re^n$ is the aircraft state vector; $\mathbf{u} \in \Re^l$ denotes the control surfaces; $A \in \Re^{n\times n}$ and $B \in \Re^{n\times l}$ represent the system matrix and control effectiveness matrix, respectively.

Suppose that one or more control surfaces are suddenly damaged, or that the control actuators driving the control surfaces get partial loss of its control effectiveness, then the post-fault model becomes

$$\dot{\mathbf{x}} = A\mathbf{x} + B_f\mathbf{u} \tag{2}$$

Let $\mathbf{y} = C_{\mathbf{y}}\mathbf{x}$ be a selected p-dimensional controlled output vector to be used in defining the control allocation, then

$$\dot{\mathbf{y}} = C_{\mathbf{y}}\dot{\mathbf{x}} = C_{\mathbf{y}}A\mathbf{x} + C_{\mathbf{y}}B_f\mathbf{u} \tag{3}$$

The choice of $\mathbf{y}$ is not unique but it must be chosen such that the closed-loop stability and performance is ensured by the control of $\mathbf{y}$. One natural choice of $\mathbf{y}$ is $\mathbf{y} = [p, q, r]^T$ (the roll, pitch, and yaw angular rates) [5], [7], although other choices are possible.

We require in general that $l > p$ for control re-allocation. The number of operable control surfaces is assumed greater than that of the controlled variables. For example, $l = 7$ and $p = 3$ for the ADMIRE benchmark model. For the current state $\mathbf{x}(t)$, suppose that the reference baseline flight control law for the healthy aircraft would have produced input $\mathbf{u}^*$ if all of the control surfaces were healthy. Then the desired rate of $\mathbf{y}$ would be

$$\dot{\mathbf{y}}^* = C_{\mathbf{y}}A\mathbf{x} + C_{\mathbf{y}}B\mathbf{u}^* \tag{4}$$

We therefore seek a $\mathbf{u}$ that makes the right-hand side of Eq. (3) as close as possible to that of Eq. (4); that is,

$$C_{\mathbf{y}}B_f\mathbf{u} = C_{\mathbf{y}}B\mathbf{u}^* \text{ or } C_{\mathbf{y}}B_f\mathbf{u} - C_{\mathbf{y}}B\mathbf{u}^* = 0 \tag{5}$$

Thus the actual rate of $\mathbf{y}$ will approximate the desired rate of $\mathbf{y}$, i.e. $\dot{\mathbf{y}} = \dot{\mathbf{y}}^*$. Consequently, $\mathbf{y}$ will remain close to $\mathbf{y}^*$, which represents the desired performance. Such a $\mathbf{u}$ can be determined by the minimization of the following quadratic function:

$$\min_{\mathbf{u}} J = \frac{1}{2}\left[(C_{\mathbf{y}}B_f\mathbf{u} - \mathbf{v}_d)^T Q (C_{\mathbf{y}}B_f\mathbf{u} - \mathbf{v}_d)\right] \tag{6}$$

subject to

$$\mathbf{u}_{\min} \leq \mathbf{u} \leq \mathbf{u}_{\max} \tag{7}$$

where $\mathbf{v}_d = C_{\mathbf{y}}B\mathbf{u}^*$ is called as a virtual control or generalized control, while $\mathbf{u}$ is called as a physical control, which corresponds to the deflections of physical flight control surfaces. Q is a positive-definite matrix of appropriate dimensions. The $\mathbf{u}_{\min}$ and $\mathbf{u}_{\max}$ are the lower and upper bounds of the control surface positions, respectively.

III. CONTROL RE-ALLOCATION SOLUTION BASED ON PSEUDO-INVERSE METHODS

If the above control constraint (7) is not considered, an explicit solution can be obtained as follows from minimization of the above quadratic function in (6):

$$\mathbf{u} = \left[(C_{\mathbf{y}}B_f)^T Q (C_{\mathbf{y}}B_f)\right]^{-1} (C_{\mathbf{y}}B_f)^T Q \mathbf{v}_d \tag{8}$$

The above control re-allocation method is referred to as Pseudo-Inverse Method (PIM). However, the above solution may not be feasible for all virtual control input $\mathbf{v}_d$ in the presence of actuator position or rate constraints, which is even more often being occurred during the initial reconfiguration period of control re-allocation. Various ways to accommodate the constraints have been proposed in the literature. The simplest alternative is to truncate $\mathbf{u}$ by clipping those components that violate some constraints [5]. However, since this typically causes only a few control inputs to saturate, it seems natural to use the remaining control inputs to produce the desired moment.

Virnig and Bodden [18] propose a Redistributed Pseudo-Inverse (RPI) scheme, in which all control inputs that violate their bounds in the pseudo-inverse solution are saturated and removed from the optimization. Then, the control allocation problem is resolved with only the remaining control inputs as free variables. Bordignon [19] further proposed an iterative variant of the RPI. Instead of only redistributing the control effect once, the further redistribution of the saturated inputs is carried out. This is known as the Cascaded Generalized Inverse (CGI) approach. The method of CGI arises from the idea that if a generalized inverse generates a control signal that exceeds a position limit, then that control channel should be set at the exceeded limit, and the rest of the controls are redistributed to achieve the desired moment.

The procedure of the CGI can be described as follows. Initially, a generalized inverse is computed using Eq. (8). This matrix is used to allocate the controls given in response to the desired moments. If none of the elements in the solution is saturated, then the desired moment lies within the limits of the constraints. If any of the elements in the solution exceeds their constraints, the corresponding element is set equal to its constraint, and their effects at saturation are subtracted from the desired moment. The effect of a saturated control is equivalent to the control position multiplied by the column of the $C_{\mathbf{y}}B_f$ matrix which corresponds to that control. The resulting moment is the part of the moment demand that must be satisfied by the remaining controls. In this paper, the CGI method has been used for control re-allocation implementation.

IV. MODELING CONTROL EFFECTOR FAULTS AS A LOSS IN CONTROL EFFECTIVENESS

During a normal flight, the aircraft control actuators operate exactly as directed by the flight control law and the control allocation function under normal flight conditions. We say that these control actuators are 100% effective (in executing the control commands). When faults occur in control effectors, such as partial loss of a control surface or pressure reduction in the hydraulic lines, the control effectors is not able to fulfill the control commands effectively. In such cases, we say that the effectiveness of the control effectors has been reduced.

Based on the above idea, we can quantify the severity of the control effector faults by defining a parameter expressing the reduction of the control effectiveness [20]. Such parameter represents the loss of the one-to-one relationship between the control command (output of control allocation) and the true control effector actions. Therefore, the actuator faults can be defined as an abnormal operation of any element in the control effector subsystem such that the control command from the controller output cannot be delivered to the manipulated variables with 100% efficiency. An illustration is shown in Fig. 2.

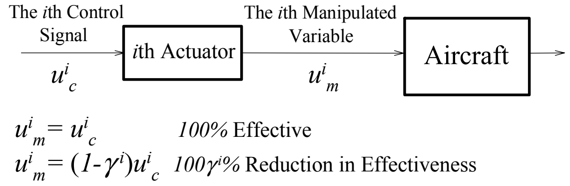

Fig. 2. Modeling control effector failures by control effectiveness factor

Based on the above modeling, the post-fault control matrix B_f in Eq. (2) can be written in relation to the nominal constant control input matrix B and the control effectiveness factors $\gamma^i, i = 1, ..., l$, as following:

$$B_f = B(I - \Gamma),\ \Gamma = \begin{bmatrix} \gamma^1 & 0 & \cdots & 0 \\ 0 & \gamma^2 & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \gamma^l \end{bmatrix} \tag{9}$$

where $\gamma^i = 0, i = 1, ..., l$, denotes the healthy ith control effector, $\gamma^i = 1$ corresponds to total failure of the ith control effector, and $0 < \gamma^i < 1$ represents partial loss in control effectiveness.

As long as the control effectiveness factor in each control effector is available or estimated based on the algorithm for example proposed in [20], the post-fault model of the aircraft can be determined for control re-allocation calculation.

V. ADMIRE IMPLEMENTATION AND PERFORMANCE EVALUATION

A. Brief Introduction to ADMIRE

ADMIRE (Aero-Data Model In Research Environment) [2], developed by the Swedish Defense Research Agency (FOI), is a nonlinear and six degree of freedom simulation aircraft model of a generic single seated, single engine fighter aircraft with a delta-canard configuration. The ADMIRE aero-data comes from Saab Aerospace and the configuration of the model corresponds to the Gripen fourth-generation combat aircraft. The model has been developed as one of the GARTEUR (Group of Aeronautical Research and Technology in EURope) benchmark models in an Action Group AG-11 project for flight control clearance investigation.

The available control effectors in the ADMIRE are: left and right canards (δ_{lc} and δ_{rc}), left and right outer elevons (δ_{loe} and δ_{roe}), left and right inner elevons (δ_{lie} and δ_{rie}), rudder (δ_r), as well as engine thrust, leading edge flaps, and landing gear. For control design purpose, only the first 7 control surfaces (δ_{lc}, δ_{rc}, δ_{loe}, δ_{roe}, δ_{lie}, δ_{rie}, δ_r) are used. For pitch control, both the canards and elevons are used. For yaw control, the rudder is used. For roll control, the rudder and the elevons are used. This means that there is a coupling between the elevon and the rudder. The inner and outer part of the elevon control surfaces always have the same deflection, both for pitch and roll control. Furthermore, the left and right canard always have the same deflection under normal flight conditions.

The ADMIRE is augmented with a Flight Control System (FCS) in order to provide stability and sufficient handling qualities within the operational envelope under normal flight conditions. The baseline flight control law contains a longitudinal part and a lateral part, which have been implemented with a pole placement design method for 29 trim conditions to cover the entire flight envelope [2]. The longitudinal controller provides pitch rate (q) control below Mach number 0.58. For Mach numbers greater than or equal to 0.62, it provides load factor control. The longitudinal controller also contains a speed controller to maintain desired aircraft speed. The lateral controller enables the pilot to perform roll control (p) and the sideslip angle control (β). These three controller outputs (i.e. three commanded moments in the pitching (q), rolling (p) and yawing direction (r)) are used as inputs for the control allocation module (denoted also as control selector) to determine the desired control surface deflections to generate the desired three axes of aircraft movement. It should be mentioned that the yawing rate r feedback is equivalent to β feedback when ϕ and β are zero or relatively small. The function of control allocation is to distribute the three control channel signals to the seven control actuators. A scheduling of the control allocation for different flight conditions is done by using the Mach number and the altitude.

In the original ADMIRE benchmark model, however, fault models were not included since fault-tolerant control was not investigated. Partial losses of the 7 control effectors have been implemented for reconfigurable flight control allocation design and evaluation purposes in this study. Simulations are conducted in order to analyze whether the fault-tolerant control system possesses the ability to maintain an acceptable command tracking performance even in the presence of control effector faults and control surface damages.

B. Fault Scenarios and Simulation Results

In the following, two fault scenarios are simulated: 1) a 50% loss of control effectiveness in rudder; 2) a 50% loss of control effectiveness in all elevons. Simulation of rudder fault is to demonstrate the effectiveness of control reconfiguration for lateral-directional control, while the simulation of elevon fault is to show longitudinal control. The faults occur at 2 sec. It is assumed that the fault detection and diagnosis information (time of the fault occurrence and the magnitude of the fault) are available for control re-allocation.

1) Rudder partial fault: As shown in Fig. 3, without control re-allocation, the angular rates (p, q, r) cannot track those under normal flight conditions. With the proposed control re-allocation, the reconfigured angular rates track the normal responses with zero steady-state error.

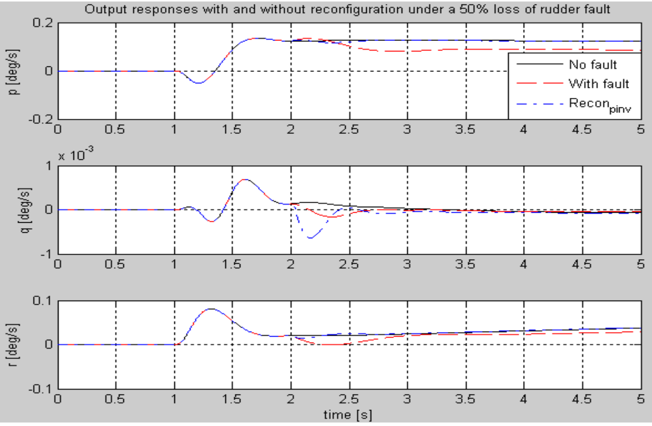

Fig. 3. Responses of angular rates (p, q, r)

Time histories of Euler angles (ϕ, θ, ψ) are shown in Fig. 4. While the reconfigured responses maintain almost the same trajectories as those under normal flight condition, the responses without reconfiguration show the trend of increased deviation from the normal flight condition.

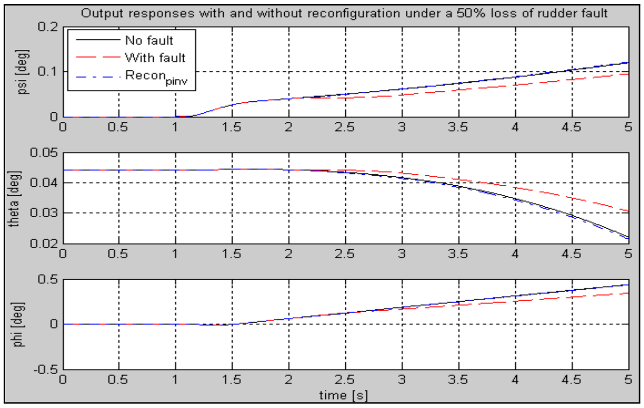

Fig. 4. Responses of Euler angles (ϕ, θ, ψ)

Fig. 5 shows the responses of angle of attack (α), sideslip angle (β), and flight path angle (γ). As can be seen, significantly improved tracking performance with control reconfiguration have been achieved in sideslip angle where zero steady-state error with very small transient is obtained. The reconfigured angle of attack and flight path angle are also improved with reconfiguration. However, it is not so significant since the fault is in the lateral direction, compensation in the longitudinal direction is relatively small.

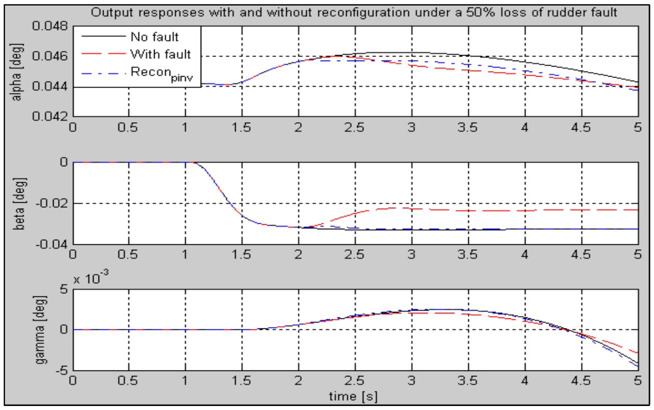

Fig. 5. Responses of aerodynamic angles (α, β, γ)

Figs. 6 and 7 show the history of corresponding seven control surface deflections. Almost the same amount but with opposite direction of deflections in left and right canards have been generated to compensate for the rudder fault. All four segments of the elevon are also used to compensate for the rudder fault. Deflection of the rudder has no significant difference between those with and without control re-allocation since the primary concept of control re-allocation is to make the best use of the remaining healthy control surfaces, so the adjustment to the faulty control surface is less weighted.

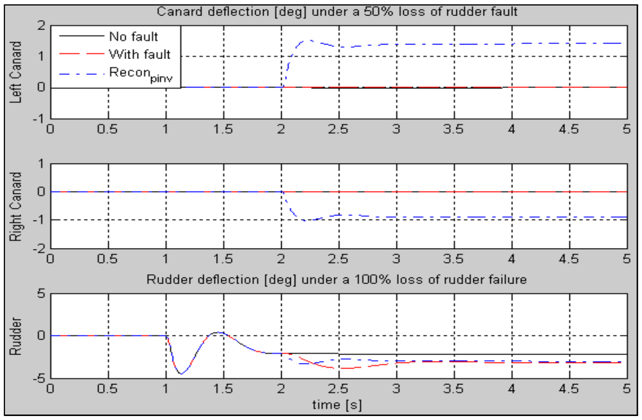

Fig. 6. Deflections in canard and rudder

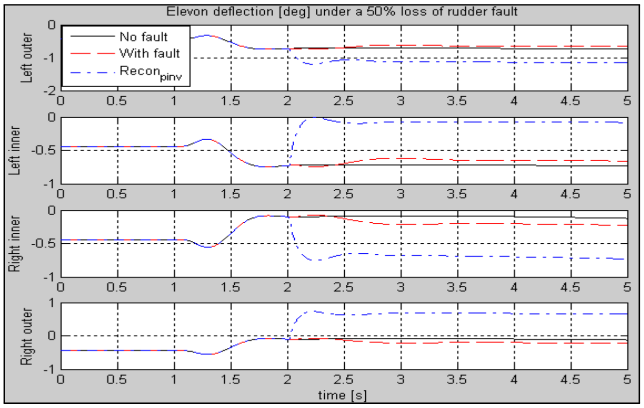

Fig. 7. Control surface deflections in elevon

2) Elevon partial fault: As can be seen in Figs. 8-12, with control re-allocation, responses of angular rates, Euler angles, and aerodynamic angles can track closely to those under normal flight conditions, while the responses without control reconfiguration cannot track the desired responses. To compensate for the elevon fault, both left and right canards have been re-commanded accordingly to compensate for the effects of the elevon fault. However, as expected the rudder deflection has little change since physically the longitudinal elevon fault should be mainly compensated by canard. The required deflections in elevon are also reduced compared to the case without reconfiguration.

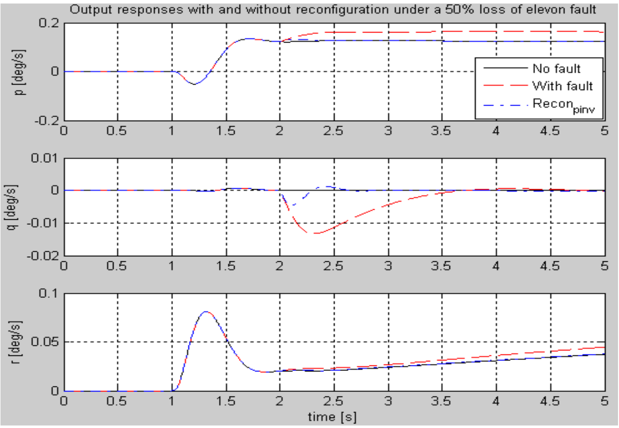

Fig. 8. Responses of angular rates (p, q, r)

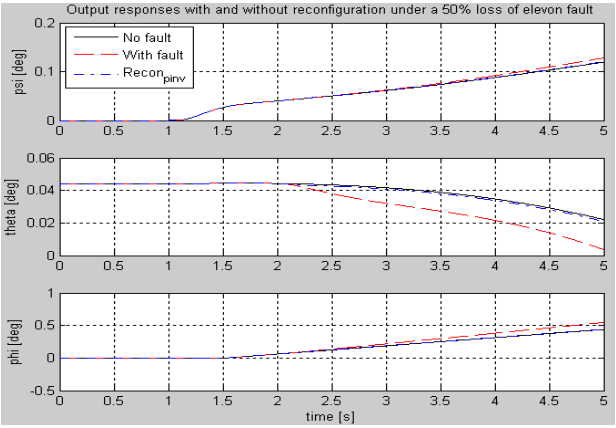

Fig. 9. Responses of Euler angles (ϕ, θ, ψ)

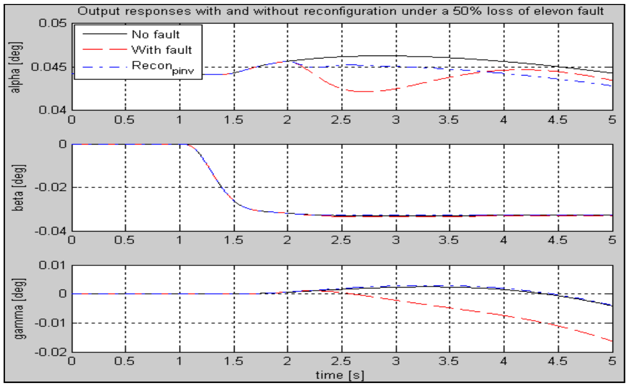

Fig. 10. Responses of aerodynamic angles (α, β, γ)

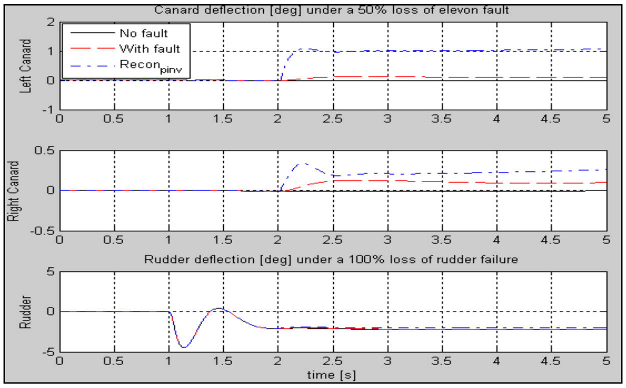

Fig. 11**.** Deflections in canard and rudder

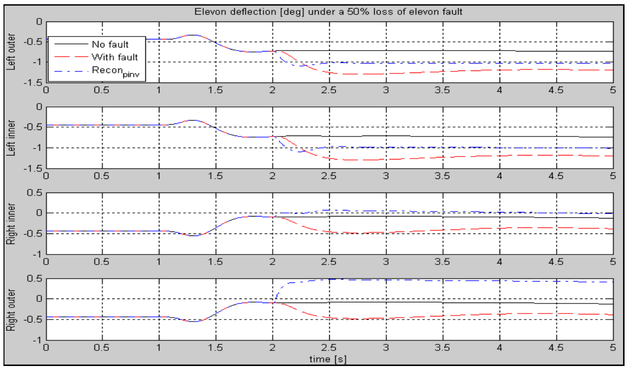

Fig. 12. Control surface deflections in elevon

Different levels of partial faults having occurred in rudder and elevon, and partial faults having occurred in other control effectors are also simulated and tested. Due to space limit, those results are omitted in the paper.

VI. ACKNOWLEDGMENT

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VII. CONCLUSIONS

In this paper, as an initial study of reconfigurable control allocation applied to a realistic and nonlinear aircraft model, a pseudo-inverse method has been implemented and tested under an ADMIRE aircraft benchmark model. Partial control effector faults represented by partial loss of control effectiveness are implemented in the ADMIRE benchmark model and used for evaluating the control re-allocation scheme. Simulation results have shown satisfactory results for accommodating the partial loss of control effectiveness. Future works include incorporation of fault detection and diagnosis schemes for actuator faults in the ADMIRE environment. In addition to the partial faults, other types of faults such as stuck, runaway, or floating faults will be considered for control re-allocation implementation and evaluation. New and more effective control re-allocation algorithms are also to be investigated.

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