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Quasi-decentralized Networked Control of Process Systems

Yulei Sun and Nael H. El-Farra†
Department of Chemical Engineering & Materials Science
University of California, Davis, CA 95616 USA

***Abstract*— This paper develops a quasi-decentralized control framework for plants with distributed, interconnected units that exchange information over a shared communication network. In this architecture, each unit in the plant has a local control system that communicates with the plant supervisor – and with other local control systems – through a shared communication medium. The objective is to design an integrated control and communication strategy that enforces the desired closed-loop stability and performance for the plant while minimizing network utilization and communication costs. The idea is to reduce the exchange of information between the local control systems as much as possible without sacrificing stability of the individual units and the overall plant. To this end, dynamic models of the interconnected units are embedded in the local control system of each unit to provide it with an estimate of the evolution of its neighbors when measurements are not transmitted through the network. The use of a model to recreate the interactions of a given unit with one of its neighbors allows the sensor suite of the neighboring unit to send its data in a discrete fashion since the model can provide an approximation of the unit's dynamics. The state of each model is then updated using the actual state of the corresponding unit provided by its sensors at discrete time instances to compensate for model uncertainty. By formulating the networked closed-loop plant as a hybrid system, an explicit characterization of the maximum allowable update period (i.e., minimum cross communication frequency) between each control system and the sensors of its neighboring units is obtained in terms of the degree of mismatch between the dynamics of the units and the models used to describe them. The developed control strategy is illustrated using a network of interconnected chemical reactors with recycle.**

I. Introduction

Modern industrial and commercial systems, such as chemical plants and manufacturing processes, are large-scale dynamical systems that involve complex, distributed arrangements of interconnected subsystems which are tightly integrated through mass, energy and information flows. Traditionally, control of plants with geographically-distributed interconnected units has been studied within either the centralized or decentralized control frameworks. In the centralized setting, all measurements are collected and sent to a central unit for processing, and the resultant control commands are then sent back to the plant. While centralized control is known to provide the best performance – because it imposes the least constraints on the control structure – the computational and organizational complexity associated with centralized controllers often makes their implementation impractical, especially for plants with complex dynamics. Also, the consequences of failures in a centralized controller can be detrimental to the entire plant.

These considerations have motivated significant work on decentralized control. In this paradigm, the plant is decomposed into a number of simpler subsystems (typically based on functional and/or time-scale differences of the unit operations) with interconnections, and a set of local controllers are connected to the distributed subsystems with no signal transfer taking place between the local controllers. Decentralized control of multi-unit plants can reduce complexity in the controller design and implementation, and can also provide flexibility in dealing with local controller failures. However, since in this structure the interconnections between the constituent subsystems are totally neglected, the closed-loop performance of the plant may deteriorate, and in some cases stability may be lost. Significant research work has explored in depth the benefits and limitations of decentralized controllers as well as possible ways of overcoming some of their limitations (e.g., see [1], [2], [3], [4], [5], [6], [7], [8] and the references therein). In recent times, there has also been some interest in studying plant-wide control problems within the distributed model predictive control framework (e.g., [9], [10], [11]). Other examples of recent works on control of integrated process networks can be found in [12], [13], [14], [15].

To solve the problem where a decentralized control structure cannot provide the required stability and performance properties, and to avoid the complexity and lack of flexibility associated with traditional centralized control, a quasi-decentralized control strategy with cross communication between the plant units offers a suitable compromise. The term quasi-decentralized control refers to a situation in which most signals used for control are collected and processed locally - although some signals (the total number of which is kept to a minimum) still need to be transferred between local units and controllers to adequately account for the interactions between the different units and minimize the propagation of disturbances and process upsets from one unit to another. One of the key problems that need to be addressed in the design of quasi-decentralized control systems is how to coordinate the control and communication functions and how to account for possible limitations of the communication medium in the formulation and solution of the control problem. This is an important problem given the increased reliance in the process industries in recent years on sensor and control systems that are accessed over

† To whom correspondence should be addressed: E-mail: nhelfarra@.ucdavis.edu. Financial support by NSF CAREER Award, CBET-0747954, is gratefully acknowledged.

shared communication networks rather than dedicated links (e.g., [16], [17]). The transition from dedicated, point-to-point connections to multi-purpose shared communication networks is driven in part by the reduced installation and maintenance time and costs as well as the flexibility and enhanced fault-tolerance of networked control systems.

The design of a quasi-decentralized control strategy that enforces the desired closed-loop objectives with minimal cross communication between the component subsystems is an appealing goal since it reduces reliance on the communication medium and helps save on communication costs. This is an important consideration particularly when the communication medium is a (potentially unreliable) wireless sensor network where conserving network resources is key to prolonging the service life of the network. While the emerging paradigm of control over networks (e.g., see [18], [19], [20], [21], [22], [23], [24], [25], [26]) provides a natural framework to address the issues of control and communication integration, the majority of research studies on networked control systems have focused on single-unit processes using a centralized control architecture, which is not always the best choice for the structure of the controller in a plant-wide setting. By comparison, results on networked control of multi-unit plants with tightly interconnected units have been more limited.

Motivated by these considerations, we develop in this work a quasi-decentralized control framework for multi-unit plants with tightly interconnected units that exchange information over a shared communication network. In this architecture, each unit in the plant has a local control system that communicates with the plant supervisor - and with other local control systems - through a shared communication medium. We address the problem of designing an integrated control and communication policy that enforces the desired closed-loop stability and performance while keeping communication through the network to a minimum in order to save on communication costs. To this end, we embed in the local control system of each unit a set of dynamic models that provide an approximation of the interactions between the given unit and its neighbors in the plant when communication is suspended and measurements are not transmitted through the network. The state of each model is updated using actual measurements from the corresponding unit when communication is re-established. The rest of the paper is organized as follows. Following some preliminaries in Section II, the networked quasi-decentralized control architecture is presented in Section III. The closed-loop system is then cast as a hybrid system in Section IV and its stability properties are analyzed leading to an explicit characterization of the maximum allowable update period (i.e., minimum cross communication frequency) between each control system and the sensors of its neighboring units in terms of the accuracy of the models and the choice of control laws. The proposed framework is illustrated in Section V using a simulated process example. Finally, concluding remarks are given in Section VI.

II. PRELIMINARIES

We consider a large-scale distributed plant composed of n interconnected processing units, each of which is modeled by a continuous-time linear system, and represented by the following state-space description:

$$\begin{aligned} \dot{x}_1 &= A_1 x_1 + B_1 u_1 + \sum_{j=2}^{n} A_{1j} x_j \\ \dot{x}_2 &= A_2 x_2 + B_2 u_2 + \sum_{j=1, j\neq 2}^{n} A_{2j} x_j \\ &\vdots \\ \dot{x}_n &= A_n x_n + B_n u_n + \sum_{j=1}^{n-1} A_{nj} x_j \end{aligned} \tag{1}$$

where $x_i := [x_i^{(1)} \ x_i^{(2)} \ \cdots \ x_i^{(p_i)}]^T \in \mathbb{R}^{p_i}$ denotes the vector of process state variables associated with the i-th processing unit, $u_i := [u_i^{(1)} \ u_i^{(2)} \ \cdots \ u_i^{(q_i)}]^T \in \mathbb{R}^{q_i}$ denotes the vector of manipulated inputs associated with the i-th processing unit, x^T denotes the transpose of x, A_i, B_i and A_{ij} are constant matrices. The interconnection term $A_{ij} x_j$, where $i \neq j$, describes how the dynamics of the i-th unit are influenced by the j-th unit in the plant. Note from the summation notation in Eq.1 that each processing unit can in general be connected to all the other units in the plant. Note also that even though each subsystem is referred to as a unit for simplicity, each subsystem can comprise a collection of unit operations depending on how the plant is decomposed. Our main objective is to devise an integrated control and communication strategy that stabilizes the individual units (and the overall plant) at the origin and accounts simultaneously for the presence of the communication network. To illustrate the main ideas and simplify the presentation of the results, we will focus in this work on the full state feedback problem where all the states of all the units are available as measurements. Extensions to the output feedback case are possible and the subject of other research work.

III. QUASI-DECENTRALIZED MODEL-BASED CONTROL OVER COMMUNICATION NETWORKS

A. Distributed feedback controller synthesis

To realize the quasi-decentralized control structure, the first step is to synthesize for each unit a stabilizing feedback controller of the general form:

$$u_i(x) = K_i x_i + \sum_{j=1, j\neq i}^{n} K_{ij} x_j \tag{2}$$

where $K_i x_i$ is the local feedback component responsible for stabilizing the i-th subsystem in the absence of interconnections, and $K_{ij} x_j$ is a "feedforward" component that compensates for the effect of the j-th neighboring subsystem on the dynamics of the i-th unit. Note that the implementation of the control law of Eq.2 requires the availability of state measurements from both the local subsystem being controlled and the connected units. Note also that a choice of $K_{ij} = O$ reduces the control strategy to a fully decentralized one where only measurements of the process variables of the i-th unit are collected and processed with no signal transfer taking place across the network. Without loss of generality, we consider the case when measurements of x_i are available to the local controller of unit i more frequently than measurements from the other connected plant units, x_j. This is a reasonable scenario given that the local information is typically transmitted over a dedicated control network, while transmission of measurements from the neighboring units involves using a shared medium. However, it is possible to generalize the control structure to account for the local networks.

B. Implementation over networks: a model-based approach

To reduce the transfer of information between the local control systems without sacrificing closed-loop stability, a dynamic model of each connected unit is included in the local control system of the i-th unit to provide it with an estimate of the evolution of the states of those units when measurements are not sent over the network. This allows the sensors of the neighboring unit to send their data at discrete time instants since the model can provide an approximation of the unit's dynamics. "Feedforward" from one unit to another is performed by updating the state of each model using the actual states of the corresponding unit provided by its sensors at discrete time instances. In-between consecutive transmission times, the control action for each unit relies on a collection of models that are embedded in the local control system and are running for a certain period of time. A schematic of this model-based control architecture is shown in Figure 1.

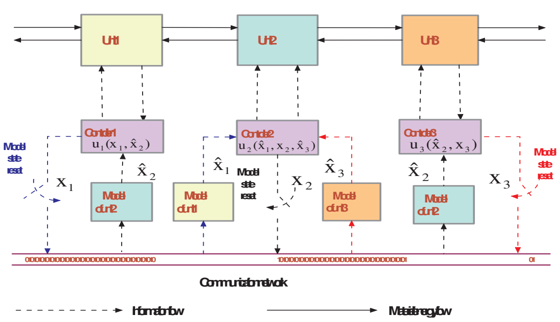

Fig. 1. Model-based quasi-decentralized control of a plant of interconnected units over a communication network.

Within this architecture, the local control law for each unit is implemented as follows:

$$
\begin{aligned}
u_i(t) &= K_i x_i(t) + \sum_{j=1, j\neq i}^{n} K_{ij}\widehat{x}_j^i(t),\ i = 1, 2, \cdots, n \\
\dot{\widehat{x}}_j^i(t) &= \widehat{A}_j \widehat{x}_j^i(t) + \widehat{B}_j \widehat{u}_j^i(t) + \widehat{A}_{ji} x_i(t) \\
&\quad + \sum_{l=1, l\neq i, l\neq j}^{n} \widehat{A}_{jl}\widehat{x}_l^i(t),\ \ t \in (t_k, t_{k+1}) \\
\widehat{u}_j^i(t) &= K_j \widehat{x}_j^i(t) + K_{ji} x_i(t) + \sum_{l=1, l\neq i, l\neq j}^{n} K_{jl}\widehat{x}_l^i(t),\ t \in (t_k, t_{k+1}) \\
\widehat{x}_j^i(t_k) &= x_j(t_k),\ \ j = 1, \cdots, n, j \neq i,\ k = 0, 1, 2, \cdots
\end{aligned} \tag{3}
$$

where $\widehat{x}_j^i$ is an estimate of x_j, used by the local control system of the i-th unit, $\widehat{A}_j$, $\widehat{B}_j$, $\widehat{A}_{jl}$ are constant matrices that model the dynamics of the j-th unit. Note from Eq.3 that even if a unit l is not be directly connected to the i-th unit, a model describing the dynamics of that unit needs to be embedded in the local control system of the i-th unit as long as unit l is connected to (at least) one immediate neighbor of unit i. Note also that the models used by the i-th controller to recreate the behavior of the neighboring units do not necessarily match the actual dynamics of those processes, i.e., in general $\widehat{A}_j \neq A_j$, $\widehat{B}_j \neq B_j$, $\widehat{A}_{jl} \neq A_{jl}$. Furthermore, a choice of $\widehat{A}_j = O$, $\widehat{B}_j = O$, $\widehat{A}_{jl} = O$ corresponds to the special case where in between consecutive transmission times, the corresponding model acts as a zero-order hold by keeping the last available measurement from neighboring units until the next one is available from the network.

A key parameter in the analysis of the control law of Eq.3 is the update period $h := t_{k+1} - t_k$, which determines the frequency at which a given unit receives measurements from the other units through the network to update the corresponding model estimates. To simplify the analysis, we consider the case when the update periods are constant and the same for all the units, i.e., we require that all units communicate their measurements concurrently every h seconds. This assumes that the sensors of all the units are given access to the network and can successfully transmit their data simultaneously. Extensions to the case where the different units transmit their data at different rates and the case when the update period is time-varying (or stochastic) are the subject of other research work.

IV. Closed-loop stability analysis

A. A hybrid system formulation

The successful implementation of the proposed quasi-decentralized control architecture requires characterizing the maximum allowable update period (equivalently, the minimum transmission frequency) between the controller of one unit and the sensor suite of its neighboring units, which is the time between information exchanges. To this end, we define the following estimation errors:

$$
e_j^i = \begin{cases} x_j - \widehat{x}_j^i, & j \neq i, \\ 0, & j = i \end{cases},\ \ i, j = 1, 2, \cdots, n \tag{4}
$$

where e_j^i represents the difference between the state of the j-th unit and the state of its model used in the local control system of the i-th unit. Note that since measurements of x_i are assumed to be available to the local control system of the i-th unit at all times, we always have $e_i^i = 0$. Introducing the augmented vectors $\mathbf{e}_j := [(e_j^1)^T\ (e_j^2)^T\ \cdots\ (e_j^n)^T]^T$, $\mathbf{e} := [\mathbf{e}_1^T\ \mathbf{e}_2^T\ \cdots\ \mathbf{e}_n^T]^T$, $\mathbf{x} := [x_1^T,\ x_2^T,\ \cdots,\ x_n^T]^T$, it can be shown that the overall closed-loop plant of Eq.1 subject to the control law of Eq.3 can be formulated as a hybrid (switched) system of the following form:

$$
\begin{aligned}
\dot{\mathbf{x}}(t) &= \Lambda_{11}\mathbf{x}(t) + \Lambda_{12}\mathbf{e}(t) \\
\dot{\mathbf{e}}(t) &= \Lambda_{21}\mathbf{x}(t) + \Lambda_{22}\mathbf{e}(t),\ \ t \in (t_k, t_{k+1}) \\
\mathbf{e}(t_k) &= \mathbf{0},\ \ k = 0, 1, 2, \cdots,
\end{aligned} \tag{5}
$$

where the process states evolve continuously in time and the estimation errors are reset to zero at each transmission instance since the state of each model in each unit is updated every h seconds. Referring to Eq.5, the matrices Λ_{11}, Λ_{12}, Λ_{21}, and Λ_{22} are linear combinations of A_i, B_i, A_{ij}, $\widehat{A}_i$, $\widehat{B}_i$, $\widehat{A}_{ij}$, K_i, K_{ij}, which are the matrices used to describe the dynamics, the models, and the control laws of the different units. The explicit forms of these matrices can be obtained by substituting Eq.3 into Eq.1 (see the simulation study in Section V for the explicit forms of these matrices in the case of a two-unit plant). Defining the augmented state $\xi(t) := [\mathbf{x}^T(t)\ \mathbf{e}^T(t)]^T$, we can re-write the closed-loop dynamics of the overall plant as:

$$\dot{\xi}(t) = \Lambda\xi(t), t \in [t_k, t_{k+1}),\ \xi(t_k) = [\mathbf{x}^T(t_k)\ \mathbf{0}]^T, \quad (6)$$

where $k = 1, 2, \cdots$, and $\Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix}$.

B. Necessary and sufficient condition for stability

Following [20], it can be shown that the system described by Eq.6 with initial conditions $\xi(t_0) = [\mathbf{x}^T(t_0)\ \mathbf{0}]^T = \xi_0$, has the following response:

$$\xi(t) = e^{\Lambda(t-t_k)}\left(I_s e^{\Lambda h} I_s\right)^k \xi_0, \quad (7)$$

for $t \in [t_k,\ t_{k+1})$, with $t_{k+1} - t_k = h$, where $I_s = \begin{bmatrix} I_{m\times m} & O_{m\times mn} \\ O_{mn\times m} & O_{mn\times mn} \end{bmatrix}$, $I_{m\times m}$ is the $m \times m$ identity matrix and $O_{m\times mn}$ is the $m\times mn$ zero matrix, where $m = \sum_{i=1}^{n} p_i$ and p_i is the dimension of the i-th state vector. By analyzing the closed-loop response in Eq.7, one can show that a necessary and sufficient condition for the zero solution of the system of Eq.6, $\xi = [\mathbf{x}^T\ \mathbf{e}^T]^T = [\mathbf{0}\ \mathbf{0}]^T$, to be globally exponentially stable is to have the eigenvalues of the test matrix $M = I_s e^{\Lambda h} I_s$ strictly inside the unit circle. The proof of this result is similar to that of Theorem 1 in [20] and will not be repeated here.

Owing to the dependence of the closed-loop matrix Λ on the matrices of the compensating models, the minimum stabilizing cross communication frequency is parameterized by the degree of mismatch between the dynamics of the units and the models used to describe them. This is expected given that if the models describe the behavior of the connected units exactly, the maximum allowable period for measurement updates to any unit can be arbitrarily large since there will be no need to communicate measurements in this case. In the case of large plant-model mismatch, a small update period will be necessary to compensate for modeling errors. Given bounds on the size of the uncertainty, it is possible to use the above stability criteria to determine the range of stabilizing update periods that can be used. Alternatively, if the update period is fixed by the characteristics of the communication medium, it is possible to use the stability criteria to determine the maximum size of tolerable process-model mismatch.

The maximum update period is also dependent on the choice of the control laws (both the feedback and feedforward components) for the various units. This dependence can serve as a criteria for identifying the controllers that are more robust to communication suspension (i.e., the ones that require measurement updates less frequently than others).

V. Simulation study: Application to chemical reactors with recycle

We consider a plant composed of two well-mixed, non-isothermal continuous stirred-tank reactors (CSTRs) with interconnections, where three parallel irreversible elementary exothermic reactions of the form $A \xrightarrow{k_1} B$, $A \xrightarrow{k_2} U$ and $A \xrightarrow{k_3} R$ take place, where A is the reactant species, B is the desired product and U, R are undesired byproducts. The feed to CSTR 1 consists of two streams, one containing fresh A at flow rate F_0, molar concentration C_{A0} and temperature T_0, and another containing recycled A from the second reactor at flow rate F_r, molar concentration C_{A2} and temperature T_2. The feed to CSTR 2 consists of the output of CSTR 1, and an additional fresh stream feeding pure A at flow rate F_3, molar concentration C_{A03}, and temperature T_{03}. The output of CSTR 2 is passed through a separator that removes the products and recycles unreacted A to CSTR 1. A jacket is used to remove/provide heat to both reactors. Under standard modeling assumptions, a plant model of the following form can be derived:

$$\begin{aligned}
\dot{T}_1 &= \frac{F_0}{V_1}(T_0 - T_1) + \frac{F_r}{V_1}(T_2 - T_1) + \sum_{i=1}^{3} G_i(T_1)C_{A1} + \frac{Q_1}{\rho c_p V_1} \\
\dot{C}_{A1} &= \frac{F_0}{V_1}(C_{A0} - C_{A1}) + \frac{F_r}{V_1}(C_{A2} - C_{A1}) - \sum_{i=1}^{3} R_i(T_1)C_{A1} \\
\dot{T}_2 &= \frac{F_1}{V_2}(T_1 - T_2) + \frac{F_3}{V_2}(T_{03} - T_2) + \sum_{i=1}^{3} G_i(T_2)C_{A2} + \frac{Q_2}{\rho c_p V_2} \\
\dot{C}_{A2} &= \frac{F_1}{V_2}(C_{A1} - C_{A2}) + \frac{F_3}{V_2}(C_{A03} - C_{A2}) - \sum_{i=1}^{3} R_i(T_2)C_{A2}
\end{aligned} \quad (8)$$

where $R_i(T_j) = k_{i0}\exp\left(\frac{-E_i}{RT_j}\right)$, $G_i(T_j) = \frac{(-\Delta H_i)}{\rho c_p} R_i(T_j)$, for $j = 1, 2$. T_j, C_{Aj}, Q_j, and V_j denote the temperature of the reactor, the concentration of A, the rate of heat input to the reactor, and the reactor volume, respectively, with subscript 1 denoting CSTR 1. ΔH_i, k_i, E_i, $i = 1, 2, 3$, denote the enthalpies, pre–exponential constants and activation energies of the three reactions, respectively, c_p and ρ denote the heat capacity and density of fluid in the reactor. Using typical values for the process parameters (see [27]), the plant with $Q_1 = Q_2 = 0$, $C_{A0} = C_{A0}^s, C_{A03} = C_{A03}^s$ and recycle rate $r = 0.5$, has three steady states: two locally asymptotically stable and one unstable at $(T_1^s, C_{A1}^s, T_2^s, C_{A2}^s) = (457.9\ K, 1.77\ kmol/m^3, 415.5\ K, 1.75\ kmol/m^3)$.

The control objective is to stabilize the plant at the (open-loop) unstable steady-state. Operation at this point is typically sought to avoid high temperatures, while simultaneously achieving reasonable conversion. The manipulated variables for the first reactor are chosen to be Q_1 and C_{A0}, while Q_2 and C_{A03} are used as manipulated variables for the second reactor. Linearizing the plant model around the unstable steady state yields the following system to which the quasi-decentralized control architecture will be applied:

$$\begin{aligned}
\dot{x}_1 &= A_1x_1 + B_1u_1 + A_{12}x_2 \\
\dot{x}_2 &= A_2x_2 + B_2u_2 + A_{21}x_1
\end{aligned} \quad (9)$$

where x_i and u_i are the (dimensionless) state and manipulated input vectors for the i-th unit, respectively, and A_i, B_i and A_{ij} are constant matrices. Following the methodology outlined in Section IV, we can re-write the closed-loop dynamics of the overall linearized plant in the form of Eq.6, where Λ consists of the following sub-matrices:

$$\Lambda_{11} = \begin{bmatrix} \bar{A}_1 & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_2 \end{bmatrix}, \ \Lambda_{12} = \begin{bmatrix} O & O & -B_1K_{12} & O \\ O & -B_2K_{21} & O & O \end{bmatrix}$$

$$\Lambda_{21} = \begin{bmatrix} O & O \\ \bar{A}_1 - \tilde{A}_1 & \bar{A}_{12} - \tilde{A}_{12} \\ \bar{A}_{21} - \tilde{A}_{21} & \bar{A}_2 - \tilde{A}_2 \\ O & O \end{bmatrix}, \ \Lambda_{22} = \begin{bmatrix} O & O & O & O \\ O & \tilde{A}_1 & -B_1K_{12} & O \\ O & -B_2K_{21} & \tilde{A}_2 & O \\ O & O & O & O \end{bmatrix}$$

where $\widetilde{A}_j = \widehat{A}_j + \widehat{B}_jK_j$, $\widetilde{A}_{ji} = \widehat{A}_{ji} + \widehat{B}_jK_{ji}$, with $\widehat{A}_j$, $\widehat{B}_j$, $\widehat{A}_{ji}$ estimates of A_j, B_j and A_{ji}, respectively, and K_i, K_{ij} the local feedback and "feedforward" components of each controller, respectively. The numerical values of these matrices can be found in the full version of this paper [27]. By examining the above expressions and from the fact that $M(h) = I_s e^{\Lambda h} I_s$, it can be seen that the eigenvalues of M depend on each process-model mismatch, the control gains and the update period. Since stability requires all eigenvalues of M to lie within the unit circle, it is sufficient to consider only the maximum eigenvalue of M, $\lambda_{\max}$.

A. Dependence of update period on model uncertainty

Figure 2(a) shows the magnitude of λ_{max} as a function of the update period when the control system of each reactor relies on a perfect model of the other reactor. As expected, the update time can be chosen arbitrarily large without loss of stability. To investigate the effect of model uncertainty, we consider parametric uncertainty in the enthalpy of the first reaction and define $\delta_1 = (\Delta H_1^m - \Delta H_1)/\Delta H_1$, where ΔH_1^m is a nominal value used in the models, as a measure of model accuracy. Figure 2(b) is a contour plot showing the dependence of λ_{max} on both δ_1 and h. The area enclosed by the unit contour line represents the stability region of the plant. In obtaining this plot, the local feedback gains, K_1 and K_2, were selected by placing the eigenvalues of both $\bar{A}_1$ and $\bar{A}_2$ at -5 and -1, while K_{12} and K_{21} were chosen to force $\bar{A}_{12} = \bar{A}_{21} = O$. As expected, the range of tolerable uncertainty shrinks for larger update periods.

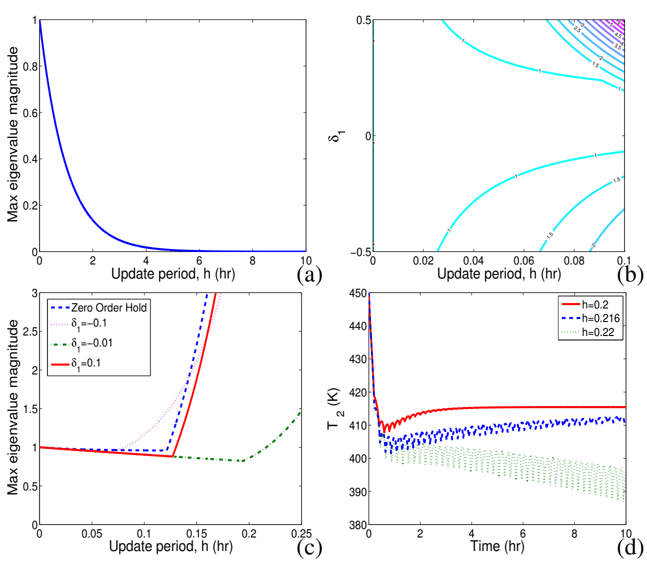

Fig. 2. Dependence of $\lambda_{\max}$ on (a) the update period using accurate models, and (b) on plant-model mismatch for various update periods. Plot (c): Maximum stabilizing update period using different compensating models and a zero-order hold scheme. Plot (d) shows the closed-loop profiles of T_2 with $\delta_1 = -0.01$ for different update periods.

Figure 2(c) shows the maximum eigenvalue magnitude versus the update period for different values of δ_1. For example, when $\delta_1 = -0.01$ (dash-dotted line), the stability condition is to have $h \leq 0.216$ hr; and for $h = 0.216$ hr the test matrix M has one eigenvalue with unit magnitude which means that the plant will be marginally stable at this communication frequency. This is further confirmed by the closed-loop temperature profile in Figure 2(d), where the linearized plant is stable for $h = 0.2$ hr, marginally stable for $h = 0.216$ hr, and unstable for $h = 0.22$ hr (for brevity, only the profile for the temperature of CSTR 2 are shown; T_1 exhibits similar tendencies). Figure 2(c) also shows that, as expected, the maximum allowable update period decreases as the model uncertainty increases. For comparison, included in this plot also is the case when a zero-order hold scheme is used. In this case, the local controller of each reactor holds the last measurement received from its neighbor until the next time a measurement is transmitted and received from the communication network. This corresponds to using models of the form $\dot{\hat{x}}_j = \tilde{A}_j\hat{x}_j + \tilde{A}_{ji}x_i$ with $\tilde{A}_j = O$ and $\tilde{A}_{ji} = O$. The dashed line in Figure 2(c) shows that the condition for stability in this case is to have $h \leq 0.123$ hr. It is clear that a model-based scheme with relatively accurate models yields larger update times than the zero-order hold scheme. In the case of large model uncertainty, however, the zero-order hold scheme outperforms its model-based counterpart.

B. Impact of controller choice on closed-loop stability

In this part, we investigate the effect of varying the controller gains on the maximum tolerable process-model mismatch for a fixed update period. To this end, we fix h at 0.1 hr, and vary the local feedback gain K_1. Different values of K_1 can be used to place the two eigenvalues of the matrix $\bar{A}_1 = A_1 + B_1K_1$ at different locations. For simplicity, we fix one of the poles at -1 and vary the other one, which we denote by λ_1. Figure 3(a) is a contour plot showing the dependence of $\lambda_{\max}$ on δ_1 and λ_1 (i.e., K_1). The stability region for the system is the region enclosed by the unit contour line. Note that as λ_1 becomes more negative, the size of tolerable model uncertainty increases. Figure 3(b) shows the dependence of $\lambda_{\max}$ on λ_1 for different values of δ_1 and for the zero-order hold scheme. The predictions of Figures 3(a)-(b) are confirmed by the state and manipulated input profiles in Figures 3(c)-(d) which show that the linearized plant is stable when we select a point inside the unit contour zone ($\delta_1 = 0.2, \lambda_1 = -10$), and unstable when the point is barely outside the unit contour zone ($\delta_1 = 0.2, \lambda_1 = -5.3$). Similar analysis can be performed by varying the feedforward components of the controllers (see [27] for the results of this analysis).

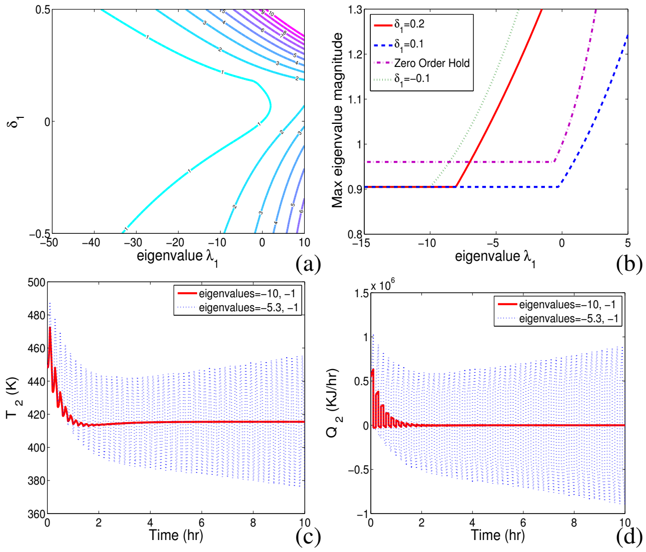

Fig. 3. Plots (a)-(b): Dependence of $\lambda_{\max}$ on local feedback controller gain, K_1, with different model uncertainties. Plots (c)-(d): Closed-loop state and manipulated input profiles with $h = 0.1$ hr and $\delta_1 = 0.2$ for different feedback gains K_1.

VI. Concluding remarks

In this work, we presented a methodology for the design of quasi-decentralized control systems for plants with distributed interconnected units. The approach is based on a hierarchical architecture in which each unit in the plant has a local control system with its sensors and actuators connected to the local controller through a dedicated communication network, and the local control systems in turn communicate with one another through a shared communication network. To achieve closed-loop stability with minimal cross-communication between the units, each control system relies on a set of models of its neighboring units to recreate the states of those units when accurate measurements of their values are not available. The models are updated at discrete time instances to compensate for modeling errors. An explicit characterization of the maximum allowable update period in terms of model uncertainty and controller design was obtained. The analysis was facilitated by the linear structure of the plants considered which allowed obtaining both necessary and sufficient conditions for the stability by applying results from the networked control systems literature. The developed quasi-decentralized control strategy was illustrated using a simulation example involving chemical reactors with recycle. Finally, we note that when the quasi-decentralized control structure is implemented on the original nonlinear plant of Eq.8, the results (not shown here) indicate that, for a given update period predicted by the linear analysis, stability can be achieved for sufficiently small initial conditions.

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